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On Moderate Analytic Hierarchy Process Pairwise Comparison Model (Model II)

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Abstract: ***Decisions usually involve getting the best solution, selecting the suitable experiments, most appropriate judgments, taking the quality result etc., using some techniques. Every decision making can be considered as the choice from the set of alternatives based on a set of criteria. The Analytic Hierarchy Process is a Multi-Criteria Decision Making and is dealing with decision making problem through pair wise comparison and priority vectors, which was introduced by Saaty(1977). This paper concerns with an alternate method of finding the priority vectors for the original AHP decision matrix with three level of hierarchy of main criteria, sub criteria and alternatives as Moderate AHP, that has the same rank as obtained in original AHP and ideal AHP decision problems.***

Key words: Multi-criteria Decision making, Analytic Hierarchy Process, pairwise comparison, AHP calculation software by CGI, Ideal AHP.

1. Introduction

Decision makers take decisions from the priorities on set of alternatives based on a set of criteria, called Multiple Criteria Decision Making (MCDM).It plays a important role in many real life problems. Each criterion induces a particular ordering of the alternatives and we need a procedure by which to construct one overall preference ordering. The number of criteria in MCDM is always assumed to be finite and we assume that the number of alternatives is also finite. A decision should also consider issues such as: cost, performance characteristics, availability of software, maintenance, expendability, etc. These may be some of the decision criteria for particular problems. In such problems we are interested in determining the best alternative. In some other situations, however, one may be interested in determining the relative importance of all the alternatives under consideration.

The AHP enforces the researchers, scientists, educationalists and industrialists for supporting tools, which can be used to solve complex decision problems. The AHP generates a weight for each evaluation criterion according to the decision maker's pairwise comparisons of the criteria. The higher the weight, the more important the corresponding criterion. Next, for a fixed criterion, the AHP assigns a score to each alternative according to the decision maker's pairwise comparisons of the alternative based on that criterion. The higher the score, the better the performance of the alternatives with respect to the considered criterion. Finally, the AHP combines the criteria weights and the alternatives scores, thus determining a global score for each alternative. The global score for a given alternative is a weighted sum of the scores it obtained with respect to all the criteria. Thus we have ranking for a set of objectives.

Data are collected from decision-makers corresponding to the hierarchical structure in the pairwise comparison of criteria and alternatives on a scale of relative importance (weight) as described below Table [1].

Table 1 : Scale of Relative Importance (According to Saaty 1980)

Intensity of Importance	*Definition*
1	Equal importance
3	Weak importance of one over another
5	Essential or strong importance
7	Demonstrated importance
9	Absolute importance
2, 4, 6, 8	Intermediate values between the two adjacent judgments
Reciprocals of above	If activity i has one of the above nonzero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i.

In order to compute the weights for the different criteria, we start creating a parwise comparison matrix A. The matrix A is a n x n real matrix, where n is the number of criteria for considered problem. Each entry a_{ij} of the matrix A represents the importance of the ith criterion, relative to the jth criterion. If $a_{ij} > 1$, then the ith criterion is more important than the jth criterion, while if $a_{ij} < 1$, then the ith criterion is less important than the jth criterion. If two criteria have the same importance, then the entry a_{ij} is 1. The entries a_{ij} and a_{ji} satisfy the following constraint:

$$a_{ij} \cdot a_{ji} = 1.$$

Obviously $a_{ii} = 1$ for all i. The relative importance between two criteria is measured according to a numerical scale from 1 to 9, as shown in Table 1 above, where it is assumed that the ith criterion is equally or more important than the jth criterion. The phrases in the "Interpretation" column of Table 1 are only suggestive, and may be used to translate the decision makers, qualitative evaluations of the relative importance between two criteria into numbers. It is also possible to assign intermediate values which do not correspond to a precise interpretation. The values in the matrix A are by construction pairwise consistent.

The values of the pairwise comparisons in the AHP are determined according to the scale introduced by Saaty (1980). According to this scale, the available values for the pairwise comparisons are members of the set: {9,8,7,6,5,4,3,2,1 , 1/2 , 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, 1/9 }.

