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Inclined Slider Bearing Under the Effects of Second Order Rotatory Theory of Hydrodynamic Lubrication

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Abstract: ***The second order rotatory theory of hydrodynamic lubrication was founded on the expression obtained by retaining the terms containing first and second powers of rotation number in the extended generalized Reynolds equation. In the present paper, there are some new excellent fundamental solutions with the help of geometrical figures, expressions, calculated tables and graphs for the plane and exponentially inclined slider bearings in the second order rotatory theory of hydrodynamic lubrication. The analysis of equations for pressure and load capacity, tables and graphs reveal that pressure and load capacity are not independent of viscosity and increase slightly with viscosity for the constant value of M in the case of fluid with small rotation. Also the pressure and load capacity both increase with increasing values of rotation number M. In the absence of rotation, the extended generalized Reynolds equation gives the classical solutions of the classical theory of hydrodynamic lubrication given by Reynolds, previously. The relevant tables and graphs confirm these important investigations in the present paper.***

Keywords: Continuity, Density, Film thickness, Reynolds equation, Rotation number, Taylor's number, Viscosity

1. Introduction

In the theory of hydrodynamic lubrication, two dimensional classical theories [4, 10] were first given by Osborne Reynolds. In 1886, in the wake of a classical experiment by Beauchamp Tower [12], he formulated an important differential equation, which was known as: Reynolds Equation. The formation and basic mechanism of fluid film was analyzed by that experiment on taking some important assumptions given as:

(1.1) The fluid film thickness is very small as compare to the axial and longitudinal dimensions of fluid film.
(1.2) If the lubricant layer is to transmit pressure between the shaft and the bearing, the layer must have varying thickness.

Later he [11] derived an improved version of Reynolds Equation known as: "Generalized Reynolds Equation", which depends on density, viscosity, film thickness, surface and transverse velocities. The rotation of fluid film about an axis that lies across the film gives some new results in lubrication problems that were derived by Banerjee *et. al* [1] in fluid mechanics. The origin of rotation can be traced by certain general theorems related to vorticity in the rotating fluid dynamics. The rotation induces a component of vorticity in the direction of rotation of fluid film and the effects arising from it are predominant, for large Taylor's Number, it results in the streamlines becoming confined to plane transverse to the direction of rotation of the film.

The new extended version of "Generalized Reynolds Equation" is said to be "Extended Generalized Reynolds Equation", which takes into account of the effects of the uniform rotation about an axis that lies across the fluid film and depends on the rotation number *M* [1], i.e. the square root of the conventional Taylor's Number. The generalization of the classical theory of hydrodynamic lubrication was given by Banerjee *et. al* [1], is known as the "Rotatory Theory of Hydrodynamic Lubrication".

The "First Order Rotatory Theory of Hydrodynamic Lubrication" and the "Second Order Rotatory Theory of Hydrodynamic Lubrication" was given by Banerjee *et.al.* [1] on retaining the terms containing up to first and second powers of *M* respectively by neglecting higher powers of *M*.

The most common form of lubricated slider bearing is Plane Inclined Pad. The geometry of plane-inclined pad is given by figure (1).

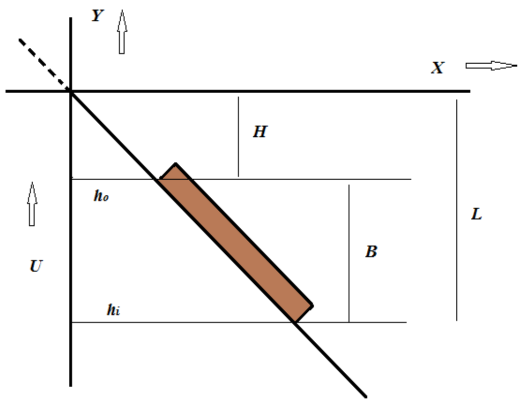

Figure 1: (Geometry of plane inclined slider)

Which shows that the gap *h* decreases with increasing *y*, hence the runner has to move towards origin in *y*-direction. Its velocity is *U*. the minimum film thickness is h_o and the minimum film thickness is h_i .The position of h_o is at a distance *H* from the origin and h_i is at a distance *L* from the origin. Taking *n* as:

$$n = \frac{h_i - h_o}{h_o} \quad (1)$$

The film thickness *h* can be expressed at any point as:

$$h = h_o\left(1 + \frac{ny}{L}\right) \quad (2)$$

$$h = y \cot\alpha, \quad (3)$$

Where α is the angle of inclination of the pad.

