Performance Analysis of Selection Combining For Differential Amplify and Forward Relaying Over Rayleigh Fading Channels

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Abstract: ***This paper investigates and puts forward the selection combining (SC) at the destination for differential amplify and forward (D-AF) relaying over slow Rayleigh fading channels. The selection combiner opts the link with the maximum magnitude of the decision variable, here no channel state information (CSI) is required at the destination for detection of the transmitted symbols. But for the Semi-Maximum Ration Combining (Semi-MRC), channel state information is required at the destination. Different from Semi-MRC, the selection combiner does not need the second-order statistic of any of the channels, which simplifies the destination's detection task. The exact average bit-error-rate (BER) of the proposed SC is verified with the simulation results. It is also shown that the performance of the SC method is very close to that of the MRC method, even though with lower complexity.***

Keywords: Channel state information, Differential amplify and forwarding, Semi-MRC, Second order statistics, Selection combining.

1. Introduction

The fact by which mobile user capacity is limited is this that within the duration of a given call, they experience severe variations in the signal attenuation, thereby demanding the use of some type of diversity [1]. Cooperation diversity is a new form of diversity where the diversity gains are achieved through the cooperation of in-cell users. The consequential gain from user cooperation has high data rate and less sensitivity to channel variations. The augmented data rate with cooperation can also be translated into reduced power for the users. But this makes an increased complexity in the receiver section.

The idea of using other wireless users as the relays in a communication networks was anticipated more than a decade ago. Cooperative communication utilizes the facts that, other user can also listen to a source, they are able to receive process and rebroadcast the received data to the destination. Depending on these strategy the relay networks using for cooperation are categorized as decode and forward (DF) and amplify and forward (AF) [2]. AF is more attractive among these two because of its computations at the relay. The relay's task is simply to multiply the received signal with an amplification factor. Depending upon the availability of the channel state information (CSI) the amplification factor can be fixed or variable. When no CSI is available at the relays, the second order statistics of source-relay channels are used to settle on the fixed amplification factor. When there is no CSI at the destination, a set of fixed weights, based on the second order statistics of all channels, have been used to merge the received signals from both links i.e., from the source-destination link and the relay-destination link. This combiner is called semi-maximal ratio combining (semi-MRC). But the precise performance analysis of the semi-MRC is very complex; the performance of a system using instantaneous combining weights is usually conducted, which is known as instantaneous MRC. This is a benchmarking for the performance of a semi-MRC system. It was shown that the performance of D-AF using semi-MRC is close to the performance of an instantaneous MRC.

Obtaining the second order statistics of all channels at the destination for combining the received signals is an issue, this call for a simpler combiner without relinquishing much of the performance.

In this work, I consider D-AF relaying over Rayleigh feeding channels using post-detection selection combining (SC) which can be viewed as the counterpart to MDPSK for point-to-point communications. At the destination, the decision variable is calculated for each link and the one with the maximum magnitude is chosen. The difference from the semi-MRC, the selection combiner does not need the second order statistics of any of the channels, which simplifies the destination's task. The probability density function (PDF) and cumulative density function (CDF) of the instantaneous signal to the noise ratio (SNR) in each link and the combiner's output are used to achieve the accurate average bit-error-rate (BER) and the outage probability of the system. The analysis is verified with the simulation. Comparison of semi-MRC and the selection combining systems shows that the performance of SC is very close to the semi-MRC, although with lower complexity.

2. Background

The various backgrounds behind this scheme are given below:-

2.1 Cooperation Diversity Protocols

The low complexity cooperative diversity protocols that combat fading induced by multipath propagation in wireless networks. The underlying techniques make use of space diversity available through cooperating terminal's relaying signals for one another. Several strategies employed by cooperating radios, including fixed relaying schemes such as amplify-and-forward and decode-and-forward, selection relaying schemes that adapt based upon channel measurements between the cooperating terminals, and incremental relaying schemes that adapt based upon limited feedback from the destination terminal. The performance characterization in terms of outage events and associated outage probabilities which measure the vigor of the transmissions, focusing on the high signal-to-noise ratio (SNR) regime. Except for fixed decode-and-forward, all of our cooperative diversity protocols are competent in the sense that they achieve full diversity and more over, are close to optimum in certain regimes. Applicable to any wireless setting, including cellular or ad hoc networks – wherever space constraints rule out the use of physical arrays – the performance characterization exposes that large power or energy savings result from the use of these protocols.

