A Representation of all Tolerances on a Set by Certain Type of Covers

Pusuluri V. N. H. Ravi¹, Nistala V. E. S. Murthy ²

¹Department of Basic Sciences and Humanities, Sri Vasavi Engneering College, Tadepalligudem-534101, AP, India
²Department of Mathematics, A.U. College of Science and Technology, Andhra University, Vishakapatnam-530003, AP, India

Abstract: Most of the correspondences, especially those in Computer Science, and Mathematics which are not a Galois correspondence are actually Lagois correspondences introduced by Austin Melton. In this paper we represent the poset of all tolerances on any set, including the infinite ones, by certain type of covers on the same via a Lagois correspondence in such a way that it is a natural extension of the complete isomorphism between the set of all partitions of a non empty set and the set of all equivalence relations on the same set.

Keywords: i-Cover, tolerance relation, Lagois connection

A.M.S. Subject Classification: Primary 04A72; Secondary 08A72, 08A02

1. Introduction

A tolerance relation or simply a tolerance on a set is a binary relation which is reflexive and symmetric but not necessarily transitive. Introduced by Poincare, Carnap and independently by Zeeman, tolerances provide a general tool for studying indiscernible phenomena. Tolerances have found applications in rough set theory, digital mathematics and other areas. Also, in Computer Science, they frequently appear under such other names as compatibility relations, similarity relations and proximity relations, depending on the field of study.

Bartol-Pioro-Rossello[3], gave a characterization of those coverings of a finite set that are families of classes of some tolerance. However, in this paper we aim to represent the poset of all tolerances on a set, not necessarily finite, by certain type of covers on the same via a Lagois connection. Further, we realize that this connection is a natural generalization of the complete isomorphism between the set of all partitions of a non empty set and the set of all equivalence relations on the same.

Let us recall that a special, namely, transitive tolerances are the equivalence relations and that there is a Galois connection which is in fact a complete isomorphism between the complete lattice of all partitions of a set and the complete lattice of all equivalence relations on the same. Now, in a Galois connection the composite maps behave differently with respect to order relations in posets, while in case of Lagois connection Austin Melton[1,2] their behavior is same (either increasing or decreasing).

In this paper we introduce the notion of an i-cover of a set and there by establish a Lagois connection between the set of all i-covers of a set and the set of all tolerance relations on the same.

2. Preliminaries

In this section, some notions and results from Lattice Theory that are useful in proving some important Theorems in the coming sections are mentioned. Further, some standard notions and results from Bartol-Pioro-Rossello[3] are recalled.

2.1 Proposition

Let \( L \) be a meet complete semi lattice with the greatest element \( 1_L \). For any subset \( \varphi \subseteq S \subseteq L \) define \( \varphi \wedge S = \bigwedge \{ \alpha \in L \mid \alpha \land \varphi \beta \beta \wedge \beta \in S \} \). Then, \( L \) is a complete lattice, where the join is given by \( \vee \).

2.2 Theorem

For any pair of posets \( L \) and \( M \) such that \( M \) is a complete lattice with 0, 1 as the least, the greatest elements respectively and for any pair of order preserving maps \( \varphi : L \rightarrow M \) and \( \psi : M \rightarrow L \) such that \( \varphi \circ \psi = \iota_L \) and \( \iota_M \) is the identity map on \( M \), \( \text{Image}(\psi) \) is a complete lattice with \( \psi(0) \) as the least element and \( \psi(1) \) as the greatest element. In particular for any \( (\beta_i)_{i \in I} \subseteq \text{Image}(\psi) \), \( \bigvee_{i \in I} \beta_i = \psi(\bigvee_{i \in I} \phi(\beta_i)) \) and \( \bigwedge_{i \in I} \beta_i = \psi(\bigwedge_{i \in I} \phi(\beta_i)) \).