$$\cot\alpha = \frac{h_o}{H} = \frac{h_i}{L} = \frac{dh}{dy} \quad (4)$$

$$\frac{dh}{dy} = \frac{nh_o}{L} = \frac{h_i - h_o}{L} \quad (5)$$

$$\frac{h_o}{H} = \frac{h_i}{L} = \frac{h_i - h_o}{L - H} = \frac{L\frac{dh}{dy}}{L - H} \quad (6)$$

2. Governing Equations

In the second order rotatory theory of hydrodynamic lubrication the "Extended Generalized Reynolds Equation", given by Banerjee et.al. in 1981 [1], is written by equation (7). Let us consider the mathematical terms as follows:

$$A = \sinh\left(h\sqrt{\frac{M\rho}{2\mu}}\right) - \sin\left(h\sqrt{\frac{M\rho}{2\mu}}\right)$$

$$B = \cosh\left(h\sqrt{\frac{M\rho}{2\mu}}\right) + \cos\left(h\sqrt{\frac{M\rho}{2\mu}}\right)$$

$$C = \sinh\left(h\sqrt{\frac{M\rho}{2\mu}}\right) + \sin\left(h\sqrt{\frac{M\rho}{2\mu}}\right)$$

$$D = \cosh\left(h\sqrt{\frac{M\rho}{2\mu}}\right) - \cos\left(h\sqrt{\frac{M\rho}{2\mu}}\right)$$

$$K = \sqrt{\frac{2\mu}{M\rho}}$$

$$F_1 = -\left(\frac{KA}{MB}\right)\frac{\partial P}{\partial x}$$

$$F_2 = -\left(\frac{KA}{MB}\right)\frac{\partial P}{\partial y}$$

$$F_3 = -\left(\frac{h}{M}\right) + \left(\frac{KC}{MB}\right)\frac{\partial P}{\partial y}$$

$$F_4 = -\left(\frac{h}{M}\right) + \left(\frac{KC}{MB}\right)\frac{\partial P}{\partial x}$$

$$F_5 = \frac{\partial(A/D)}{\partial y} - \frac{\partial(C/B)}{\partial x}$$

$$\frac{\partial(F_1 + F_3)}{\partial x} + \frac{\partial(F_2 - F_4)}{\partial y} = \frac{\rho U K F_5}{2} - \rho W^* \quad (7)$$

Where x, y and z are coordinates, U is the sliding velocity, P is the pressure, ρ is the fluid density, μ is the viscosity and W^* is fluid velocity in z-direction.

3. Boundary Conditions and Derivative Analysis

The Extended Generalized Reynolds Equation in view of second order rotatory theory of hydrodynamic lubrication, in ascending powers of rotation number M and by retaining the terms containing up to second powers of M and neglecting higher powers of M, can be written as equation (8). For the case of pure sliding $W^* = 0$, so we have the equation as given:

$$\frac{\partial(F_1 + F_3)}{\partial x} + \frac{\partial(F_2 - F_4)}{\partial y} = \frac{\rho U K F_5}{2} \quad (8)$$

Let we assume the bearing to be infinitely long in y-direction, which implies that the variation of pressure in x-direction is very small as compared to the variation of pressure in y-direction i.e.,

$$\frac{\partial P}{\partial x} \ll \frac{\partial P}{\partial y}$$

then the equation (8) will be

$$\frac{\partial F_3}{\partial x} + \frac{\partial F_2}{\partial y} = \frac{\rho U K F_5}{2} \quad (9)$$

Taking the pressure distribution as the function of the coordinate along the length of the slider only, we have $P = P(y)$, we have

$$\frac{d}{dy}\left[-\frac{h^3}{12\mu}\left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right] = -\frac{d}{dy}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \quad (10)$$

For the determination of pressure the boundary conditions are:

$P=0$ at $h=h_o$ and $P=0$ at $h=h_o(1+n)$

So we have the differential equation for the pressure will be

$$\frac{dP}{dy} = -\frac{1}{2}M\rho U\left[1 - \left({h_0}^3 - \frac{17M^2\rho^2{h_0}^7}{1680\mu^2}\right)\frac{1}{h^3} - \frac{17M^2\rho^2{h_0}^3}{1680\mu^2}h\right] \quad (11)$$

The solution of the differential equation (11) under the boundary conditions gives the pressure for plane inclined slider bearings by (12).

