Smarandache Fuzzy Strong Ideal and Smarandache Fuzzy n-Fold Strong Ideal of a BH-Algebra

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Abstract: ***In this paper, we define the concepts of a Q-Smarandache n-fold strong ideal anda Q-Smarandache fuzzy (strong, n-fold strong) ideal of a BH- algebra .Also, we study some properties of these fuzzy ideals***

Keywords: BCK-algebra , BCI/BCH-algebras, BH-algebra , Smarandache BH-algebra, Q-Smarandache fuzzystrong ideal.

1. Introduction

In 1965, L. A. Zadeh introduced the notion of a fuzzy subset of a set as a method for representing uncertainty in real physical world [7] . In 1991, O. G. Xi applied the concept of fuzzy sets to the BCK-algebras [8] . In 1993, Y. B. Jun introduced the notion of closed fuzzy ideals in BCI -algebras[11].In 1999,Y.B.Jun introduced the notion of fuzzy closed ideal in BCH-algebras [13]. In 2001, Q. Zhang, E. H. Roh and Y. B. Jun studied the fuzzy theory in a BH-algebra [10] . In 2006,C.H. Park introduced the notion of an interval valued fuzzy BH-algebra in a BH-algebra [2].In 2009, A. B. Saeid and A. Namdar, introduced the notion of a Smarandache BCH-algebra and Q-Smarandache ideal of a Smarandache BCH-algebra [1].In 2012, H. H .Abbass introduced the notion of a Q-Smarandache fuzzy closed ideal with respect to an element of a Smarandache BCH-algebra [5].In the seam year, E.M. kim and S. S. Ahn defined the notion of a fuzzy (n-fold strong) ideal of a BH-algebra[3]. In 2013, E.M. kim and S. S. Ahn defined the notion of a fuzzy (strong) ideal of a BH-algebra[4].In the same year, H. H. Abbass and S. J. Mohammed introduced the Q-Smarandache fuzzy completely closed ideal with respect to an element of a BH-algebra[6] . In this paper, we define the concepts of Q-Smarandachen-fold strong ideal anda Q-Smarandache fuzzy (strong, n-fold strong) ideal of a Smarandache BH- algebra .Also, we study some properties of these fuzzy ideals

2. Preliminaries

In this section, we give some basic concept about a BCK-algebra ,a BCI-algebra ,a BH-algebra , a BH*-algebra,a normal BH-algebra, fuzzy strong ideal, fuzzyn-fold strong ideala Smarandache BH-algebra,(Q-Smarandache ideal, Q-Smarandache fuzzyclosedideal, Q-Smarandache fuzzy completely closed ideal and Q-Smarandache fuzzy ideal of BH-algebra

Definition 1 (see[11]).A **BCI-algebra** is an algebra (X,*,0), where X is a nonempty set, "*" is a binary operation and 0 is a constant, satisfying the following axioms : for all x, y, z∈ X:
i. ((x * y) * (x * z)) *(z * y)= 0, ii. (x*(x*y))*y = 0,
iii. x * x = 0,
iv. x * y =0 and y * x = 0 ⇒ x = y.

Definition 2 (see [8]). A **BCK-algebra** is a BCI-algebra satisfying the axiom
v. 0 * x = 0 for all x ∈X.

Definition 3(see[9]). A **BH-algebra** is a nonempty set X with a constant 0 and a binary operation "*" satisfying the following conditions:i.x*x=0, ∀ x∈X
ii. x*0 =x, ∀ x∈X.
iii. x*y=0 and y*x =0⇒ x = y,:for all x,y∈ X.

Definition4.(see[4]).A BH-algebraX is called a **BH*-algebra**if (x*y)*x=0 for all x,y∈X

Definition5.(see[9]). A BH-algebra X is said to be a **normalBH-algebra** if it satisfying the following condition:
i. 0*(x*y)= (0*x)*(0*y) ,∀x,y∈X .
ii. (x*y)*x = 0* y ,∀x,y∈X.
iii. (x*(x*y))*y = 0 , ∀x,y∈ X.

Definition 6.(see[5]) . Let X be a BH-algebra.Then the set X_+={x∈X:0*x=0} is called the **BCA-part** of X .

Remark 1(see[6]). Let X and Y be BH-algebras. A mapping f : X→Y is called a **homomorphism** if f(x*y)=f(x)*` f(y) for all x, y∈X. A homomorphism f is called a **monomorphism** (resp., **epimorphism**) if it is injective (resp**., surjective**). For any homomorphism f:X →Y, the set { x∈X : f(x)=0'} is called the **kernel** of f, denoted by Ker(f), and the set { f(x) : x∈X} is called the **image** of f, denoted by Im(f). Notice that f(0)=0' for all homomorphism f.

Definition7 (see[3]). Let X be a BH-algebraand n be a positive integer**.** A nonempty subset I of X is called a ***n-fold strong ideal of X*** if it satisfies: i. 0 ∈ I , ii. ∀y ∈I and (x* y)*z^n∈I ⇒x*z^n∈I, ∀ x ,z ∈X.

