

Unsteady MHD Free Convection Flow past an Accelerated Vertical Plate with Chemical Reaction and Ohmic Heating

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Abstract: *The unsteady free convection flow of an electrically conducting fluid past an accelerated infinite vertical plate with constant heat flux is investigated under the influence of uniform transverse magnetic field fixed relative to the fluid or to the plate in the presence of chemical reaction with heat generation or absorption. Two important cases, (i) Exponentially Accelerated Plate (EAP) and (ii) Uniformly Accelerated Plate (UAP), have been considered. The governing partial differential equations have been solved numerically, using finite difference technique. The numerical solutions are obtained for the velocity, temperature, and concentration distributions. The effects of system parameters such as the Prandtl number, heat generation or absorption, Grashof number, and magnetic field parameter on the flow fields are thoroughly analyzed through graphs and tables.*

Keywords: Exponentially accelerated vertical plate, uniformly accelerated vertical plate, free convection, constant heat flux, Chemical reaction.

1. Introduction

The study of Magnetohydrodynamics boundary layer flows of electrically conducting fluids finds applications in several industrial and technological fields such as meteorology, electrical power generation, solar power technology, nuclear engineering, and geophysics. Debnath(1, 2) obtained an exact solution of the unsteady hydro magnetic boundary layer equation for a viscous incompressible and electrically conducting rotating fluid in the presence of an external magnetic field. Debnath(3) derived exact solutions of the unsteady hydrodynamic and hydro magnetic boundary layer flows including the effects of the pressure gradient and uniform suction or blowing. Georgantopoulos et al.(4) studied the magneto hydrodynamic free convection flow past an impulsively started infinite vertical plate with constant temperature. Tokis and Pande(5) investigated the unsteady two-dimensional flow of a viscous incompressible and electrically conducting fluid near a moving porous plate of an infinite extent in the presence of a transverse magnetic field.

In recent years hydro magnetic flows and heat transfer have essentially become more important because of numerous applications, for example, metallurgical processes in cooling of continuous strips through a quiescent fluid, thermonuclear fusion, aerodynamics, among others. Raptis and Singh(6) together discussed the effect of a uniform transverse magnetic field on the free convection flow of an electrically conducting fluid past an accelerated vertical infinite plate when the magnetic lines of force are fixed relative to the fluid they observed the fluid velocity attains to a non-zero steady state as the boundary layer increases. Tokis(7) investigated a class of exact solutions of the unsteady free convection flow of an electrically conducting fluid near a moving vertical plate of an infinite extent in the presence of uniform transverse magnetic field fixed to the fluid or to the plate. Several studies have been continued on

Magnetohydrodynamics free convection flows past a vertical surface under different physical situations (8 – 11).

However, it seems less attention was paid on hydro magnetic free convection flows near a vertical plate subjected to a constant heat flux boundary condition even though this situation involves in many engineering applications. Chandra et al. (12) analyzed the effects of magnetic field and buoyancy force on the unsteady free convection flow of an electrically conducting fluid when the flow was generated by uniformly accelerated motion of an infinite vertical plate subjected to constant heat flux. They obtained an exact solution with the help of Laplace transform technique and the numerical results are computed with the approximated error functions appeared in the solution. Narahari and Debnath(13) studied the unsteady magneto hydrodynamic free convection flow near an accelerated infinite vertical plate with constant heat flux and heat generation or absorption has been considered when the magnetic lines of force are fixed to the fluid or to the plate. The governing coupled linear partial differential equations are solved analytically using the Laplace transform technique without any restriction.

In the present paper, an unsteady free convection flow of an electrically conducting fluid past an accelerated infinite vertical plate, with constant heat flux is precisely investigated under the influence of uniform transverse magnetic field fixed relative to the fluid or to the plate in the presence of chemical reaction with heat generation or absorption and Ohmic heating has been considered, when the magnetic lines of force are fixed to the fluid or to the plate. The governing coupled linear partial differential equations are solved numerically, using the finite difference technique. The present problem finds typical applications in aeronautics, spacecraft design and the study of the thermal plumes into atmosphere which are responsible for

atmospheric pollution. The numerical results are presented through graphs and tables.