$$P = \frac{M\rho U}{2}\left[\begin{array}{l}\left\{\left(\frac{n(n+1)^2 - 1}{n(n+1)^2}\right)L - y + \frac{L^3}{n(ny+L)^2}\right\} \\ -\frac{17M^2\rho^2{h_0}^4}{1680\mu^2}\left\{\begin{array}{c}\frac{L^3}{n(ny+L)^2} \\ -\left(y + \frac{ny^2}{2L}\right) - \\ \left(\left(\frac{2 - n(n+1)^2(n+2)}{2n(n+1)^2}\right)L\right)\end{array}\right\}\end{array}\right] \quad (12)$$

The load capacity for plane inclined slider bearing is given by

$$W = -\int_L^0 P\,dy \quad (13)$$

$$W = \frac{M\rho U L^2}{12(n+1)^2}\left[\begin{array}{c}3(n^2 + 2n + 3) \\ -\frac{17M^2\rho^2{h_0}^4}{1680\mu^2}(2n^3 + 7n^2 + 8n + 9)\end{array}\right] \quad (14)$$

3.1 Numerical Simulation-1

Table 1 (Calculated values of pressure and load capacity by taking the values of different mathematical terms in C.G.S. system as:

$\mu = 0.0002, U = 500, \rho = 0.9, L = 15, n = 1,$
$y = 7.5, h = 0.015, h_i = 0.02, h_o = 0.01)$

S.No.	M	P	W
1.	0.1	234.3618400	3634.675200
2.	0.2	234.3675975	3634.762613
3	0.3	234.3702624	3634.803072
4.	0.4	234.3717100	3634.825050
5.	0.5	234.3725829	3634.838302
6.	0.6	234.3731494	3634.846903
7.	0.7	234.3735378	3634.852800
8.	0.8	234.3738156	3634.857018
9.	0.9	234.3740212	3634.860139
10.	1.0	234.3741775	3634.862573

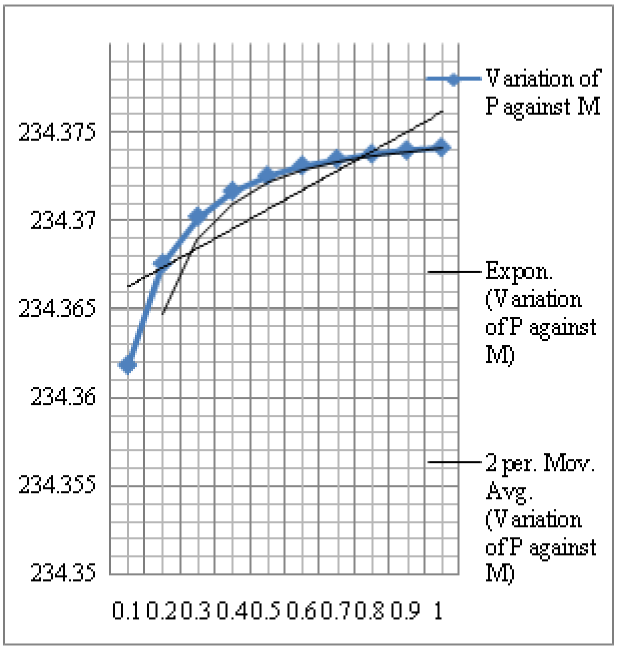

Figure 2: (Variation of pressure against *M* with exponential trend line and moving average)

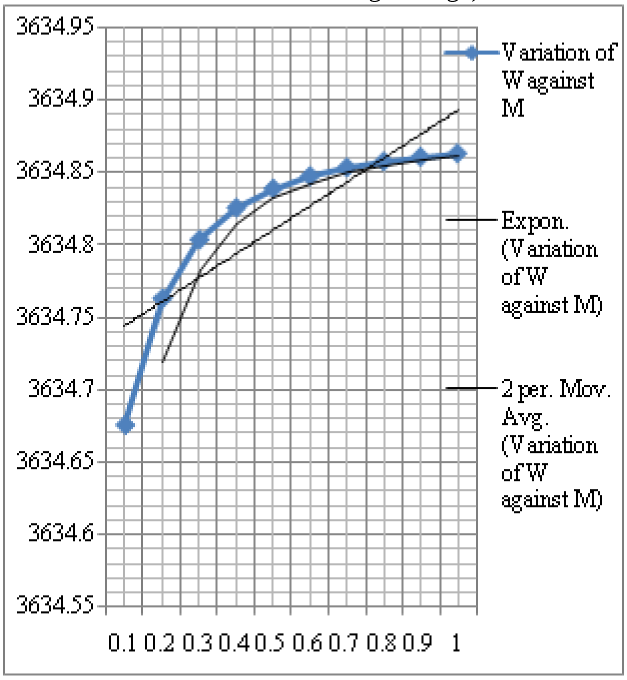

Figure 3: (Variation of load capacity against *M* with exponential trend line and moving average)

In the exponentially inclined slider [14], the film thickness h increases exponentially with the distance. The geometry of exponentially inclined slider is given by figure (4).