2.2 Multi node Differential AF scheme

The cooperation strategy is based on amplify-and-forward protocol in which each relay amplifies the received signal from the source and then forwards it to the destination. Specifically, signal transmissions of the considered cooperation system comprise two phases. We assume, in both phases, that all signals are transmitted through orthogonal channels. If we use DMPSK modulation, the information is passed on in the phase difference between two consecutive symbols. The modulated information at the source in Phase I can be described as $v_m = e^{j\varphi m}$ where $v_m = 2\pi m/M$ for m = 0, 1, …., M-1, and M is the constellation size. The source differentially encoded the information symbol v_m as

$$x^{\tau} = v_m x^{\tau-1} \quad (1)$$

where τ is the time index, and x^{τ} is the differential encoded symbol to be transmitted at the time τ. Then the source transmits x^{τ} with transmitted power P_s to the destination and the relays. In Phase II, each relay amplifies the received signal in and forwards it to the destination with transmit power. At the destination, the received signal from the source and the relays are amalgamated and then used to approximate the transmitted information.

2.3 Unified Approach

The integrated approach permits previously acquired results to be simplified both analytically and computationally and new results to be obtained for special cases that here to fore resisted solution in a simple form. Unified approach is used to assess the error rate performance of digital communication systems operating over generalized fading channels. Enables the unification is the recognition of the sought-after form for alternate representations of the Gaussian and Marcum Q-functions that are characteristic of error-probability expression for coherent differentially coherent and noncoherent forms of detection. It is shown that in the largest majority of cases, these error-rate expressions can be put in the form of a single integral with finite limits and an integrand composed of basic functions, thus readily facilitating numerical.

3. System Model

The wireless relay model under consideration has one source, one relay and one destination. The source communicates with the destination both directly and via the relay. Each node has a single antenna, and the communication between nodes is half duplex (i.e., each node is able to only send or receive in any given time). The channel coefficients at time k, from the source to the destination (SD), from the source to the relay (SR) and from the relay to the destination (RD) are shown with $h_{sd}[k]$, $h_{sr}[k]$ and $h_{rd}[k]$ respectively.

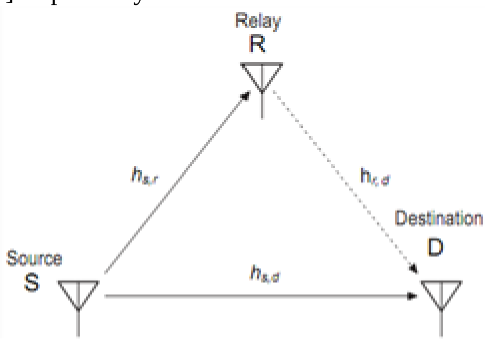

Figure 1: Wireless Relay model

All channels are CN(0,1) (i.e.' Rayleigh flat-fading) and follow jakes' correlation model. Also, the channels are spatially uncorrelated and are approximately constant for two consecutive channel uses.

3.1 Transmission Process

Let V= $\{e^{j2\pi m/M}, m = 0,1,..M-1\}$ be the set of M-PSK symbols. A group of m information bits at time k are converted to an M-PSK symbol v[k] ∈ V. before transmission, the symbols are encoded differentially as

$$s[k] = v[k]s[k-1],\ s[0] = 1 \quad (2)$$

The transmission process has two phases. Block – by- Block transmission protocol is utilized to transmit a frame of symbols in each phase as symbol-by symbol transmission causes recurrent switching between reception and transmission, which is not practical.
In phase I, symbol s[k] is transmitted by source to the relay and the destination. Let P_0 be the average source power per symbol. The received signal at the destination and the relay are

$$y_{sd}[k] = \sqrt{P_0}h_{sd}[k]s[k] + w_{sd}[k] \quad (3)$$

$$y_{sr}[k] = \sqrt{P_0}h_{sr}[k]s[k] + w_{sr}[k] \quad (4)$$

Where $w_{sd}[k], w_{sr}[k] \sim CN(0, 1)$ are noise components at the destination and the relay, respectively.