Definition 8.(see[6]).A Smarandache BH-**algebra** is defined to be a BH-algebra X in which there exists a proper subset Q of X such that
i. 0 ∈ Q and |Q| ≥ 2 .
ii. Q is a BCK-algebra under the operation of X.

Definition9 (see[6]). Let X be a Smarandache BH-algebra **.** A nonempty subset I of X is called a ***Smarandache strong ideal of X related toQ*** (or briefly**, *Q- Smarandache strong ideal*** of X) if it satisfies: i. 0 ∈ I , ii. ∀y ∈I and (x* y)*z ∈I ⇒x*z ∈I, ∀ x ,z ∈Q.

Definition 10(see[7]). Let μ be a fuzzy set in X, for all t∈[0,1]. The set μ_t={x∈X, μ(x) ≥ t} is called a ***level subset of*** μ.

Definition11(see [4]). Let A and B be any two sets, μ be any fuzzy set in A and f : A → B be any function. Setf^{-1}(y) = {x ∈A| f (x) = y} for y ∈B. The fuzzy set ν in B defined by

$\nu(y) = \begin{cases} \sup\{\mu(x) \mid x \in f^{-1}(y)\} & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$

for all y ∈B, is called the ***image*** of μ under f and is denoted by f (μ).

Definition 12(see[4]. Let A and B be any two sets, $f : A \to B$ be any function andν be any fuzzy set in f (A). The fuzzy set μ in A defined by: μ(x) = ν(f (x)) for all x ∈X is called the ***preimage*** of ν under f and is denoted by f^{-1}(ν).

Definition13.(see[4].A fuzzy set μ in a BH-algebra X is called a fuzzy strong ideal of X if i.For all x∈X, μ(0) ≥ μ(x).
ii.μ (x ∗z) ≥ min{μ ((x ∗y) ∗z) ,μ (y)}, ∀x; y ∈X.

Definition14.(see[5].A fuzzy set μ in a BH-algebra X is called a fuzzy n-fold strong idealof X if i. For all x∈X, (0) ≥ μ(x).
ii. μ (x ∗z^n) ≥ min{μ ((x ∗y) ∗z^n); μ (y)}, ∀x, y ∈X.

Definition15(see [6]).A fuzzy subset μ of a Smarandache BH-algebra X is said to be a ***Q-Smarandachefuzzy ideal***if and only if :
i. For all x∈X, μ(0) ≥ μ(x).
ii.For all x ∈ Q, y∈X, μ(x) ≥ min{ μ(x*y), μ(y)}.
Is said to be ***closed*** if (0*x) ≥ μ(x), for all x∈X.

Definition16(see[6]). Let X be a Smarandache BH-algebra and μ be a Q-Smarandache fuzzy ideal of X. Then μ is called ***a Q-Smarandachefuzzy completely closed ideal*** if μ(x*y) ≥ min{ μ(x), μ(y)},∀x,y∈X.

Proposition1(see[6]). Let X be a BH*-algebra ,and μ be a Q-Smarandache fuzzy ideal. Then
i. μ is a Q-Smarandache fuzzy closed ideal of X .
ii. μ is a Q-Smarandache fuzzy completely closed ideal of X, if X*X/{0} ⊆Q

3. The Main Results

In this paper, we give the concepts a Q-Smarandache n-fold strong ideal and a Q-Smarandache fuzzy (strong, n-fold strong) ideal of a BH- algebra .Also, we give some properties of these fuzzy ideals

Definition 1 .A fuzzy subset of a BH-algebra X is called a Q-***Smarandache*** **fuzzy *strong* ideal** ,iff i. μ (0) ≥μ (x)) ∀x∈X
ii. μ (x*z) ≥ min{ μ ((x*y)*z), μ (y)},∀x,z∈Q.

Example 1: The set X={0,1,2,3} with the following operation table

*	0	1	2	3
0	0	0	2	2
1	1	0	1	2
2	2	2	0	0
3	3	2	1	0

is a BH-algebra Q={0,1} is a BCK-algebra .Then (X,*,0) is a Smarandache BH-algebra.The fuzzy set μ which is defined by:

$\mu(x) = \begin{cases} 0.5 & x = 0,3 \\ 0.4 & x = 1,2 \end{cases}$

is a Q-Smarandache fuzzy strong ideal , since: i.μ(0) = 0.5 ≥μ(x) ∀x∈X, ii. μ (x*z) ≥ min{ μ((x*y)*z), μ(y)},∀x, z∈Q.

But the fuzzy set$\mu(x) = \begin{cases} 0.5 & x = 0,2,3 \\ 0.4 & x = 1 \end{cases}$
is not a Q-Smarandache fuzzy strong ideal sinceμ(1*0)=μ(1)=0.4 < min{ μ(1*3)*0), μ(3)}=0.5

Proposition1.Every Q-Smarandache fuzzy strong ideal of a Smarandache BH-algebra X is a Q-Smarandache fuzzy ideal of X.