After constructing pairwise comparisons matrix, the corresponding maximum left eigenvector is approximated by using the geometric mean of each row. The elements in each row are multiplied with each other and then the n-th root is taken (where n is the number of elements in the row). Next the numbers are normalized by dividing them with their sum. Hence obtaining the corresponding final priority vector.

The procedure for obtaining the following values of the criterion is as follows:

In the AHP the pairwise comparison matrix is considered to be adequately consistent if the corresponding consistency ratio (CR) is less than 10% (Saaty, 1980). The CR coefficient is calculated as follows. First the consistency index (CI) needs to be estimated. This is done by adding the columns in the pairwise comparison matrix of the criterion and multiply the resulting vector by the vector priorities obtained. This yields an approximation of the maximum eigenvalue, denoted byλ. Then, the CI value is calculated by using the formula: $CI = (\lambda - n)/(n-1)$. Next the consistency ratio CR is obtained by dividing the CI value by the Random Consistency index (RCI) as given in table [2]. ie. CR = $\frac{CI}{RCI}$, If CR > 0.10, we must re-evaluate the pairwise comparison for the criterion.

Table 2: RCI Values For Different Values of N

N	1	2	3	4	5	6	7	8	9
RCI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

Similar procedure mentioned above should be followed for all the n-alternatives

After the alternatives are compared with each other in term of each one of the decision criteria and the individual priority vectors are derived, the synthesis step is taken. The priority vectors become the columns of the decision matrix. The weights of importance of the criteria are also determined by using pairwise comparisons. Therefore, if a problem has m alternatives and n criteria, then the decision maker is required to construct n judgment matrices (one for each criterion) of order mxm and one pairwise comparison matrix of order nxn (for the n criteria). Finally, given a decision matrix the final priorities, denoted, by A^{i}_{AHP}, of the alternatives in terms of all the criteria combined are determined according to the following formula below.

$$A^{i}_{AHP} = \sum_{j-1}^{n} a_{ij} W_j, \text{ for i } = 1,2,,3,,\ldots.M. \quad \ldots$$

Recall that in the case of the ideal mode AHP the columns of the decision matrix are normalized by dividing by the largest entry in each column.

Some of the industrial engineering applications of the AHP include its use in integrated manufacturing (Putrus, 1990), in the evaluation of technology investment decision (Boucher and McStravic, 1991), in flexible manufacturing system (Wabalickis,1988), layout design (Cambron and Evans, 1991), in other engineering problems (Wang and Raz, 1991) and the selection of preference in the field of academic activities.

2. Geometrical Interpretation

The structure of the typical problem can consist of Criteria, sub criteria with respect to criteria and the alternatives with respect to the sub criteria. Each alternative can be evaluated interms of the sub critiera with main criteria and the relative importance of each criterion can be estimated as well. Suitable performance values for criteria, subcriteria and alternatives are given. The problem has three level hierarchy of alternatives and critiera.

3. Model of the Problem

Three departments D_1,D_2 and D_3 in the ABC institution Each department has two masters with subjects M,P and C respectively (Figure 1). Each one handling the three students S_1, S2 and S_3 with their respective subjects. We find the effective and best candidate with master in the particular department in order to get the best performance of the institution.

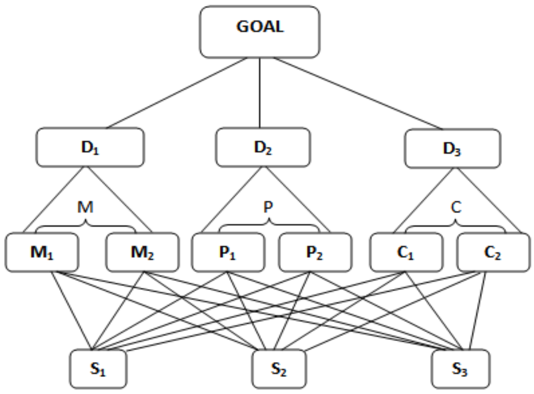

Figure 1: Main Criteria, Sub Criteria and alternatives

Construct the pair wise comparison matrix for the main criteria D_1 D_2 and D_3 and obtain their weight vectors, λ, CI and CR [saaty (1980)].