The received signal at the relay is then multiplied by an amplification factor, and retransmitted to the destination. The common amplification factor, based on the variance of SR channel, is generally used in the literature as $A = \sqrt{P_1/(P_0+1)}$, where P_1 is the average power per symbol at the relay. However, A can be any randomly fixed value. The corresponding received signal at the destination is

$$y_{rd}[k] = Ah_{rd}[k]y_{sr}[k] + w_{rd}[k] \quad (5)$$

Where $w_{rd}[k] \sim CN(0, 1)$ is the noise at the destination. Substituting (4) into (5) yields

$$y_{rd}[k] = A\sqrt{P_0}h[k]s[k] + w[k] \quad (6)$$

where $h[k] = h_{sr}[k]h_{rd}[k]$ is the equivalent double-Rayleigh channel with zero mean and variance one and $w[k] = Ah_{rd}[k]w_{sr}[k] + w_{rd}[k]$ is the equivalent noise. It should be noted that for a given $h_{rd}[k]$, $w[k]$ is complex Gaussian random variable with zero mean and variance $A^2|h_{rd}[k]|^2 + 1$.

3.2 Selection combining

By substituting (2) into (3) and (6), and using the slow fading assumption, $h_{sd}[k] \approx h_{sd}[k-1]$ and $h[k] \approx h[k-1]$, one has

$$y_{rd}[k] = v[k]y_{sd}[k-1] + n_{sd}[k] \quad (7)$$

$$n_{sd}[k] = w_{sd}[k] - v[k]w_{sd}[k-1] \quad (8)$$

$$y_{rd}[k] = v[k]y_{rd}[k-1] + n_{rd}[k] \quad (9)$$

$$n_{rd}[k] = w[k] - v[k]w[k-1] \quad (10)$$

Note that, the equivalent noise components $n_{sd}[k]$ and $n_{rd}[k]$ (for a given $h_{rd}[k]$) are combinations of complex Gaussian random variables, and hence they are also complex Gaussian with variances equal 2 and $2(1+A^2|h_{rd}[k]|^2)$, respectively.

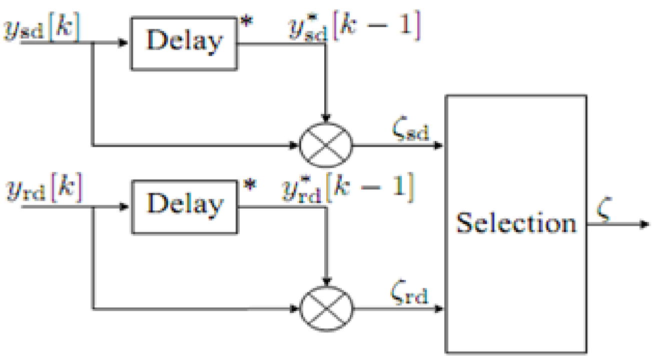

Figure 2: Block Diagram of SC at the destination

To obtain the cooperative diversity, the received signals from the two phases should be united using some combining technique. For the semi-MRC, the variance of n_{sd} and the expected value of the variance of n_{rd} were employed to fuse the signals

$$\zeta = \frac{1}{2}y^*_{sd}[k-1]y_{sd}[k] + \frac{1}{2(1+A^2)}y^*_{rd}[k-1]y_{rd}[k] \quad (11)$$

Nevertheless, instead of the semi-MRC which needs the second order statistics of all channels, I propose to combine the received signals using a selection combiner as illustrated in Figure 2. As it is seen, the decision statistics for the direct link, $\zeta_{sd} = y^*_{sd}[k-1]y_{sd}[k]$, and the cascaded link, $\zeta_{sd} = y^*_{rd}[k-1]y_{rd}[k]$, are computed and compared to pick the link with a higher magnitude. The output of the combiner is therefore

$$\zeta = \begin{cases} \zeta_{sd} & if \quad |\zeta_{sd}| > |\zeta_{rd}| \\ \zeta_{rd} & if \quad |\zeta_{rd}| > |\zeta_{sd}| \end{cases} \quad (12)$$

.

3.3 Detection

Evidently, no channel state information is needed at the destination. Finally, the well known minimum Euclidean distance (ED) detection is applied to detect the transmitted signal as

$$\hat{v}[k] = \arg\min_{x \in V} |\zeta - x|^2 \quad (13)$$

where the minimization is taken over all symbols x of the constellation V

4. Error Performance Analysis

In order to evaluate the performance of the system, the distribution of the instantaneous SNR at the output of the selection combiner is derived and used in the integrated approach to obtain the BER. To simplify the notation, the time index of the channels is omitted in this section.