2. Formulation of the problem

The unsteady free convection flow of an electrically conducting, viscous, incompressible fluid past an infinite non conducting vertical plate with Ohmic heating is considered under the following assumptions:

1. The x' - axis is taken along the plate in the upward direction and the y' - axis is perpendicular to the plate into the fluid by choosing an arbitrary point on this plate as the origin.
2. A uniform magnetic field of strength B_0 is applied in the horizontal direction that is in the y' - direction.
3. Initially, at time $t' \leq 0$, the plate and the fluid are at rest and at the same temperature T_∞' . At time $t' > 0$,

suddenly the plate accelerated with velocity $u_0 f(t')$ in its own plane along the x' - axis against the gravitational field and heat is supplied to the plate at a

constant rate in the presence of temperature dependent heat generation or absorption.

4. All the physical properties of the fluid are assumed to be constant, except the density variations with temperature in the body force term.
5. The magnetic Reynolds number of the flow is assumed to be small. So that, the induced magnetic field is neglected in comparison with applied magnetic field (B_0).
6. The Ohmic heating is considered.
7. As the plate is infinite extent in x' direction, all the physical quantities are the functions of the space coordinate y' and time t' only and therefore the inertia terms are negligible
8. Here the plate is subjected to exponential and uniform acceleration in the fluid, so that, there arise two cases :
 - a) Exponentially Accelerated Plate Case
 - b) Uniformly Accelerated Plate

For Exponentially Accelerated Plate (EAP) Case

Using the finite difference scheme, the governing equations for the case of an exponentially accelerated plate turn into

$$\frac{u(i, j+1) - u(i, j)}{\Delta t} = \frac{u(i+1, j) - 2u(i, j) + u(i-1, j)}{\Delta y^2} - M \left[u(i, j) - K \exp(a_0 t) \right] + Gr \theta(i, j) + Gc \phi(i, j) \quad (1)$$

$$\frac{\theta(i, j+1) - \theta(i, j)}{\Delta t} = \frac{1}{Pr} \frac{\theta(i+1, j) - 2\theta(i, j) + \theta(i-1, j)}{\Delta y^2} - \frac{Q}{Pr} \theta(i, j) - M Ec \left[u(i, j) \right]^2 \quad (2)$$

$$\frac{\phi(i, j+1) - \phi(i, j)}{\Delta t} = \frac{1}{Sc} \frac{\phi(i+1, j) - 2\phi(i, j) + \phi(i-1, j)}{\Delta y^2} \quad (3)$$

The initial and boundary conditions are represented as

$$u(i, 0) = 0, \quad \theta(i, 0) = 0, \quad \phi(i, 0) = 0 \quad \text{for all } i \quad (4)$$

$$u(0, j) = \exp(a_0 t), \quad \theta(0, j) = 1, \quad \phi(0, j) = 1 \quad \text{for all } i$$

$$u(i, j) \rightarrow 0, \quad \theta(i, j) \rightarrow 0, \quad \phi(i, j) \rightarrow 0 \quad \text{for all } j$$

For Uniformly Accelerated Plate (UAP) Case

Using the explicit finite difference scheme, the governing equations for the case of an exponentially accelerated plate become

$$\frac{u(i, j+1) - u(i, j)}{\Delta t} = \frac{u(i+1, j) - 2u(i, j) + u(i-1, j)}{\Delta y^2} - M \left[u(i, j) - K t \right] + Gr \theta(i, j) + Gc \phi(i, j) \quad (5)$$

$$\frac{\theta(i, j+1) - \theta(i, j)}{\Delta t} = \frac{1}{Pr} \frac{\theta(i+1, j) - 2\theta(i, j) + \theta(i-1, j)}{\Delta y^2} - \frac{Q}{Pr} \theta(i, j) - M Ec [u(i, j)]^2 \quad (6)$$

$$\frac{\phi(i, j+1) - \phi(i, j)}{\Delta t} = \frac{1}{Sc} \frac{\phi(i+1, j) - 2\phi(i, j) + \phi(i-1, j)}{\Delta y^2} \quad (7)$$

The initial and boundary conditions are represented as

$$u(i, 0) = 0, \quad \theta(i, 0) = 0, \quad \phi(i, 0) = 0 \quad \text{for all } i$$

$$u(0, j) = K t, \quad \theta(0, j) = 1, \quad \phi(0, j) = 1 \quad \text{for all } i$$

$$u(i, j) \rightarrow 0, \quad \theta(i, j) \rightarrow 0, \quad \phi(i, j) \rightarrow 0 \quad \text{for all } j$$

The suffix, i corresponds to y and j corresponds to t and $\Delta t = t(j+1) - t(j)$ and $\Delta y = y(i+1) - y(i)$.

the given system parameters. The results are shown in graphs and tables.