Proof : Let μ be a fuzzy strong ideal of X .
i. Let x ∈ X ⇒ (0) ≥μ(x) .
[By definition 1(i)]
ii. let x, z ∈X and y ∈ X ⇒x,z∈ Q
⇒μ(x*z) ≥ min{μ((x*y)*z) , μ(y)}
[By definition 1(ii)]
When z=0⇒μ(x*0) ≥ min{μ((x*y)*0) , μ(y)}⇒μ(x) ≥ min{μ(x*y) , μ(y)}
⇒μ is a Q-Smarandache fuzzy ideal of X. ■

Proposition2. Let Q_1 and Q_2 be a BCK-algebras contained in a Smarandache BH- algebra X and $Q_1 \subseteq Q_2$. Let μ be Q_2-Smarandache fuzzy strong ideal of X then μ is a Q_1-Smarandache fuzzy strong ideal of X .

Proof : Let μ be a Q_2-Smarandache fuzzy strong ideal of X .
i. Let x ∈ X ⇒ μ(0) ≥μ(x) .
[Since μ is a Q_2 -Smarandache fuzzy strong ideal. By definition 1(i)]
ii. Let x,z∈ Q_1,y∈X ⇒x,z∈ Q_2
⇒ μ(x*z) ≥ min{ μ((x*y)*z) ,μ(y)}
[Since μ is a Q_2 -Smarandache fuzzy strong ideal. By definition 1(ii)]
⇒μ is a Q_1-Smarandache fuzzy strong ideal of X. ■

Proposition3. Every fuzzy strong ideal of a Smarandache BH-algebra X is a Q-Smarandache fuzzy strong ideal of X.

Proof : Letμ be a fuzzy strong ideal of X .
i. Let x ∈ X ⇒ μ(0) ≥μ(x) .
[By definition 13(i)]
ii. let x, z ∈X and y ∈ X ⇒x,z∈ Q
⇒μ(x*z) ≥ min{μ((x*y)*z) , μ(y)}
[By definition13(ii)]
⇒μ is a Q-Smarandache fuzzy strong ideal of X. ■

Theorem 1. Let X be SmarandacheBH-algebra and let μ be a fuzzy set.Then μ is a Q-Smarandache fuzzy strong ideal if and only if v(x) = μ (x) / μ (0) is a Q-Smarandache fuzzy strong ideal.

Proof: Let μ be a Q-Smarandache fuzzy strong ideal,
1) v(0)= μ (0) / μ (0),⇒ v(0)=1
⇒ v(0) ≥ v(x) ∀x∈X

2) $v(x*z)= \mu(x*z)/\mu(0)$
$\geq \min\{\mu((x*y)*z), \mu(y)\}/\mu(0)\}$
[Since μ is a Q-Smarandache fuzzy strong ideal. .By definition1(ii)]
$\geq \min\{\mu((x*y)*z)/\mu(0), \mu(y)/\mu(0)\}$
$\geq \min\{v((x*y)*z), v(y)\}$
$\Rightarrow v(x*z) \geq \min\{v((x*y)*z), v(y)\}$
$\Rightarrow v$ is a Q-Smarandache fuzzy strong ideal.**Conversely.**Let v be a Q-Smarandache fuzzy strong ideal.
i.$\mu(0)=v(0).\ \mu(0), \Rightarrow \mu(0) \geq v(x).\mu(0)$
$\Rightarrow \mu(0) \geq \mu(x)\ \forall x \in X$
ii.$\mu(x*z)= v(x*z).\mu(0) \geq \min\{v(x*(y*z)), v(y)\}.\mu(0)$
[Since μ' is a Q-Smarandache fuzzy ideal.By definition1(i)]
$\geq \min\{v((x*y)*z).\mu(0), v(y).\mu(0)\}$
$\geq \min\{\mu((x*y)*z), \mu(y)\}$
$\Rightarrow \mu(x) \geq \min\{\mu((x*y)*z), \mu(y)\}$
$\Rightarrow \mu$ is a Q-Smarandache fuzzy strong ideal.■

Proposition 4. Let X be a BH*-algebra ,and μ be a Q-Smarandache fuzzy strong ideal. Then
i.μ is a Q-Smarandache fuzzy closed idealof X.
ii. μ is a Q-Smarandache fuzzy completely closed ideal of X,if $X*X/\{0\} \subseteq Q$.

Proof : Let μ be a fuzzy strong ideal of X.
[ByProposition1]
$\Rightarrow \mu$ is a Q-Smarandache fuzzy ideal of X.
i. $\Rightarrow \mu$ is a Q-Smarandache fuzzy closed ideal of X. [By proposition 1]
ii. Let $x,y \in X \Rightarrow \mu$ is a Q-Smarandache fuzzy completely closed ideal of X.
[By proposition 1].■

Theorem 2. Let A be a non-empty subset of a Q-Smarandache BH-algebra X and let μ be a fuzzy set in X defined by:

$$\mu(x)=\begin{cases}\alpha_1 & x \in Q\\ \alpha_2 & \text{otherwise}\end{cases}$$

where $\alpha_1 > \alpha_2$ in [0, 1].Then μ is a Q-Smarandache fuzzy strong ideal X

Proof : Let μ be a fuzzy set of X.
i. $0 \in Q \Rightarrow \mu(0) = \alpha_1. \Rightarrow \mu(0) \geq \mu(x)$ [Since$\alpha_1 > \alpha_2$]
ii. let $x,z \in Q$ and $y \in X \Rightarrow x*z \in Q$
$\Rightarrow \mu(x*z) = \alpha_1$
Then we have for cases.