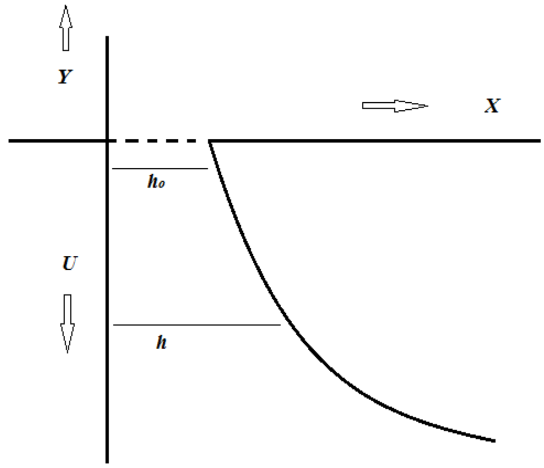

Figure 4: (Geometry of exponentially inclined slider)

The film thickness at any point is given by

$$h = h_o e^{-\alpha y} \quad (15)$$

For the determination of pressure the boundary conditions are:

$$P = 0, at\ h = h_o\ or\ y = 0$$
$$and\ P = 0, at\ h = h_o e^{-\alpha y}\ or\ y = -L \quad (16)$$

So we have the differential equation for the pressure will be

$$\frac{dP}{dy} = -\frac{1}{2} M\rho U \left[1 - \left({h_0}^3 - \frac{17M^2\rho^2{h_0}^7}{1680\mu^2}\right)\frac{1}{{h_0}^3 e^{-3\alpha y}} - \frac{17M^2\rho^2{h_0}^3}{1680\mu^2} h_0 e^{-\alpha y}\right] \quad (17)$$

The solution of the differential equation (7) under the boundary condition (6) gives the pressure for exponentially inclined slider bearings by (8).

$$P = \frac{M\rho U}{6\alpha}\left[\frac{(e^{3\alpha y} - e^{-3\alpha L}) + \frac{17M^2\rho^2 h_0^4}{1680\mu^2}}{(e^{-3\alpha L} + 3\,e^{\alpha L} - 3\,e^{-\alpha y} - e^{3\alpha y})}\right] \quad (18)$$

The load capacity for exponentially inclined slider bearing is given by

$$W = -\int_L^0 P\,dy \quad (19)$$

$$W = \frac{M\rho U}{6\alpha}\left[\left(\frac{1-e^{-3\alpha L}}{3\alpha} - Le^{-3\alpha L}\right) + \frac{17M^2\rho^2 h_0^4}{1680\mu^2}\begin{Bmatrix} Le^{-3\alpha L} + 3Le^{\alpha L} \\ +\frac{3}{\alpha}(1-e^{\alpha L}) \\ -\frac{1}{3\alpha}(1-e^{-3\alpha L}) \end{Bmatrix}\right] \quad (20)$$

3.2 Numerical Simulation-2

By taking the values of different mathematical terms in C.G.S. system there are calculated tables and graphical representations as follows:

Table 2: (The calculated values of pressure and load capacity against the rotation number by assuming the values of mathematical terms as: $U = 500, \rho = 0.9,$ $L = 15, y = 7.5, h = 0.015, h_o = 0.01, \mu = 0.0002$

in the equation (18) and (20))

S.No.	M	P	W
1.	0.1	1536.413947	14781.27479
2.	0.2	3072.827575	29550.61501
3	0.3	4609.240567	44296.08607
4.	0.4	6145.652604	59005.75339
5.	0.5	7682.063366	73667.68240
6.	0.6	9218.472537	88269.93852
7.	0.7	10754.87980	102800.5872
8.	0.8	12291.28483	17247.6938
9.	0.9	13827.68731	131599.3237
10.	1.0	15364.08693	145843.5425

Table 3: (The calculated values of pressure and load capacity against the viscosity of the fluid by assuming the values as:
$U = 500, \rho = 0.9, \ L = 15, y = 7.5, h = 0.015, h_o = 0.01, M = 0.1$)

S.No.	μ	P	W
1.	0.00015	1536.413906	14779.72772
2.	0.00020	1536.413947	14781.27479
3	0.00025	1536.413966	14781.99087
4.	0.00030	1536.413976	14782.37985
5.	0.00035	1536.413983	14782.61439
6.	0.00040	1536.413987	14782.76662
7.	0.00045	1536.413990	14782.87098
8.	0.00050	1536.413992	14782.94563
9.	0.00055	1536.413993	14783.00087
10.	0.00060	1536.413994	14783.04288