From the velocity, temperature, and concentration fields, the expressions for skin friction coefficient, the rate of heat transfer coefficient in terms of Nusselt number, and the rate of mass transfer in terms of Sherwood number are derived using

$$\tau = \frac{\tau'}{\rho u_0^2} = - \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad (9)$$

$$Nu = - \frac{1}{\theta(0, t)} \left(\frac{\partial \theta}{\partial y} \right)_{y=0} \quad (10)$$

$$Sh = - \left(\frac{\partial \phi}{\partial y} \right)_{y=0} \quad (11)$$

For Exponentially Accelerated Plate (EAP) Case

The velocity profiles, for an Exponentially Accelerated Plate (EAP), are discussed through the graphs 1 to 8, when the magnetic field is being fixed to the fluid ($K = 0$) and to the moving plate ($K = 1$), for various physical parameters such as Hartmann number (M), Eckert number (Ec), thermal Grashoff number (Gr), the Solutal Grashoff number (Gc), the acceleration parameter (a_0), and the heat generation or absorption parameter (Q) etc.

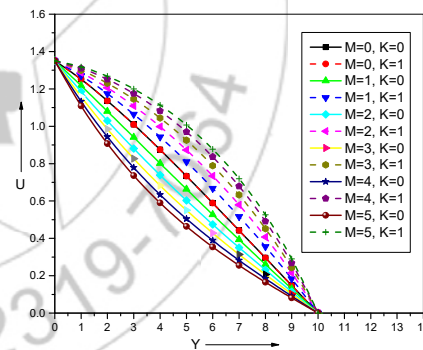


Fig. 1
Velocity profiles for different values of M (EAP)

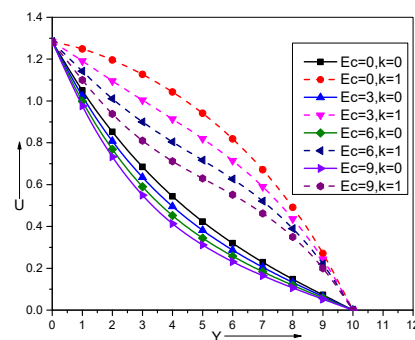


Fig. 2
Velocity profiles for different values of Ec(EAP)

Stability Analysis

The computations are carried out for different values of the various physical parameters. The procedure is repeated until the steady state. During computation Δt was chosen as 0.001. These computations are carried out for $Pr = 0.71, 1, 7$ and 11 and $Ec = -1, -0.7, 0, 0.7, 1$ and $Ec = 0, 3, 6, 9$. To judge the accuracy of the convergence of the finite difference scheme, the same program was run with the $\Delta t = 0.0009$ and no significant change was observed. Hence, we conclude the finite difference scheme is stable and convergent.

Discussion of Results

The numerical solutions for the velocity, temperature and mass diffusion are computed for various physical parameters such as Hartmann number (M), Eckert number (Ec), Prandtl number (Pr), time (t), accelerating parameter (a_0), and heat generation or absorption coefficient (Q), and Schmidt number (Sc) etc. for two cases (i) Exponentially Accelerated Plate (EAP) and (ii) Uniformly Accelerated Plate (UAP). The skin - friction, the rate of heat transfer in terms of the Nusselt number (Nu), and the rate of mass transfer in terms of the Sherwood number (Sh) are also derived in terms of

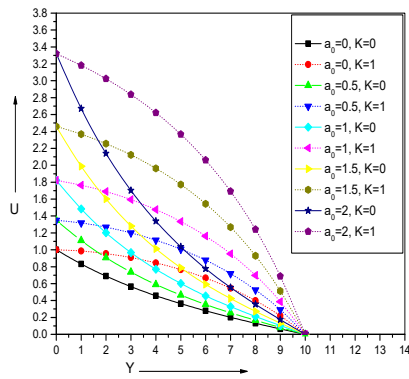


Fig. 3
Variation of velocity profiles for different values of a_0 (EAP)