Case1 If $((x*y)*z)= \alpha_1$and $\mu(y) =\alpha_1$
$\Rightarrow \min\{\mu((x*y)*z), \mu(y)\} =\alpha_1$
$\Rightarrow \mu(x*z) \geq \min\{\mu((x*y)*z), \mu(y)\}$

Case2 If $((x*y)*z)= \alpha_2$and $\mu(y) =\alpha_1$
$\Rightarrow \min\{\mu((x*y)*z), \mu(y)\} =\alpha_2$
$\Rightarrow \mu(x*y) = \min\{\mu(x*y), \mu(y)\}$

Case3 If $((x*y)*z)= \alpha_1$and $\mu(y) =\alpha_2$
$\Rightarrow \min\{\mu((x*y)*z), \mu(y)\} =\alpha_2$
$\Rightarrow \mu(x*y) = \min\{\mu(x*y), \mu(y)\}$

Case3 If $((x*y)*z)= \alpha_2$and $\mu(y) =\alpha_2$
$\Rightarrow \min\{\mu((x*y)*z), \mu(y)\} =\alpha_2$
$\Rightarrow \mu(x*y) = \min\{\mu(x*y), \mu(y)\}$
$\Rightarrow$μ is a Q-Smarandache fuzzy strong ideal of X. ■

Theorem 3. Let X be BH-algebra and let μ be a fuzzy set.Thenμ is a Q-Smarandache fuzzy strong ideal if and only if $\mu'(x)= \mu(x)+1-\mu(0)$ is a Q-Smarandache fuzzy strong ideal.

Proof: Let μ be a Q-Smarandache fuzzy strong ideal,
i. $\mu'(0)= \mu(0)+1-\mu(0), \Rightarrow \mu'(0)=1$
$\Rightarrow \mu'(0) \geq \mu'(x)\ \forall x \in X$
ii. $\mu'(x*z)= \mu(x*z)+1-\mu(0)$
$\geq \min\{\mu((x*y)*z)), \mu(y)\}+1-\mu(0)$[Since μ is a Q-Smarandache fuzzy strong ideal. .By definition1]
$\geq \min\{\mu((x*y)*z)) +1-\mu(0), \mu(y) +1-\mu(0)\} \geq \min\{\mu'((x*y)*z)), \mu'(y)\}$
$\Rightarrow \mu'(x*z) \geq \min\{\mu'((x*y)*z)), \mu'(y)\}$
$\Rightarrow \mu'$ is a Q-Smarandache fuzzy strong ideal.

ConverselyLet μ' be a Q-Smarandache fuzzy strong ideal.
1) $\mu(0)= \mu'(0)-1+\mu(0), \Rightarrow \mu(0) \geq \mu'(x)-1+\mu(0) \Rightarrow \mu(0) \geq \mu(x)\ \forall x \in X$
2) $\mu(x*z)= \mu'(x*z)-1+\mu(0)$
$\geq \min\{\mu'((x*y)*z)), \mu'(y)\}-1+\mu(0)$ [Since μ' is a Q-Smarandache fuzzy strong ideal. By definition1]
$\geq \min\{\mu'((x*y)*z)) -1+\mu(0), \mu'(y) -1+\mu(0)\}$
$\geq \min\{\mu((x*y)*z)), \mu(y)\}$
$\Rightarrow \mu(x) \geq \min\{\mu((x*y)*z)), \mu(y)\}$
$\Rightarrow \mu$ is a Q-Smarandache fuzzy strong ideal.■

Theorem4. Let X be a Smarandache BH-algebra such that $X = X_+$ and μ be a Q-Smarandachestrong ideal of X. Then μ is a Q-Smarandache closed ideal of X .

Proof : Let μ be a Q-Smarandache strong ideal $\Rightarrow \mu$I is a Q-Smarandache ideal of X. [Byproposition1]
Now, let $x \in I$
$\Rightarrow (0*x) = \mu(0) \geq \mu(x)$ [By definition 6]
$\Rightarrow \mu$ is a Q-Smarandache closed ideal of X . ■

Proposition5. Let X be a Smarandache normal BH-algebra such that $X=X_+$ and let μ be a Q-Smarandache fuzzy strong ideal such that $x*y \in Q, \forall x,y \in X$ and $y \neq 0$.Then μ is a Q-Smarandache fuzzy completely closed ideal of X.