The instantaneous received SNRs of two links are given as

$$\gamma_{sd} = P_0|h_{sd}|^2 \quad (14)$$

$$\gamma_{rd} = c|h_{sr}|^2 \quad (15)$$

Where $c = A^2P_0 \ |h_{rd}|^2/(1 + A^2|h_{rd}|^2)$. Since, $|h_{sd}|^2$ has an exponential distribution, γ_{sd} is also exponentially distributed with the following pdf and cdf:

$$f_{\gamma_{sd}}(\gamma) = (1/P_0)e^{-\gamma/P_0}, \ F_{\gamma_{sd}}(\gamma) = 1 - e^{-\gamma/P_0} \quad (16)$$

Since, the quantity c conditioned on h_{rd} is a constant.

The instantaneous SNR at the output of the combiner is defined as $\gamma_{max} = \max(\gamma_{sd}, \gamma_{rd})$. Thus, its cdf, conditioned on h_{rd}.Using unified approached BER

$$P_b(E|\gamma_{max}, h_{rd}) = (1/4\pi)\int_{-\pi}^{\pi} g(\theta)e^{-\alpha(\theta)\gamma_{max}}\,d\theta \quad (17)$$

The unconditioned BER is given as

$$P_b(\mathrm{E}) \propto [2/((1+\propto(\theta)P_0)(2+\propto(\theta)P_0))] \propto 1/P_0^2 \quad (18)$$

which shows that the diversity order two can be obtained in high SNR region. Before closing this section, it is pointed out that the outage probability. Particularly, the probability that the instantaneous SNR at the output of the SC combiner drops below an SNR

$$P_{out} = \Pr(\gamma_{max} \le \gamma_{th}) = \int_0^{\infty} F_{\gamma_{max}|h_{rd}}(\gamma_{th})e^{-\lambda}d\lambda \quad (19)$$

5. Simulation Results

To verify the BER performance analysis, computer simulation was carried out. In the simulation, the channels $h_{sd}[k]$, $h_{sr}[k]$, and $h_{rd}[k]$, are produced independently according to the simulation. The normalized Doppler frequency of all channels is set to 0.001, so that the channels are slow fading. Binary data is differentially encoded for M=2,4 constellations. At the destination, the received signals are mixed using the Sc technique and the decision variable is used to recover the transmitted signal using the minimum Euclidean distance detection. The simulation is run for various values of the total power in the network, whereas the amplification factor at the relay is fixed to $A=\sqrt{P_1/(P_0+1)}$, to normalized the average relay power to P_1 .

First, to fined the optimum power allocation between the source and the relay, the expression of BER is examined for different values of power allocation factor $c=P_0/P$, where $P = P_0 + P_1$ is the total power in the system. The BER curves are plotted versus c in figure 3 for P=15,20,25 dB and when DBPSK and DQPSK are employed.

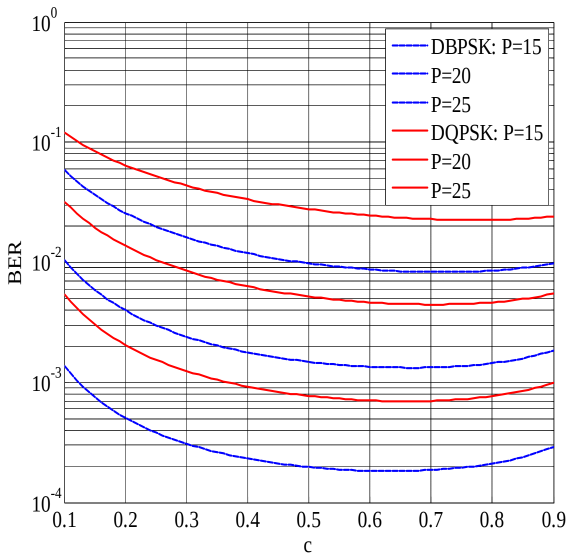

Figure 3: BER as a function of power allocation factor c for P=15,20,25 dB.

The figure shows that more power should be allocated to the source than the relay and the BER is minimized at c ≈ 0.7. This observation is similar to the semi-MRC technique. Based on Figure 3 the power allocation factor c = 0.7 is used in all the simulations.