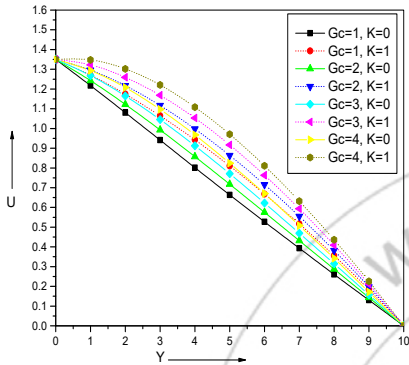


Fig. 4
Variation of velocity profiles for different values of G_c (EAP)

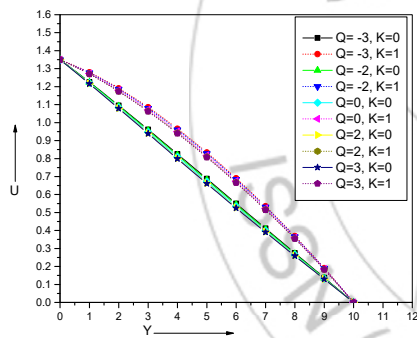


Fig. 5
Variation of velocity profiles for different values of Q (EAP)

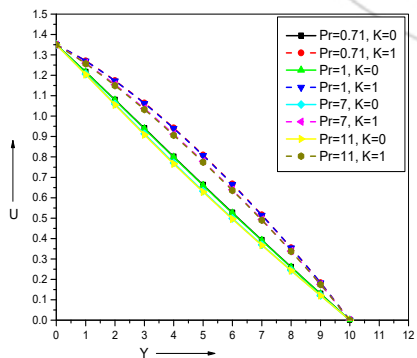


Fig. 6
Velocity profiles for different values of Pr (EAP)

shown in the figure 1. From the figure, the decrease of velocity is observed with the increase of the magnetic field. The velocity increases with the moving plate.

The variation of velocity profiles for different values of Eckert number (Ec) are shown in figure 2, when the magnetic field is being fixed to the fluid ($K = 0$) and to the moving plate ($K = 1$). From the figure 2, it is clearly observed that, as Eckert number increases, velocity decreases when the magnetic field is being fixed to the fluid ($K=0$) as well as to the moving plate ($K=1$). The effect of the exponential accelerated parameter (a_0) on the velocity profiles is presented in figure 3. It is observed that the velocity increases with an increase in exponential accelerated parameter in the presence of heat absorption.

The variation of velocity distribution to the Solutal and thermal Grashoff numbers is discussed in the figures 4. The thermal Grashoff number is the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. The positive values of Grashoff number indicates the cooling of the plate. The rise in the velocity is observed due to the enhancement of the thermal buoyancy force. As thermal buoyancy increases, the velocity increases rapidly near the plate and gradually decreases to free stream velocity. Similar to the thermal Grashoff number, the Solutal Grashoff number effect is also to increase in the velocity. The rise in the velocity distribution is observed from the figure.

The velocity profiles are presented in figure 5 for different values of Q . It is observed that the velocity decreases with increasing heat generation effect close to the plate while the velocity away from the plate increases. On the other hand, the velocity increases with increasing heat absorption effects in the vicinity of the plate and an opposite trend is observed away from the plate. From the figure 6, the decrease of velocity with the increase of Prandtl number (Pr) is shown.

The substantial change, in the temperature of the flow and concentration, with variation of parameters like Prandtl number, Eckert number, Heat generation or absorption coefficient (Q) etc. is discussed through the figures 7 to 12.

The presence of a magnetic field in an electrically conducting fluid introduces the Lorentz force, which acts against the flow results in the decrease of the velocity with the increase of magnetic field (M). The effect of the Hartmann number (M), when the magnetic field is being fixed to the fluid ($K = 0$) and to the moving plate ($K = 1$), is

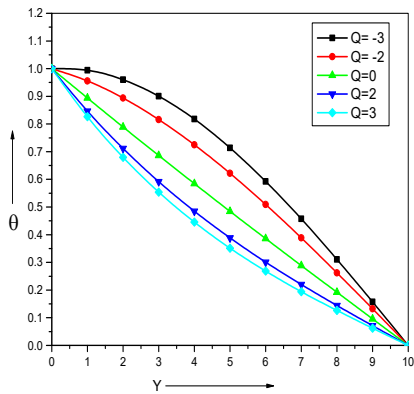


Fig. 7
 Temperature profiles for different values of Q (EAP)