Proof: It follows from \( \varphi \circ \psi = \iota_M \) that \( \phi \) is onto and \( \psi \) is one-one. Now for any \( (\beta_i)_{i \in I} \subseteq \text{Image}(\psi) \), \( \exists (\alpha_i)_{i \in I} \subseteq M \ni \psi(\alpha_i) = \beta_i \). This implies \( \phi(\alpha_i) = \phi(\beta_i) \), which in turn gives \( \alpha_i = \phi(\beta_i) \forall i \in I \). Thus \( \beta_i \leq \psi(\bigvee_{i \in I} \phi(\beta_i)) \forall i \in I \). Thus \( \psi(\bigvee_{i \in I} \phi(\beta_i)) \) is an upper bound of \( (\beta_i)_{i \in I} \subseteq \text{Image}(\psi) \). Further, it is easy to see that \( \psi(0) \) and \( \psi(1) \) are the least and the greatest elements respectively of \( \text{Image}(\psi) \). Now, for \( (\beta_i)_{i \in I} \subseteq \text{Image}(\psi) \), let \( \beta \in \text{Image}(\psi) \) be \( \beta_i \leq \beta \forall i \in I \). Then it follows that \( \psi(\bigvee_{i \in I} \phi(\beta_i)) \leq \beta \).

Then \( \bigvee_{i \in I} \beta_i = \psi(\bigvee_{i \in I} \phi(\beta_i)) \). Similarly, we can show that \( \bigwedge_{i \in I} \beta_i = \psi(\bigwedge_{i \in I} \phi(\beta_i)) \).
2.3 Definitions and Statements

(a) For any nonempty set $U$, a family $\mathcal{A}$ of nonempty subsets of $U$ is called a cover of $U$, if $\bigcup_{A \in \mathcal{A}} A = U$. The set of all covers of $U$ is denoted by $\Sigma(U)$.

(b) For any pair of sets $U$ and $V$, a relation from $U$ to $V$ is any subset of $U \times V$. We write some times $U^2$ in place of $U \times U$. If $R$ is a relation from a set $U$ to itself, then $R$ is a binary relation on $U$. For any nonempty set $U$, the relation $\{(u, u) \mid u \in U\}$ is the identity relation on $U$ and is denoted by $\Delta_U$. The set of all binary relations on $U$ denoted by $R^2(U)$ is a meet complete semi lattice where the $\wedge$ is given by the usual set intersection and the underlying partial order is given by the set inclusion. The least element is $\emptyset$ and the greatest element is $U \times U$. In fact, for any set $U$, $R^2(U)$ is a complete lattice with the infimum and the supremum given by $\inf R = \cap_{(u,v) \in R} R$, $\sup R = \cup_{(u,v) \in R}$, where $R = \{(u,v) \mid (u,v) \in R\}$ is any subset of $R^2(U)$.

(c) For any nonempty set $U$, and for any binary relation $R$ on $U$: (1) $R$ is reflexive if for each $u \in U$, $(u, u) \in R$ (2) $R$ is symmetric iff for each $u, v \in U$, $(u, v) \in R$ implies $(v, u) \in R$ (3) $R$ is transitive iff $(u, v), (v, w) \in R$ implies $(u, w) \in R$.

(d) A binary relation $R$ on the nonempty set $U$ is called a tolerance relation or simply a tolerance if it is reflexive and symmetric. The set of all tolerances on $U$ is denoted by $R^T(U)$.

(e) For any nonempty set $U$, $(R^T(U), \subseteq)$ is a poset with the greatest element $U \times U$ as the least element and $U \times U$ as the greatest element. In fact, $(R^T(U), \subseteq)$ is a complete lattice with $\Delta_U$ as the least element and $U \times U$ as the greatest element. In what follows some notions and results from Bartol-Piro-Rossello[3] are recalled.

(f) For any nonempty set $U$, for any tolerance relation $R$ on $U$, a nonempty subset $A$ of $U$ is called a T-preblock (or simply preblock) of $R$ if $A \times A \subseteq R$.

(g) For any nonempty set $U$, for any tolerance relation $R$ on $U$, a T-preblock $A$ of $R$ is called a T-block (or simply block) of $R$ if $A \subseteq B$ such that $B \times B \subseteq R$, then we must have $B = A$. A T-block of the tolerance relation $R$ is usually denoted by $RB$.

(h) For any nonempty set $U$, for any tolerance relation $R$ on $U$, a subset $A$ of $U$ is a T-preblock of $R$ if and only if it is contained in some T-block of $R$.

3. Main Results

In this section, the notion of i-cover of a non empty set is introduced and a Lagois Connection between the set of all i-covers and the set of all tolerance relations is established.

3.1 Definitions and Statements

(a) For any non empty set $U$, a cover $\mathcal{A}$ of $U$ is said to be an i-cover of $U$, if $A, B \in \mathcal{A}$ such that $A \subseteq B$ implies $A \cap B \subseteq B$ (or equivalently $A \subseteq B$ implies $A \subseteq B$). The set of all i-covers of $U$ is denoted by $\Sigma_i(U)$.