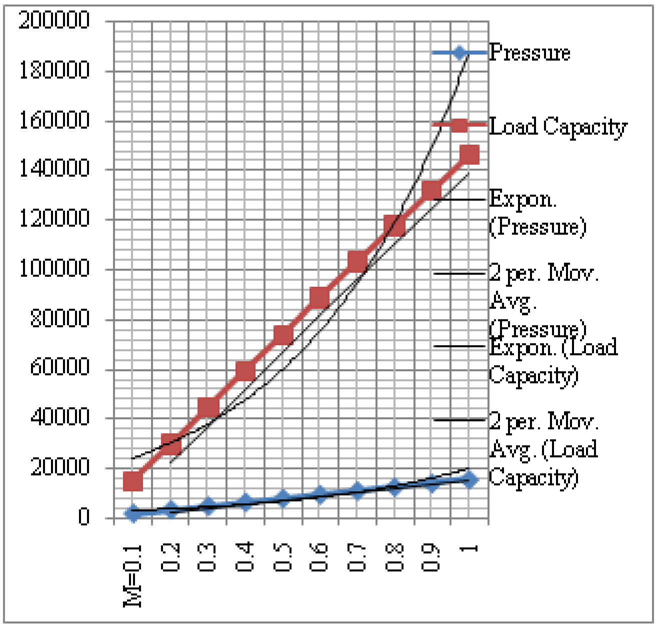

Figure 5: (Variation of *P* and *W* against *M* for *μ*=0.0002)

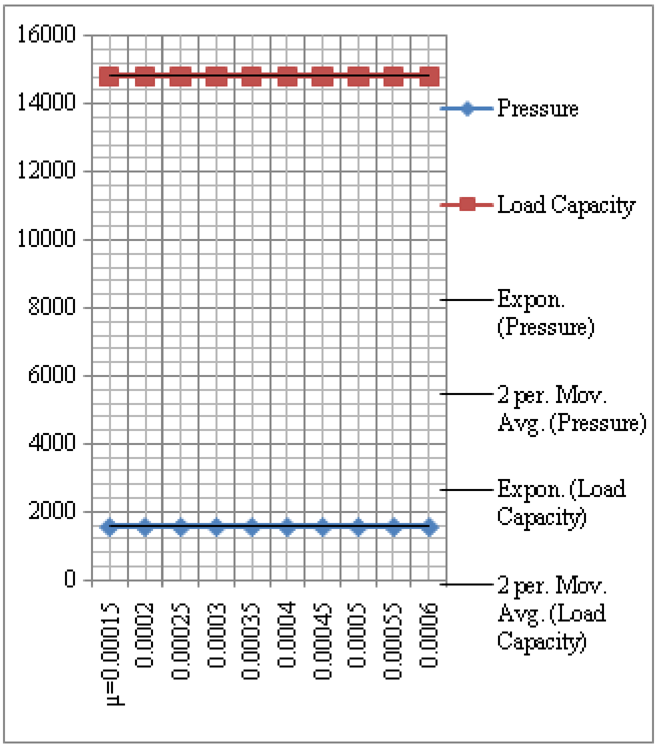

Figure 6: (Variation of *P* and *W* against *μ* for *M*=0.1**)**

4. Discussions and Results

The variation of pressure and load capacity for plane inclined slider bearings with respect to rotation number *M*, when viscosity is constant; are shown by table-1 and graphs-2, 3. That shows the variation of pressure and load capacity. Hence in the second order rotatory theory of hydrodynamic lubrication, the pressure and load capacity for plane inclined slider bearings both increase with increasing values of *M*, when viscosity is taken as constant. The equations of pressure and load capacity also show that they are not independent of viscosity *μ* and slightly increase with *μ*, when *M* is constant. On taking (*M*=0) in the extended generalized Reynolds equation, we will find the classical solutions given by Reynolds. Also the variation of pressure and load capacity for exponentially inclined slider bearings with respect to rotation number *M*, when viscosity is constant; are shown by table-2 and figure-4, which shows the variation of pressure and load capacity. Hence in the second order rotatory theory of hydrodynamic lubrication, the pressure and load capacity for exponentially inclined slider bearings both increase with increasing values of *M*, when viscosity is taken as constant. The equations of pressure and load capacity, table-3 and figure-5 also show that they are not independent of viscosity *μ* and slightly increase with *μ*, when *M* is constant. On taking (*M*=0) in the extended generalized Reynolds equation, we will again find the classical solutions given by Reynolds.

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