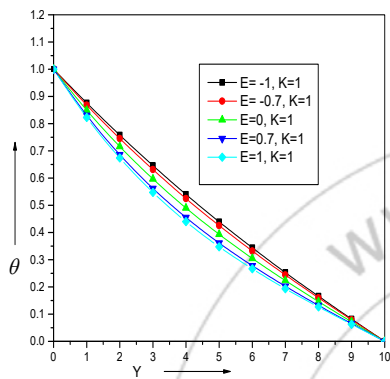


Fig. 10
 Variation of temperature profiles for different values of Ec (EAP)

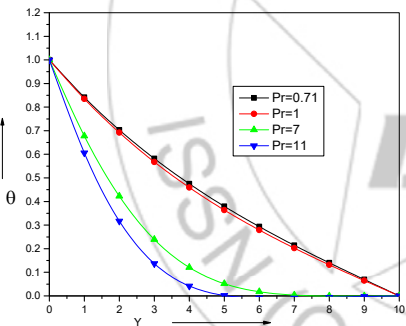


Fig. 9
 Variation of temperature profiles for different values of Pr when K=1 (EAP)

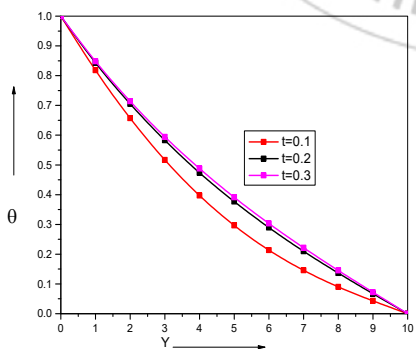


Fig. 10
 Variation of temperature profiles for different values of t when Q=2 (EAP)

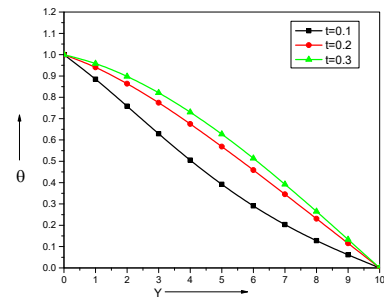


Fig. 11
 Variation of temperature profiles for different values of t when Q = -2 (EAP)

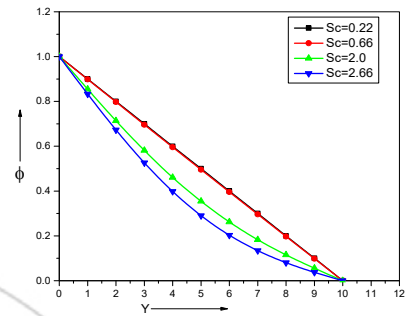


Fig. 12
 Variation of concentration profiles for different values of Sc (EAP)

The effect of the heat generation ($Q < 0$) or absorption ($Q > 0$) on the temperature is shown in the figure 7. From the figure it is precisely observed that near the plate, the temperature increases with the heat generation while with the heat absorption within the boundary layer produces opposite effect. The temperature increases 106.89% with the heat generation i.e., as Q varies from 0 to -2 and temperature increases 111.19% as Q varies from 0 to -3. The temperature decreases 5.30% and 7.53% in the case of heat absorption i.e., as Q varies from 0 to 2 and 0 to 3 respectively. From figure 8, for fixed to the fluid ($K = 0$) and to the moving plate ($K = 1$), temperature decreases with an increase in Ec. From the figures 9, the decrease of temperature in the boundary layer is observed for an increase in the Prandtl number.

The temperature profiles at different time (t) are shown in the figure 10, for heat absorption ($Q > 0$). The attaining of steady state temperature with increasing time is seen from the figure. The reason is that the heat supplied from the plate is balanced by the heat absorption in the boundary layer as the time progresses. The temperature profiles at different time (t) are shown in the figure 11, for heat generation ($Q < 0$). The increase of temperature with an increase in time can be seen from the figure. The reason for this is the heat supplied from the plate is aided by the heat generated in the boundary layer to increase the fluid temperature.

The concentration distribution is vastly affected by the presence of foreign species such as Hydrogen ($Sc = 0.22$), Oxygen ($Sc = 0.66$), Sc = 2.0 (Pentane), and Sc = 2.66 (Octane) which are given in the figure 12. The concentration is decreased with the increase of the Schmidt number i.e. with the presence of heavy foreign species.