(b) For any nonempty set $U$, $\{\Sigma_i(U), \subseteq\}$ is a poset with the i-cover $\{(u,v) \mid u, v \in U\}$ as the least element and the i-cover consisting of $\{U\}$ alone as the greatest element, where $\subseteq$ is given by: $A \subseteq B$. Thus, $\Sigma_i(U)$ and for any binary relation $R$ be any three i-covers of the $U$. Is a poset with the i-cover $\{\Sigma_i(U), \subseteq\}$.

Proof: Let $\mathcal{A}_1, \mathcal{A}_2$ and $\mathcal{A}_3$ be any three i-covers of the $L$-set $U$.

To show that $\subseteq$ is reflexive. We know that $A \subseteq A \forall A \in \mathcal{A}_2$. Thus, $\mathcal{A}_1 \subseteq \mathcal{A}_2$. Thus, $\subseteq$ is reflexive.

To show that $\subseteq$ is antisymmetric. Let $\mathcal{A}_1 \subseteq \mathcal{A}_2$ and $\mathcal{A}_2 \subseteq \mathcal{A}_1$. Then for $A_1 \in \mathcal{A}_1$, we have $A_1 \subseteq A_2$ for some $A_2 \in \mathcal{A}_2$ because $\mathcal{A}_1 \subseteq \mathcal{A}_2$. Now for this $A_2$, we have $A_2 \subseteq A_1$ for some $A_1 \in \mathcal{A}_1$ because $\mathcal{A}_2 \subseteq \mathcal{A}_1$. This implies $A_1 \subseteq A_1$. This implies $1 = 1$. Otherwise $A_1 \neq A_1 \forall A_1 \in \mathcal{A}_1$ which is a contradiction. This implies $\mathcal{A}_1 \subseteq \mathcal{A}_2$. Similarly we can show that $\mathcal{A}_2 \subseteq \mathcal{A}_1$. Thus, $\mathcal{A}_1 \subseteq \mathcal{A}_2$. Thus, $\subseteq$ is antisymmetric.

To show that $\subseteq$ is transitive. Let $\mathcal{A}_1 \subseteq \mathcal{A}_2$ and $\mathcal{A}_2 \subseteq \mathcal{A}_3$. Now for $A_1 \in \mathcal{A}_1$, we have $A_1 \subseteq A_2$ for some $A_2 \in \mathcal{A}_2$ because $\mathcal{A}_1 \subseteq \mathcal{A}_2$. Now for this $A_2$, we have $A_2 \subseteq A_3$ for some $A_3 \in \mathcal{A}_3$ because $\mathcal{A}_2 \subseteq \mathcal{A}_3$. This implies $A_1 \subseteq A_3$ for some $A_3 \in \mathcal{A}_3$. This implies $\mathcal{A}_1 \subseteq \mathcal{A}_3$. Thus, $\subseteq$ is transitive.

It is clear that the i-cover $\{u,v\} \subseteq U$ is the least element the i-cover consisting of $U$ alone is the greatest element of $\Sigma_i(U)$. Thus, $\{\Sigma_i(U), \subseteq\}$ is a poset with the i-cover $\{u,v\}$ as the least element and with the i-cover consisting of $U$ alone, namely $\{U\}$ as the greatest element.

The above relation need not be a partial order when it is defined on $\Sigma(U)$ as can be seen in the following example.

Example: For the set $U = \{u,v\}$ and for the covers $\mathcal{A} = \{\{u\}, \{v\}, \{u,v\}\}$ of $U$, we have $\mathcal{A} \subseteq \mathcal{B}$ but $\mathcal{B} \neq \mathcal{A}$. Further observe that $\mathcal{B}$ is an i-cover while $\mathcal{A}$ is a mere cover but not an i-cover.

(c) For any nonempty set $U$, an i-cover $\mathcal{A}$ of $U$ is said to be an s-cover of $U$, if $B \subseteq U$ such that every pair of elements of $B$ belongs to some member of $\mathcal{A}$ then we must have $B \subseteq A$ for some $A \in \mathcal{A}$. The set of all s-covers of $U$ is denoted by $\Sigma_s(U)$. However an i-cover need not be an s-cover as can be seen in the following example.

Example: For the set $U = \{u, v, w, x\}$, the cover $\mathcal{A} = \{\{u, v\}, \{w, x\}\}$ is an i-cover but not an s-cover.