Proof: Let μ I be a Q-Smarandache fuzzy strong ideal of X.
$\Rightarrow \mu$ is a Q-Smarandache fuzzy ideal of X. [By remark3]
Now, let $x,y \in X \Rightarrow x*y \in Q$
[Since $x*y \in Q, \forall x,y \in X$]
We have
$\mu(x*y) = \mu((x*y)*0) \geq \min\{\mu(((x*y)*x)*0)), \mu(x)\}$ [By definition 3(ii)]
$= \min\{\mu((x*y)*x), \mu(x)\}$[By definition 3(ii)] $= \min\{\mu(0*y), \mu(x)\}$ [By definition 5(ii)]
$= \min\{\mu(0), \mu(x)\} = \mu(x)$
$\Rightarrow \mu(x*y) \geq \min\{\mu(y), \mu(x)\}$
$\Rightarrow \mu$ is a Q-Smarandache fuzzy completely closed ideal. ■

Proposition 6. Let X be a normal BH-algebra such that $X*X/\{0\} \subseteq Q$ .Then every Q-Smarandache fuzzy strong ideal and closed of X is a Q-Smarandache fuzzy completely closed ideal of X.

Proof : Let μ be a Q-Smarandache fuzzy strong ideal of X.

$\Rightarrow\mu$ is a Q-Smarandache fuzzy ideal of X. [By proposition1]
Now, let x, y ∈X. ⇒ x*y ∈Q [Since X*X /{0} ⊆Q]
$\Rightarrow\mu(x*y) \geq \min\{\mu((x*y)*x)*0), \mu(x)\}$ [By definition1]
$= \min\{\mu(0*y), \mu(x)\}$[By definition 5(ii)]
$\geq \min\{\mu(y), \mu(x)\}$[By definition14]
$\Rightarrow\mu$ is a Q-Smarandache fuzzy completely closed ideal of X. ■

Remark 1. Let μ be a fuzzy set of a Smarandache BH-algebra X and w ∈ X .The set {x∈X: (w) ≤μ(x)} is denoted by ↑μ(w) .

Theorem 5. Let X be a Smarandache BH-algebra, w∈X and μ is a Q-Smarandache fuzzy strong ideal of X. Then↑(w) is a Q-Smarandachestrong ideal of X.

Proof : Letμ be a Q-Smarandache fuzzy strong ideal of X.
To prove that ↑(*w*) is a Q-Smarandache strong ideal of X.
1) Let x ∈↑(w) ⇒$\mu(0) \geq \mu(x)$ [By definition 1(i)]
⇒(0) ≥μ(w) ⇒ 0 ∈↑μ(w)
2) Let x,z∈ Q, y ∈↑μ(w) and (x*y)*z∈↑μ(w).
⇒(w) ≤μ(y) and μ(w) ≤μ((x*y)*z)
⇒(w) ≤ min{ μ(y) , μ((x*y)*z)}
But μ(x*z) ≥ min{μ((x *y)*z), μ(y)} [By definition 1(ii)]
⇒(w) ≤μ(x*z) ⇒ x*z∈↑μ(w)
⇒↑(*w*) is a Q-Smarandache strong ideal of X. ■

Corollary1. Let X be a Smarandache BH-algebra . Then μ is a Q-Smarandache fuzzy strong ideal of X if and only if μ_t is a Q-Smarandache strong ideal of X, for all t ∈ $[0, \sup_{x\in X}\mu(x)]$

Proof *:*Let t ∈ $[0, \sup_{x\in X}\mu(x)]$. To prove that μ_t is a Q-Smarandache strong ideal of X.Since μ is a Q-Smarandache fuzzy strong ideal of X.
Now, let y ∈ μ_t and x*(y*z) ∈ μ_t⇒μ(y) ≥ t and μ((x*y)*z)) ≥ t .
To prove that x*z ∈ μ_t
We have μ(x*z) ≥min{μ((x*y)*z)) ,μ(y)} [By definition 1]
Since ((x*y)*z)) ≥t and μ(y) ≥t ⇒min{μ((x*y)*z)) ,μ(y)}≥ t
⇒μ(x*z) ≥t ⇒ x*z ∈ μ_t
⇒μ_t is a Q-Smarandache strong ideal of X.
Conversely,
To prove that μ is a Q-Smarandache fuzzy strong ideal of X.
Since μ_t is a Q-Smarandache strong ideal of X.
Let $t = \sup_{x\in X}\mu(x)$,x,z∈Q and(x*y)*z, y ∈ μ_t
⇒ x*z ∈ μ_t[By definition9]
⇒μ(x*z) ≥t ⇒μ(x*z) = t [Since $t = \sup_{x\in X}\mu(x)$]
Similarly,((x*y)*z)=t and μ(y)=t
⇒t = min {((x*y)*z),μ(y)}
⇒μ(x*z) ≥ min {((x*y)*z),μ(y)}
⇒μ is a Q-Smarandache fuzzy strong ideal of X.■

Proposition7 .Let f : (X, *, 0) →(Y, $*'$, $0'$) be a Smarandache BH-epimorphism. If μ is a Q-Smarandache fuzzy strong ideal of X, then f (μ) is a f (Q)-Smarandache fuzzy strong ideal of Y .