For Uniformly Accelerated Plate (UAP)

The velocity profiles, for Uniformly Accelerated Plate (EAP), are discussed through the graphs 13 to18, when the magnetic field is being fixed to the fluid ($K = 0$) and to the

moving plate ($K = 1$), for various physical parameters such as Hartmann number (M), thermal Grashoff number (Gr), the Solutal Grashof number (Gc), the heat generation or absorption parameter (Q), and time (t) etc.

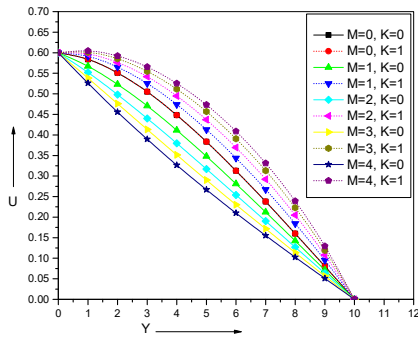


Fig. 13
Variation of velocity profiles for different values of M (UAP)

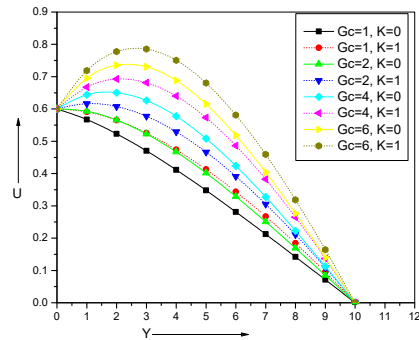


Fig. 14
Variation of velocity profiles for different values of Gc (UAP)

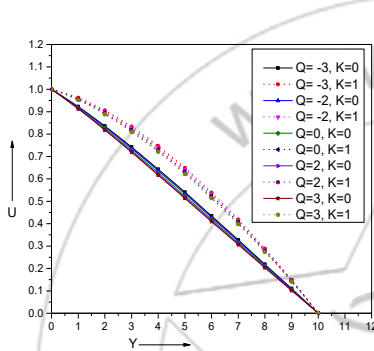


Fig. 15
Variation of velocity profiles for different values of Q (UAP)

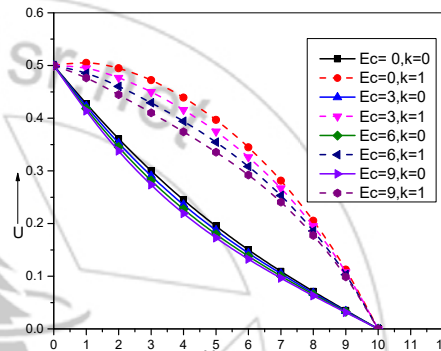


Fig. 16
Velocity profiles for different values of Ec (UAP)

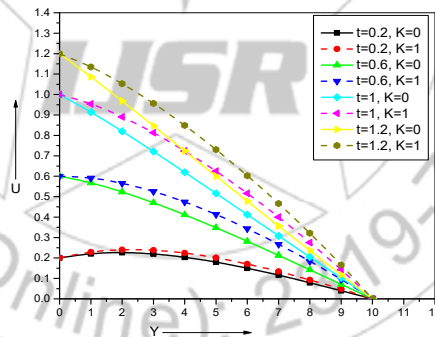


Fig. 17
Velocity profiles for different values of t (UAP)

The velocity variation with the magnetic field (M) is shown in the figure 13 near a Uniformly Accelerated Plate (UAP). The velocity decreases with the increasing transverse magnetic field when $K=0$ whereas it exhibits the opposite effect away from the plate when $K=1$. The variation of velocity distribution to the Solutal and thermal Grashoff numbers is meticulously discussed in the figures 14. The rise in the velocity is carefully observed due to the enhancement of the thermal buoyancy force. As thermal buoyancy increases, the velocity increases rapidly near the plate and gradually decreases to free stream velocity. Similar to the thermal Grashoff number, the Solutal Grashoff number effect is also to increase in the velocity. The rise in the velocity distribution is seen from the figures.

From the figure 15, it is observed that the velocity decreases with increasing heat generation close to the plate and an

opposite trend is observed away from the plate. The fluid velocity increases with increasing heat absorption. From figure 16, it is clear that an increase in Eckert number Ec leads to decrease in velocity for fixed to the fluid ($K=0$) and to the moving plate ($K=1$). The effect of t on the velocity field is given in figure 17 in the presence of heat absorption, when the other parameters are being fixed. It is observed that the velocity of the fluid increases with increasing values of t . This is because of an increase in t leads to an increase in the buoyancy force which causes an increase in the fluid velocity.