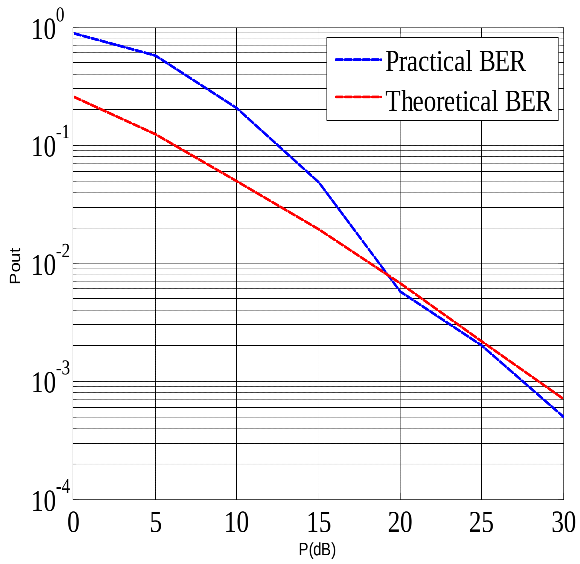

Figure 4: Outage probability versus total power for m=2

The simulated graph shows the outage probability at m=2 , 4 for both the theoretical and practical BER

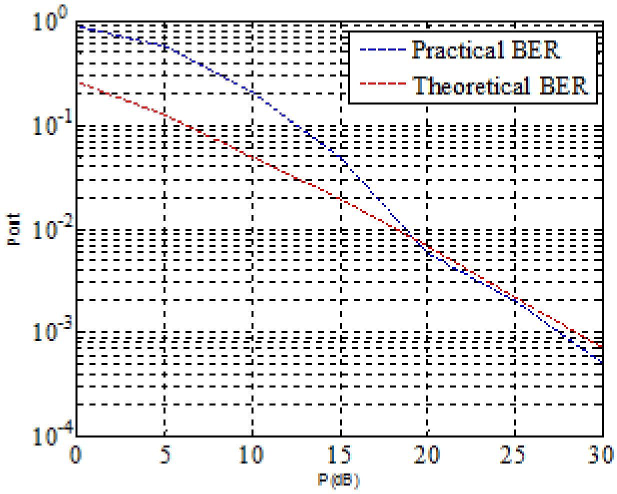

Figure 5: Outage probability versus total power for M=4

Figure 7 plots the BER curves versus the total power P that are obtained with the SC technique (both theoretical and simulation results) and the semi-MRC techniques, and for both DBPSK(lower plots) and DQPSK(upper plots). As can be seen, the simulation results of SC techniques are very close to the theoretical values. Moreover, the diversity order of two is achieved for both SC and semi-MRC methods and their results are also very close to each other. The small difference between the two methods can be accepted in may practical applications which seek a trade off between simplicity and performance.

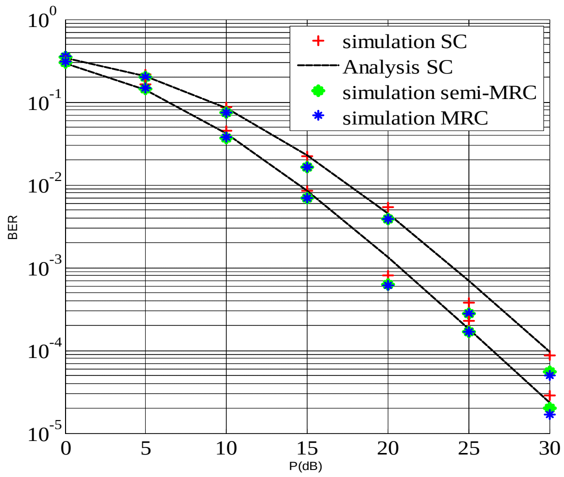

Figure 6: Theoretical and simulation BER of the D-AF system with semi-MRC, MRC and SC methods using DBPSK (lower) and DQPSK (upper)

6. Conclusion

A selection combining of the received signals at the destination of a D-AF relay network was studied. Thanks to the differential encoding and selection combiner, no channel state information is needed at the destination for detection of the transmitted symbols. The distribution of the instantaneous SNR at the output of the combiner was derived and the exact bit error rate and the outage probability of the system have been obtained. It was shown that the desired diversity order of two can be achieved by the SC system. Simulation results verified the analysis and show that the selection combiner performs very close to the morecomplicated semi-MRC technique (which needs the second-order statistics of all channels).

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