Proof : Let μ be a Q-Smarandache fuzzy strong ideal of X.
i. Let y ∈f (μ) such thaty = f (x) .
(f (μ))($0'$)=sup {μ(x) | x ∈ f^{-1}($0'$)}
=(0) ≥ μ(x) [By definition 1(i)]
=(f (μ))(f (x))
= (f (μ))(y)⇒(f (μ))($0'$)≥(f (μ))(y)
ii. Let y_1, y_3 ∈f (Q), y_2 ∈ Y, there exists x_1, x_3 ∈Q and x_2 ∈X such that $y_1 = f(x_1), y_3 = f(x_3)$ and$y_2 = f(x_2)$⇒ (f (μ))($y_1 * y_3$) = sup{$\mu(x_1*x_3)$ | x ∈ $f^{-1}(y_1 * y_3)$}
(f (μ))(y_1* y_1) ≥($x_1 * x_3$)≥ min{ $\mu((x_1 * x_2)*x_3)$,$\mu(x_2)$} [By definition 1(ii)]
= min {(f (μ))(f ((x_1*x_2)*x_3)) , (f (μ))(x_2)} =min{(f (μ))((f (x_1) $*'$ f (x_2)) $*'$ f(x_3)),(f (μ))(f (x_2)}= min{(f (μ))(y_1 $*'$ y_2) $*'$ y_3),(f (μ))(y_2)}
⇒ (f (μ))(y_1) ≥ min { (f (μ))((y_1 $*'$ y_2) $*'$ y_2),(f (μ))(y_2)}
⇒ f (μ) is a f (Q)-Smarandache fuzzy strong ideal of Y. ■

Theorem 6. Let f :(X,*,0)→(Y, $*'$, $0'$)be a Smarandache BH-epimorphism.If$\nu$is a Q -Smarandache fuzzy strong ideal of Y, then $f^{-1}(\nu)$ is af^{-1}(Q)-Smarandache fuzzy strong ideal of X

Proof *:*i. Let x ∈X. Since f (x) ∈Y and ν is a Q-Smarandachestrong fuzzy ideal of Y .
$(f^{-1}(\nu))(0) = \nu(f(0)) = \nu(0') \geq \nu(f(x)) = (f^{-1}(\nu))(x)$
ii. Let x ∈ f^{-1}(Q), y ∈X .
$f^{-1}(\nu)(x*z)$=(f (x*z)) [By definition 12]
≥ min{ν(f (x) $*'$ f (y)) $*'$ f (z)), ν(f (y))} [By remark1]
=min{ν(f ((x * y)*z), ν(f (y))}
⇒$f^{-1}(\nu)$(x)≥min{$f^{-1}(\nu)$((x*y)*z),$f^{-1}(\nu)$(y)}[By definition1]
⇒$f^{-1}(\nu)$ is a Q-Smarandache fuzzy strong ideal of X . ■

Definition 2. Let X be a Smarandache BH-algebra and n be apositive integer**.** A nonempty subset I of X is called a ***Smarandachen-fold strong ideal of X related toQ*** (or briefly**,** ***Q- Smarandachen-fold strong ideal*** of X) if it satisfies: i. 0 ∈ I , ii. ∀y ∈I and $(x* y)*z^n$∈I ⇒x*z ∈I, ∀ x ,z ∈Q.

Example 2. Consider the set I={0,3} in example 1 is a Q-Smarandachen-fold strongideal of X.But the set I={0,2,3} is not a Q-Smarandachen-fold strongideal since$(1*3)*0^n$= 2∈I,but $1*0^n$=1∉ I.

Remark 2. Every n-fold strong ideal of a Smarandache BH-algebra X is a Q-Smarandachen-fold strong ideal of X.

Remark3. Every Q-Smarandachen-fold strong ideal of a Smarandache BH-algebra X is a Q-Smarandachestrong ideal of X.**Remark4.**Every Q-Smarandachen-fold strong ideal of a Smarandache BH-algebra X is a Q-Smarandache ideal of X.

Proposition8 . Let X be a Smarandache BH-algebra . Then every Q-Smarandachen-foldstrong ideal which is contained in Q is a Q-Smarandache completely closed ideal of X.

Proof : Let I be a Q-Smarandachen-foldstrong ideal of X⇒ I is a Q-Smarandache ideal of X. [By remark4]
Now, let x,y∈I ⇒x,y∈Q ⇒ x*y∈Q [Since I⊆ Q]

Where ((x*y)*x)*0 =((x*x)*y)*0 [Since (x*y)*z =(x*z)*y. By definition 1(iii)]
=(0*y)*0 [By definition 3(i)]
=0*y [By definition 3(ii)]
=0∈I [Since 0*x=0.By definition 2(v)]
$\Rightarrow$((x*y)*x)*0^n∈I and x∈I.$\Rightarrow$ (x*y)*0^n∈ I $\Rightarrow$ x*y∈ I[By definition 3(ii)]
$\Rightarrow$ I is a Q-Smarandache completely closed ideal of X. ■

Definition 3.A fuzzy subset of a Smarandache BH-algebra Xand n be a
positive integer is called a Q-***Smarandache* fuzzy *n-fold strong* ideal** ,iff i. μ (0) ≥μ (x)) ∀x∈X. ii. μ ($x*z^n$) ≥ min{ μ ((x*y)*z^n), μ (y)},∀x,z∈Q.