The substantial change in the temperature of the flow and concentration with variation of parameters like Heat generation or absorption parameter (Q), Prandtl number, and time (t) etc., are discussed through the figures 18 to 23.

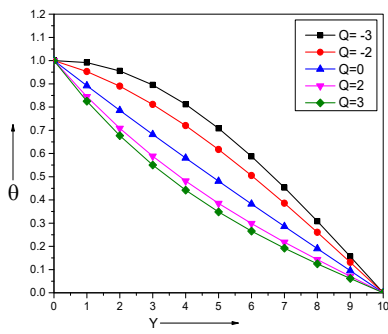


Fig. 18
Variation of temperature profiles for different values of Q (UAP)

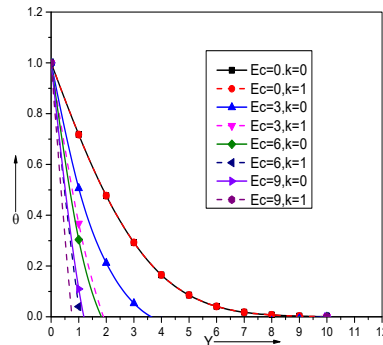


Fig. 19
Temperature profiles for different values of Ec (UAP)

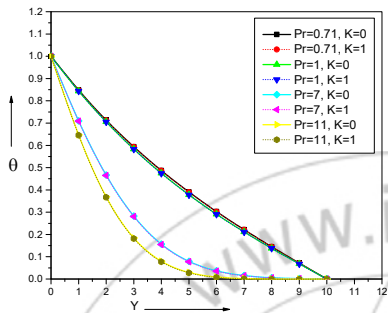


Fig. 20
Temperature profiles for different values of Pr (UAP)

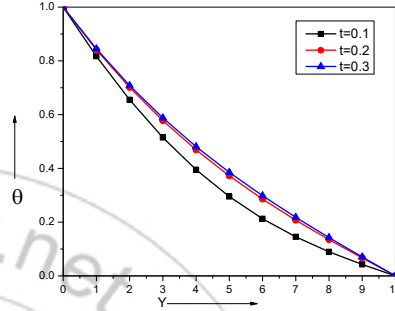


Fig. 21
Temperature profiles for different values of t when Q=2 (UAP)

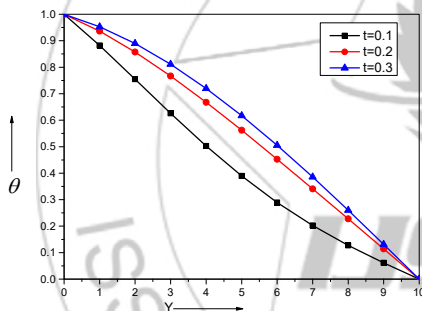


Fig. 22
Temperature profiles for different values of t when Q = -2 (UAP)

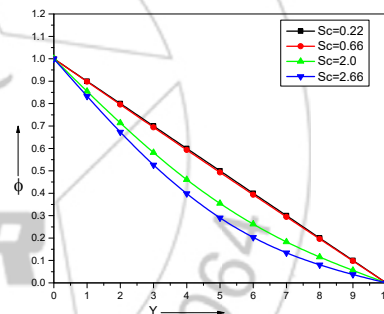


Fig. 23
Concentration profiles for different values of Sc (UAP)

The effect of the heat generation ($Q < 0$) or absorption ($Q > 0$) on the temperature is shown in the figure 18. From the figure, it is observed that near the plate the temperature decreases with the heat generation while with the heat absorption within the boundary layer produces opposite effect. The temperature increases 106.89% with the heat generation i.e., as Q varies from 0 to -2 and temperature increases 111.19% as Q varies from 0 to -3. The temperature decreases 5.30% and 7.53% in the case of heat absorption i.e., as Q varies from 0 to 2 and 0 to 3 respectively.

Figure 19 shows that the temperature decreases with increase in Eckert number Ec when $K=0$ and $K=1$. The temperature of the fluid decreases with the increase of Prandtl number is given in the figure 20 when $K = 0$ and $K = 1$. The temperature profiles at different time (t) are shown in the figures 21 and 22. From the figures, it is clear that the temperature of the fluid increases with the increase of time for both heat generation and absorption.