(d) For any nonempty set $U$, for any tolerance relation $R$ on $U$, the set of all T-preblocks (T-blocks) of $R$, denoted by $\mathcal{E}(R)(\mathcal{E}(R))$, is a cover (an i-cover) of $U$.

Volume 4 Issue 2, February 2015

www.ijsr.net


International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Licensed Under Creative Commons Attribution CC BY

Paper ID: SUB151422

1234
Proof: First notice that both $\bigcup_{x \in U} B$ and $\bigcup_{y \in \mathcal{C}(R)} B$ are subsets of $U$ always. For $u \in U$, we have $(u, u) \in R$ because $R$ is reflexive. Thus $\{u\}$ is a T-preblock for $R$. This implies $u \in \bigcup_{x \in U} B$. Now by 2.3.h, there exists a T-block $RB_1$ for $R$ such that $\{u\} \subseteq RB_1$. Thus $u \in \bigcup_{x \in \mathcal{C}(R)} B$ and $u \in \bigcup_{y \in \mathcal{C}(R)} B$. Therefore, we have $U = \bigcup_{x \in \mathcal{C}(R)} B = \bigcup_{y \in \mathcal{C}(R)} B$. Further, it is easy to see that $RB_1 \cap RB_2 \cap \mathcal{C}(R) = \bigcap_{x \in \mathcal{C}(R)} B \cap \bigcap_{y \in \mathcal{C}(R)} B$ implies $RB_1 \cap RB_2$. Thus $\mathcal{C}(R)$ and $\mathcal{C}(\mathcal{C}(R))$ are cover and i-cover respectively for $U$.

(e) For any tolerance relation $R$ on $U$, $\mathcal{C}(R)\mathcal{C}(R))$ is called an associated cover (i-cover) for $R$.

(f) For any nonempty set $U$, for any tolerance relation $R$ on $U$, we have $(u, v) \in R$ if and only if $u, v \in A$ for some $A \in \mathcal{C}(R)$. In fact $\mathcal{C}(R)$ is an s-cover of $U$.

(g) For any nonempty set $U$, for any tolerance relation $R$ on $U$ and for any $u \in U$, the T-class of $u$ for $R$, denoted by $uR$, is defined by $uR = \{v \in U | (u, v) \in R\}$. For any nonempty set $U$, for any tolerance relation $R$ on $U$, the family of T-classes for $R$ is denoted by $\mathcal{J}(R)$.

(h) For any nonempty set $U$, for any tolerance relation $R$ on $U$, the family of T-classes $\mathcal{J}(R)$ is a cover. However, the family of T-classes $\mathcal{J}(R)$ for a tolerance relation $R$ on a nonempty set $U$ need not be an i-cover as can be seen in the following example:

Example: For the set $U = \{u, v, w, x\}$, for the tolerance relation $R$ on $U$ given by:

\[
\begin{array}{cccc}
u & v & w & x \\ u & 1 & 1 & 1 \\ v & 1 & 1 & 0 \\ w & 1 & 1 & 0 \\ x & 1 & 0 & 1 \\
\end{array}
\]

$\mathcal{J}(R) = \{U, \{u, v, w\}, \{u, x\}\}$, which is clearly not an i-cover. Further $uR = U$ and $U \times U \subseteq R$. Thus a T-class of a tolerance relation need not be even a T-preblock.

(i) For any nonempty set $U$, for any tolerance relation $R$ on $U$, the following are true:

- $uR = \bigcup_{x \in \mathcal{C}(R)} B$, where $u \in U$
- $B = \bigcap_{x \in \mathcal{C}(R)} uR$, where $B \in \mathcal{C}(R)$

(j) For any cover (i-cover) $\mathcal{A}$ of a nonempty set $U$, $\bigcup_{x \in \mathcal{A}} A \times \mathcal{A} \times \mathcal{A}$ is a tolerance relation on $U$ and is denoted by $T(\mathcal{A})$.

(k) For any cover (i-cover) $\mathcal{A}$ of a nonempty set $U$, $T(\mathcal{A})$ is called the associated tolerance relation for $\mathcal{A}$.