Example3. Consider thefuzzy set μ which is defined by:

$$\mu(x) = \begin{cases} 0.5 & x = 0,2 \\ 0.4 & x = 1,3 \end{cases}$$

is a Q-Smarandache fuzzy n-foldstrong ideal , since: i.μ(0) = 0.5 ≥μ(x) ∀x∈X, ii. μ ($x*z^1$) ≥ min{ μ((x*y)*z^1), μ(y)},∀x, z∈Q.

But the fuzzy set$\mu(x) = \begin{cases} 0.5 & x = 0,2,3 \\ 0.4 & x = 1 \end{cases}$

is not a Q-Smarandache fuzzy n-foldstrong ideal since$\mu(1*0^3)=\mu(1)=0.4 < \min\{\mu((1*3)*0^3), \mu(3)\}=0.5$

Remark5. Every fuzzy n-fold strong ideal of a Smarandache BH-algebra X is a Q-Smarandache fuzzy n-fold strong ideal of X.

Proposition 9 . Every Q-Smarandache fuzzy n-fold strong ideal of a Smarandache BH-algebra X is a Q-Smarandache fuzzy strong ideal of X.

Proof : Let μ be a Q-Smarandache fuzzy n-fold strong ideal of X .**i.** Let x ∈ X $\Rightarrow$ (0) ≥μ(x) .[By definition 3(i)]
ii. let x,z∈Q and y ∈ X $\Rightarrow\mu(x*z^n) \geq \min\{\mu((x*y)*z^n), \mu(y)\}$[By definition 3(ii)]
When n=1$\Rightarrow\mu(x*z) \geq \min\{\mu((x*y)*z), \mu(y)\}$
$\Rightarrow$μ is a Q-Smarandache fuzzy strong ideal of X. ■

Theorem 7. Let f :(X,*,0)→(Y, $*'$,$0'$)be a Smarandache BH-epimorphism. If $\nu$is a Q-Smarandache fuzzy n-fold strongideal of Y, then $f^{-1}(\nu)$ is af^{-1}(Q)-Smarandache fuzzy n-fold strongideal of X.

Proof :i.Let x ∈X. Since f (x) ∈Y and ν is a Q-Smarandache fuzzy n-fold strongideal of Y.
$(f^{-1}(\nu))(0)= \nu(f(0))= \nu(0') \geq \nu(f(x)) =(f^{-1}(\nu))(x)$
ii. Let x,z∈ f^{-1}(Q), y ∈X .
$f^{-1}(\nu)(x*z^n)=(f(x*z^n))$ [By definition11]
$\geq \min\{\nu(f(x) *' f(y)) *' f(z^n)), \nu(f(y))\}$
[By remark1]
$=\min\{\nu(f((x*y)*z^n), \nu(f(y))\}$
$\Rightarrow f^{-1}(\nu)(x*z^n) \geq \min\{f^{-1}(\nu)((x*y)*z^n), f^{-1}(\nu)(y)\}$
$\Rightarrow f^{-1}(\nu)$ is a Q-Smarandache fuzzyn-foldstrong ideal of X .
■

Theorem 8. Let X be Smarandache BH-algebra. If μ is a fuzzy set such that Q= X_μ={x∈X: μ(x)=μ(0)}and μ(0)≥ μ(x) ∀x∈X, then μ is a Q-Smarandache fuzzy n-fold strong ideal of X.

Proof :Let μ be a fuzzy set of X, such that Q =X_μ and μ(0)≥ μ(x) ∀x∈X.
i. (0)≥μ(x) ∀x∈X.
ii. Let x,z∈Q and y∈X .
(0)≥μ(y) and $\mu(0) \geq \mu((x*y)*z^n)$[Since$\mu(0)\geq\mu(x)$ ∀x∈X]
$\Rightarrow(0) \geq \min\{\mu((x*y)*z^n),\mu(y)\}$
But $\mu(x*z^n)=\mu(0)$[Since Q= X_μ]
$\Rightarrow\mu(x*z^n) \geq \min\{\mu((x*y)*z^n),\mu(y)\}\Rightarrow\mu$ is a Q-Smarandache fuzzy n- fold strong ideal of X.■

Proposition10. Let $\{\mu_\alpha:\alpha\in\lambda\}$ be a family of Q-Smarandache fuzzy n- fold strongideals of a Smarandache BH-algebra X.

Then $\bigcap_{\alpha\in\lambda} \mu_\alpha$ is a fuzzy n-fold strongideal of X.