The concentration distribution is vastly affected by the presence of foreign species such as Hydrogen ($Sc = 0.22$), Oxygen ($Sc = 0.66$), $Sc = 2.0$ (Pentane), and $Sc = 2.66$ (Octane) which is given in the figure 23. The

concentration is decreased with the increase of the Schmidt number i.e. with the presence of heavy foreign species.

It is interesting to note that the fluid velocity approaches to zero with the increase of the boundary layer y when the magnetic field is fixed relative to the plate whereas the velocity approaches to a non-zero steady state, when the magnetic field is fixed to the fluid, which was keenly observed by Raptis and Singh (6).

In the absence of mass transfer and for $Ec=0$, the effects, of the various physical parameters, for $Pr = 0.71$, are in exact agreement with the results of Narahari and Debnath (13).

Table 1: Variation of Skin friction coefficient for t , Q , and a_0

| t | Q | a_0 | $\tau - EAP$ | | $\tau - UAP$ | |
|-----|----|-------|--------------|------------|--------------|-----------|
| | | | K = 0 | K = 1 | K = 0 | K = 1 |
| 0.2 | 2 | 0.5 | -0.9641629 | -0.4702732 | 0.2896599 | 0.3791163 |
| 0.6 | -3 | 1 | -1.303466 | -0.7003891 | -0.2641329 | 0.0041414 |

From the table I, for an increase in time (t), the acceleration parameter (a_0), and the heat generation or absorption coefficient (Q) the shear stress at the wall decreases for both the cases UAP and EAP. When the accelerating parameter

(a_0) increases, the shear stress decreases at the plate and also away from the plate.

Table 2: Variation of Nusselt number for t , Q , and a_0

| t | Q | a_0 | Nu - EAP | | Nu - UAP | |
|-----|----|-------|-----------|-----------|-----------|-----------|
| | | | K = 0 | K = 1 | K = 0 | K = 1 |
| 0.2 | 2 | 0.5 | -1.657682 | -1.667701 | -1.604162 | -1.604626 |
| 0.6 | -3 | 1 | -1.683353 | -1.697998 | -1.619741 | -1.622955 |

For an increase in time (t), the rate of heat transfer decreases for both the cases i.e. for UAP and EAP, and also when the magnetic field is fixed relative to the fluid ($K = 0$) and with the moving plate ($K = 1$). This observation is shown in the table II.

Observations

- In the absence of mass transfer and magnetic dissipation the governing equations are solved using Laplace Transform technique and the results are in agreement with the results of Narahari and Debnath (13).
- When the magnetic field is fixed to the fluid the velocity approaches to a non-zero steady state, which was observed by Raptis and Singh (6).

4. Conclusions

A mathematical model of an unsteady free convection flow of an electrically conducting fluid past an accelerated infinite vertical plate, with constant heat flux is investigated under the influence of uniform transverse magnetic field fixed relative to the fluid or to the plate in the presence of chemical reaction with heat generation or absorption and Ohmic heating has been considered. The following main results are concluded from the study:

I. For Exponential Accelerated Plate (EAP) case, when the magnetic field is fixed relative to the fluid ($K = 0$) and with the moving plate ($K = 1$).

- Velocity increases with increasing in a_0 , G_c , Gr , t and M .
- Increase in Q and Pr results in decrease in velocity.
- Temperature increases with increase in t when $Q=2$ and $Q=-2$.
- An increase in Q, E and Pr leads to decrease in temperature.
- Concentration decreases with increase in Sc .

II. For Uniformly Accelerated Plate (UAP), when the magnetic field is fixed relative to the fluid ($K = 0$) and with the moving plate ($K = 1$).

- Velocity increases with increase in Gr , G_c , and t .
- Increase in M , Q , and Pr leads to a decrease in velocity.
- Temperature increases with increase in t when $Q = 2$ and $Q = -2$.
- An increase in Q and Pr results in a decrease in the temperature.
- Concentration decreases with increase in Sc .

III. An increase in time (t), the acceleration parameter (a_0), and the heat generation or absorption coefficient (Q), the shear stress at $K=0$ and $K=1$, decreases for both the cases EAP and UAP.

IV. With an increase in time (t), the rate of heat transfer decreases for both EAP and UAP, at $K=0$ and $K=1$.

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