An Alternative Estimator for the Population Mean In PPS Sampling

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Abstract: ***In this paper some composite estimators depending on estimated optimum value of a constant based on sample values under PPS sampling scheme have been proposed. The estimators are better in sense (minimum variance) than PPS estimator, ratio estimator or difference estimator under certain conditions.***

Keywords: PPS estimator, Ratio estimator & Difference estimator

1. Introduction

It is well know that at large scale surveys use of multiple auxiliary characteristics improves the precision of the estimators. This can be utilized in the construction of the estimator or at the stage of selecting the sample from the population. In this paper two auxiliary characters have been used in different ways as one for the selection of the sample and other for the purpose of estimation to estimate the population mean.

2. Notations

Let y_i, x_{1i} and x_{2i} be two values of characters y under study and values of the two auxiliary characters x_1 and x_2 for the i^{th} unit in the population (i=1,2 …. N) of the population of size N. Let a sample of size n be drawn with PPSWR sampling (based on x_1) and Y, X_1 , X_2 be population total of y, x_1, x_2 respectably.

$$u_{i=\frac{y_i}{NP_{1i}}},\ v_i = \frac{x_{2i}}{N\ p_{1i}},\ p_{1i} = \frac{x_{1i}}{X_1}$$

$$\bar{u}_n = \frac{1}{n}\sum_{l=1}^{n} u_i = \bar{y}_{1pps}$$

$$\bar{v}_n = \frac{1}{n}\sum_{l=1}^{n} v_i = \bar{x}_{2pps}$$

$$\sigma_u^2 = \sum_{i=1}^{N} p_{1i}\left(u_i - \bar{y}\right)^2 = \frac{\sigma_u^2}{\bar{y}^2} = C_u^2$$

$$\sigma_v^2 = \sum_{i=1}^{N} p_{1i}\left(v_i - \bar{X}_2\right)^2 = \frac{\sigma_v^2}{\bar{X}_2^{\,2}} = C_v^2$$

$$\bar{y}_D = \bar{y} + k\left(\bar{X} - \bar{x}\right)^2$$

$$\bar{y}_R = \frac{\bar{u}_n}{\bar{v}_n}\bar{X}_2$$

$$\rho_{uv} = \frac{\sum_{i=1}^{N} p_{1i}\left(u_i - \bar{y}\right)\left(v_i - \bar{X}_2\right)}{\sigma_u\ \sigma_v}$$

3. Proposed Estimators

For estimating the population mean $\bar{Y}$, when the information on x_1, x_2 for every unit of the population is available, the proposed estimators of $\bar{Y}$ are

$\bar{Y}_{D,PPS} = a\bar{y}_D + (1-a)\bar{y}_{PPS}$ .... (1)

$\bar{Y}_{PPS,D} = a\bar{y}_{PPS} + (1-a)\bar{y}_D$ .... (2)

$\bar{Y}_{D,R} = a\bar{y}_D + (1-a)\bar{y}_R$ .... (3)

Where a is constant to be determined so that $v(\bar{y}_{D,PPS})$ and $v(\bar{y}_{D,R})$ is minimum

$$v(\bar{y}_{D,PPS}) = a^2 v(\bar{y}_D) + (1-a)^2 v(\bar{y}_{PPS}) + 2a(1-a)\operatorname{cov}(\bar{y}_D, \bar{y}_{PPS})$$

$$v(\bar{y}_{PPS,D}) = a^2 v(\bar{y}_{PPS}) + (1-a)^2 v(\bar{y}_D) + 2a(1-a)\operatorname{cov}(\bar{y}_{PPS}, \bar{y}_D)$$

$$v(\bar{y}_{D,R}) = a^2 v(\bar{y}_D) + (1-a)^2 M(\bar{y}_R) + 2a(1-a)\operatorname{cov}(\bar{y}_D, \bar{y}_R)$$

Where $v(\bar{y}_D) = \frac{\bar{y}^2}{n}\left(C_u^2 + k^2 C_v^2 - 2k\rho_{uv} C_u C_v\right)$

$$M(\bar{y}_R) = \frac{\bar{y}^2}{n}\left(C_u^2 + C_v^2 - 2\rho_{uv} C_u C_v\right)$$

$$\operatorname{cov}(\bar{y}, \bar{y}_{PPS}) = \frac{\bar{y}^2}{n}\rho_{uy} C_u C_y$$

$$\operatorname{cov}(\bar{x}, \bar{y}_{PPS}) = \frac{\bar{y}^2}{n}\rho_{ux} C_u C_x$$

$$\operatorname{cov}(\bar{y}, \bar{y}_R) = \frac{\bar{y}^2}{n}\left(C_u^2 - C_{uv}\right)$$

$$\operatorname{cov}(\bar{x}, \bar{y}_R) = \frac{\bar{y}^2}{n}\left(C_{uv} - C_v^2\right)$$