Construct the pairwise comparison matrices for the sub-criteria M_1 & M_2, P_1 & P_2 and C_1 & C_2 with respective to D_1 , D_2 and D_3 respectively and obtain the weight vectors for all sub criteria, λ CI and CR [saaty (1980)].

Construct the pairwise comparison matrices for the alternatives S_1, S_2 and S_3 with respect to M , P and C respectively and obtain the priority vectors for all alternatives λ, CI and CR [saaty (1980)].

To get original AHP decision matrix, multiply the weight vectors of main criteria with corresponding weight vectors of the sub criteria to get resulting criteria weights. Multiply these with corresponding priority Vectors of Alternatives. The sum of these values is the final priority Vector for respective alternative. In such a way, we find the final priority vectors for the remaining alternatives.

Also we can get the ideal AHP decision Matrix, by dividing the entires in the column of the original AHP matrix for the corresponding criterion with the largest entry in that particular column. Multiply these values of the alternatives with corresponding the resulting criterion weights. Sum these Values to get the final priority vector for the respective alternative. In such a way we find the final priority vectors for the remaining alternatives. After normalizing the final priority Vectors, to have the values with ranking.

It can be extended to find the finial alternative priority vectors for all alternatives from the original AHP decision matrix. It can be obtained from the following formula

$$MS_i = \sum_{j=1}^{m} W_j (W_j + S'_{ij})$$

Where W_j is the weight vector for corresponding resulting criteria weight and S'_{ij} is the weight vector of the ith alternative and jth resulting criterion of the original AHP decision matrix. We get moderate AHP decision matrix.

After normalization, we have ranked the alternatives. Finally we have the same ranking for original AHP decision matrix, Ideal AHP decision matrix and moderate AHP decision matrix, even though different values of the final priority vectors of respective alternatives for these 3 methods.

4. Numerical Example.

Pairwise comparison matrix for Main criteria

Alternative/criterion	D_1	D_2	D_3	Priority Vector
D_1	1	8	7	0.7766
D_2	1/8	1	1/3	0.0704
D_3	1/7	3	1	0.1530

λ =3.1044 , CI = 0.0522, CR = 0.09.

Pairwise comparison matrix for Sub criteria with respect to D_1

D_1	M_1	M_2	Priority Vector
M_1	1	2	0.6667
M_2	1/2	1	0.3333

λ = 2 , CI = 0, CR = 0

Pairwise comparison matrix for Sub criteria with respect to D_2

D_2	P_1	P_2	Priority Vector
P_1	1	3	0.7500
P_2	1/3	1	0.2500

λ = 2 , CI = 0, CR = 0.

Pairwise comparison matrix for Subcriteria with respect to D_3

D_3	C_1	C_2	Priority Vector
C_1	1	7	0.8750
C_2	1/7	1	0.1250

λ = 2 , CI = 0, CR =0

Pairwise comparison matrix for S_1,S_2 & S_3 with respect to M_1

M_1	S_1	S_2	S_3	Priority Vector
S_1	1	8	5	0.7418
S_2	1/8	1	1/3	0.0752
S_3	1/5	3	1	0.1830

λ = 3.0441 , CI = 0.0220, CR = 0.0379

Pairwise comparison matrix for S_1,S_2 & S_3 with respect to M_2

M_2	S_1	S_2	S_3	Priority Vector
S_1	1	7	3	0.6491
S_2	1/7	1	1/5	0.0719
S_3	1/3	5	1	0.2790

λ = 3.0649 , CI = 0.0324, CR = 0.0559

Pairwise comparison matrix for S_1,S_2 & S_3 with respect to P_1

P_1	S_1	S_2	S_3	Priority Vector
S_1	1	5	½	0.3522
S_2	1/5	1	1/5	0.0888
S_3	2	5	1	0.5591

λ = 3.0536 , CI = 0.0268, CR = 0.0462

Pairwise comparison matrix for S_1,S_2 & S_3 with respect to P_2

P_2	S_1	S_2	S_3	Priority Vector
S_1	1	7	5	0.7306
S_2	1/7	1	1/3	0.0810
S_3	1/5	3	1	0.1884