Proof : Let $\{\mu_\alpha:\alpha\in\lambda\}$be a family of Q-Smarandache fuzzy n-foldstrong ideals of X.

i. Let x∈X. $\bigcap_{\alpha\in\lambda} \mu_\alpha(0) = \inf\{\mu_\alpha(0), \alpha\in\lambda\} \geq \inf\{\mu_\alpha(x), \alpha\in\lambda\}$

[Since μ_α is a Q-Smarandache fuzzy n-foldideal, ∀$\alpha\in\lambda$. By definition 3(i)]

$= \bigcap_{\alpha\in\lambda} \mu_\alpha(x) \Rightarrow \bigcap_{\alpha\in\lambda} \mu_\alpha(0) \geq \bigcap_{\alpha\in\lambda} \mu_\alpha(x)$

ii. Let x,z∈ Q and y∈X

$\bigcap_{\alpha\in\lambda} \mu_\alpha(x*z^n) = \inf\{\mu_\alpha(x*z^n), \alpha\in\lambda\} \geq \inf\{$ $\min\{\mu_\alpha((x*y)*z^n), \mu_\alpha(y)\}, \alpha\in\lambda \}$[Since μ_α is a Q-Smarandache fuzzy n-foldstrong ideal, ∀$\alpha\in\lambda$. By definition3(ii)]= $\min\{ \inf\{\mu_\alpha((x*y)*z^n), \alpha\in\lambda \}, \inf\{\mu_\alpha(y), \alpha\in\lambda \} \}$

$= \min\{ \bigcap_{\alpha\in\lambda} \mu_\alpha((x*y)*z^n), \bigcap_{\alpha\in\lambda} \mu_\alpha(y)$

$\Rightarrow \bigcap_{\alpha\in\lambda} \mu_\alpha(x*z^n) \geq \min\{ \bigcap_{\alpha\in\lambda} \mu_\alpha((x*y)*z^n), \bigcap_{\alpha\in\lambda} \mu_\alpha(y)\} \Rightarrow$

$\bigcap_{\alpha\in\lambda} \mu_\alpha$ is a Q-Smarandache fuzzy n-foldstrong ideal of X. ■

Proposition 11. Let $\{\mu_\alpha: \alpha\in\lambda\}$ be a chain of Q-Smarandache fuzzy n-fold strongideals of a Smarandache BH-algebra X.

Then $\bigcup_{\alpha\in\lambda} \mu_\alpha$ is a Q-Smarandache fuzzy n-foldstrong ideal of X.

Proof : Let $\{\mu_\alpha: \alpha\in\lambda\}$ be a chain of Q-Smarandache fuzzy n-fold strongideals of X.

i. Let x∈X. $\bigcup_{\alpha\in\lambda} \mu_\alpha(0)=\sup\{\mu_\alpha(0),\alpha\in\lambda\} \geq \sup\{\mu_\alpha(x),\alpha\in\lambda\}$

[Since μ_α is a Q-Smarandache fuzzy n-foldstrong ideal,∀$\alpha\in\lambda$.Bydefinition 3(i)]

$= \bigcup_{\alpha\in\lambda} \mu_\alpha(x) \Rightarrow \bigcup_{\alpha\in\lambda} \mu_\alpha(0) \geq \bigcup_{\alpha\in\lambda} \mu_\alpha(x)\ \forall\ x\in X$.

ii. Let x,z∈Q,y∈X. $\bigcup_{\alpha\in\lambda} \mu_\alpha(x*z^n) = \sup\{\mu_\alpha(x), \alpha\in\lambda\} \geq \sup\{\min\{\mu_\alpha((x*y)*z^n), \mu_\alpha(y)\}, \alpha\in\lambda\}$

[Since μ_α is a Q-Smarandache fuzzy n-foldstrong ideal, $\forall\alpha\in\lambda$. By definition 3(ii)]

But $\{_\alpha, \alpha\in\lambda\}$ is a chain $\Rightarrow$ there exist, $j\in\lambda$ such that $\sup\{\min\{\mu_\alpha((x*y)*z^n), \mu_\alpha(y)\}, \alpha\in\lambda\} = \min\{\mu_j((x*y)*z^n), \mu_j(y)\}$

$=\min\{\sup\{\mu_\alpha((x*y)*z^n), \alpha\in\lambda\}, \sup\{\mu_\alpha(y), \alpha\in\lambda\}\}$

$\Rightarrow \bigcup_{\alpha\in\lambda} \mu_\alpha(x*z^n) \geq \min\{\mu_j((x*y)*z^n), \mu_j(y)\}$

$\geq \min\{\sup\{\mu_\alpha((x*y)*z^n), \alpha\in\lambda\}, \sup\{\mu_\alpha(y), \alpha\in\lambda\}= \min\{\bigcup_{\alpha\in\lambda} \mu_\alpha((x*y)*z^n), \bigcup_{\alpha\in\lambda} \mu_\alpha(y)\}$

$\Rightarrow \bigcup_{\alpha\in\lambda} \mu_\alpha(x*z^n) \geq \min\{\bigcup_{\alpha\in\lambda} \mu_\alpha((x*y)*z^n), \bigcup_{\alpha\in\lambda} \mu_\alpha(y)\} \Rightarrow$

$\bigcup_{\alpha\in\lambda} \mu_\alpha$ is a Q-Smarandache fuzzy n-fold strong ideal of X.

■

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