So that

$$v(\bar{y}_D, \bar{y}_{PPS}) = \frac{\bar{y}^2}{n}\Big[a^2\left(c_u^2 + k^2 c_v^2 - 2k\rho_{uv} c_u c_v\right) + (1-a)^2 c_u^2 + 2a(1-a)\rho_{uy} c_u c_y - 2a(1-a)k\rho_{xu} c_x c_u\Big]$$

$$v(\bar{y}_{PPS}, \bar{y}_D) = \frac{\bar{y}^2}{n}\Big[a^2 c_u^2 + (1-a)^2\left(c_u^2 + k^2 c_v^2 - 2k\rho_{uv} c_u c_v\right) + 2a(1-a)\rho_{uy} c_u c_y - 2a(1-a)k\rho_{xu} c_x c_u\Big]$$

$$v(\bar{y}_D, \bar{y}_R) = \frac{\bar{y}^2}{n}\Big[a^2\left(C_u^2 + k^2 C_v^2 - 2k\rho_{uv} C_u C_v\right) + (1-a)^2\left(C_u^2 + C_v^2 - 2\rho_{uv} C_u C_v\right) + 2a(1-a)\left(C_u^2 - C_{uv}\right) - 2a(1-a)k\left(C_{uv} - C_v^2\right)\Big]$$

4. Optimum Values of a

The optimum values of a for which $v\left(\bar{y}_{D,PPS}\right)$ $v\left(\bar{y}_{PPS,D}\right)$ and $v\left(\bar{y}_{D,R}\right)$ are minimized; are respectively

$$a_{opt} = \frac{C_{uv}}{kc_v^2} \quad a_{opt} = 1 - \frac{C_{uv}}{KC_v{}^2}$$

$$a_{opt} = \frac{C_v^2 - C_{uv}}{(1-k)C_v^2}$$

For these optimum values of a

$$\mathrm{V}(\bar{y}_D, pps) = \frac{\sigma_u{}^2}{n}(1-\rho^2)$$

$$\mathrm{V}(\bar{y}_D, R) = \frac{\sigma_u{}^2}{n}(1-\rho^2)$$

5. Empirical Study

To see the performance of the proposed estimator in comparison to other estimators the description of population data are given below:-

Population (Source: Singh and Mangat(1996) .pp-220

N=240 N_1 =70 N_2=120 N_3=50
n =24 n_1 = 7 n_2=12 n_3=5
$\bar{X}_1 = 15.28$ $\bar{X}_2 = 17.25$ $\bar{X}_3$=17.8
$\bar{y}$ =17.42 C_u=0.299 C_v=0.318
$C_u{}^2 = 0.089401$ $C_v{}^2 = 0.101124$
$\rho_{uv} = 0.76$ $\rho_{uv}\ C_u C_v$=0.072262

	k=1	k=1	k=1
$\mathrm{V}(\bar{y}_D, pps)$	a_{opt} =.71	a=.5	a=.6
	.03778	.04242	.039091
	k=.2	k=.2	k=.2
$\mathrm{V}(\bar{y}_D, R)$	a_{opt}=.35	a =.5	a=.3
	.03776	.039090	.037971

$\mathrm{V}(\bar{y}_{pps})$ =.089401
$\mathrm{V}(\bar{y}_D)$ =.06454
$\mathrm{V}(\bar{y}_R)$=.04600
As $\mathrm{V}(\bar{y}_D, pps) < \mathrm{V}(\bar{y}_{pps})$, $\mathrm{V}(\bar{y}_D)$ & $\mathrm{V}(\bar{y}_R)$
$\mathrm{V}(\bar{y}_D, R) < \mathrm{V}(\bar{y}_{pps})$, $\mathrm{V}(\bar{y}_D)$ & $\mathrm{V}(\bar{y}_R)$

Thus the variances of our proposed estimators are less than the usual estimators.

6. Conclusion

When a_{opt} put in proposed estimator the resulting proposed estimator attains minimum variance, Thus for practical applications proposed estimator may be preferred.

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