λ = 3.0649, CI = 0.0324, CR = 0.0559

Pairwise comparison matrix for S_1,S_2 & S_3 with respect to C_1

C_1	S_1	S_2	S_3	Priority Vector
S_1	1	1/7	1/3	0.0809
S_2	7	1	5	0.7307
S_3	3	1/5	1	0.1884

λ = 3.0649 , CI = 0.0324, CR = 0.0559

Pairwise comparison matrix for S_1,S_2 & S_3 with respect to C_2

C_2	S_1	S_2	S_3	Priority Vector
S_1	1	6	5	0.7258
S_2	1/6	1	½	0.1020
S_3	1/5	2	1	0.1721

λ = 3.0291, CI = 0.0145, CR = 0.025

We get the original AHP decision matrix

Alternative /criterion	D_1		D_2		D_3		Final Priority vector
Criterion	0.7766		0.0704		0.1530		
Sub Criterion	0.6667	0.3333	0.7500	0.2500	0.8750	0.1250	

S1	0.7418	0.6491	0.3522	0.7306	0.0809	0.7258	0.6083
S2	0.0752	0.0719	0.0888	0.0810	0.7306	0.1020	0.1633
S3	0.1830	0.2790	0.5591	0.1884	0.1884	0.1721	0.2283

Modified original AHP decision matrix

Alternative/criterion	M_1	M_2	P_1	P_2	C_1	C_2	Final Priority Vector	Ranking
Criterion Weight	0.5178	0.2588	0.0528	0.0176	0.1339	0.0191		

S_1	0.7418	0.6491	0.3522	0.7306	0.0809	0.7258	0.6079	1
S_2	0.0752	0.0719	0.0888	0.0810	0.7306	0.1020	0.1633	3
S_3	0.1830	0.2790	0.5591	0.1884	0.1884	0.1721	0.2283	2

The Ideal AHP decision Matrix

Alternative/Criterion	M_1	M_2	P_1	P_2	C_1	C_2	Final Priority vector	After Normalization	Ranking
Criterion Weight	0.5178	0.2588	0.0528	0.0176	0.1339	0.0191			
S_1	1.0000	1.0000	0.6299	1.0000	0.1107	1.0000	0.8614	0.6045	1
S_2	0.1014	0.1108	0.1588	0.1109	1.0000	0.1405	0.2282	0.1602	3
S_3	0.2467	0.4298	1.0000	0.2579	0.2579	0.2371	0.3352	0.2353	2

The moderate AHP decision matrix

Alternative/Criterion	M_1	M_2	P_1	P_2	C_1	C_2	Final Priority Vector	After Normalization	Ranking
Criterion Weight	0.5178	0.2588	0.0528	0.0176	0.1339	0.0191			
S_1	0.6522	0.2350	0.0214	0.0132	0.0288	0.0142	0.9648	0.4662	1
S_2	0.3071	0.0856	0.0075	0.0017	0.1158	0.0023	0.5200	0.2512	3
S_3	0.3629	0.1392	0.0323	0.0036	0.0432	0.0037	0.5849	0.2826	2

Therefore, the best selection is S_1 followed by S_3 and S_3 is followed by S_2. Hence, the selection of the best student is S_1 under the M_1 master with respect to the D_1 department in order to get the best performance of the institution. Finally, we observe that the original AHP, the ideal AHP and the moderate AHP decision matrices have the same ranking for the said 3 alternatives, even though they assigned different final priority vectors for these alternatives.

5. Conclusion

The AHP is used for ranking with priority vectors of pairwise comparison matrices. It provides an effective solution for solving MCDM problem. It should be noted that, there is a AHP calculation software (web system) by CGI system that calculates the weights and CI values of the AHP models from pairwise comparison matrices. we can involve any relative importance of criteria and that of alternatives in the moderate AHP. It has always the moderate values compared with other two values. (Original AHP and Ideal AHP). Also moderate AHP allows for a sensitivity analysis in term of the relative priorities, by adjusting the ranking values. We can develop this problem with any number of main criteria, sub-criteria and alternatives in further research level.

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