3.2 Lagois Connection Between The set of all i-covers and The set of all Tolerance Relations:

For any nonempty set $U$, define $\phi: \mathcal{J}(U) \rightarrow R(\mathcal{A})$ by $\phi(\mathcal{A}) = T(\mathcal{A})$, the associated tolerance relation for $\mathcal{A}$ and $\psi: R(\mathcal{A}) \rightarrow \mathcal{J}(U)$ by $\psi(R) = \mathcal{C}(R)$, the associated i-cover for $R$. Then, the following are true:

1. $\mathcal{A}_1 \leq \mathcal{A}_2 \Rightarrow \phi(\mathcal{A}_1) \subseteq \phi(\mathcal{A}_2)$
2. $R_1 \leq R_2 \Rightarrow \psi(R_2) \subseteq \psi(R_2)$
3. $I(\mathcal{A}_1) \subseteq \psi \circ \phi$.

4. $I(\mathcal{A}_1) = \phi \circ \psi$
5. Image($\psi$) = $\sum_{i}(U)$
6. $\sum_{i}(U)$ is a complete lattice with $\phi \circ \psi$ as the least element and $\phi \times \psi$ as the greatest element.

Further for any family of s-covers $\mathcal{A}_i$ of $U$, we have $\mathcal{A}_i = \psi(\mathcal{A}_i)$ and $\mathcal{A}_i = \psi(\mathcal{A}_i)$.

7. $\phi \circ \psi = \phi \circ \psi$
8. $\psi \circ \psi = \psi \circ \psi$

Proof: (1): Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be any two i-covers for $U$ such that $\mathcal{A}_1 \leq \mathcal{A}_2$. Let $R_1 = T(\mathcal{A}_1)$ and $R_2 = T(\mathcal{A}_2)$. Of course both $R_1$ and $R_2$ are tolerance relations by 3.1(j) above. Now for $(u, v) \in R_1$, we have $(u, v) \in A \times A$ for some $A \in \mathcal{A}_2$. This implies $\exists B \in \mathcal{A}_1$ such that $A \subseteq B$ because $\mathcal{A}_2 \leq \mathcal{A}_2$. Thus $(u, v) \in B \times B$. Now it follows that $R_1 \subseteq R_2$.

(2): let $R_1$ and $R_2$ be any two tolerance relations on $U$. Let $\mathcal{A}_1 = \mathcal{C}(R_1)$ and $\mathcal{A}_2 = \mathcal{C}(R_2)$. Of course both $\mathcal{A}_1$ and $\mathcal{A}_2$ are both i-covers for $U$ by 3.1(d) above. Now let $R_1$ be a T-block of $R_1$. Clearly $R_1 \subseteq R_1 \subseteq R_1$. Clearly $R_1 \subseteq R_1 \times R_1$. By definition of $\mathcal{C}(R_1)$, but $R_1 \subseteq R_1 \subseteq R_1 \subseteq R_1$. Thus $R_1$ is a T-preblock of $R_1$. Now $\mathcal{C}(R_1) \subseteq \mathcal{A}_1$ such that $R_1 \subseteq R_1 \subseteq R_1 \subseteq R_1$. Thus $(u, v) \in B \times B$. Now it follows that $R_1 \subseteq R_2$.

(3): Let $\mathcal{A}$ be an i-cover for $U$. Let $R = T(\mathcal{A})$. Clearly $R$ is a tolerance relation by 3.1(i) above and $\mathcal{A}$ is a T-preblock of $R$. Fix $A \in \mathcal{A}$. Now $\exists B \in \mathcal{C}(R)$ such that $A \subseteq B$ by 2.3(h) above. Thus $\mathcal{A} \leq \mathcal{C}(R)$.

(4): It is enough to show that $R = T(\mathcal{C}(R))$ for any tolerance relation $R$ on $U$. Let $(u, v) \in R$. This implies $(u, v) \in \mathcal{C}(R)$.

Further $uR = U$ and $U \times U \subseteq R$. Thus $\mathcal{A} \leq \mathcal{C}(R)$.

(5): Clear.

(6): Follows from Theorem 2.2.

(7): $\phi \circ \psi = \phi \circ \psi$ is obvious because of (3). Let $\mathcal{A}$ be an i-cover for $U$. It is enough to show that $T(\mathcal{C}(T(\mathcal{A}))) \subseteq T(\mathcal{A})$. Let $(u, v) \in T(\mathcal{C}(T(\mathcal{A})))$. This implies $(u, v) \in B \times B$ for some $B \in \mathcal{C}(T(\mathcal{A}))$. Thus $(u, v) \in T(\mathcal{A})$ above.

(8): Follows from (4).

Strict inequality in (3) of the above theorem can hold as shown in the following example:

Example: Let the set $U = \{u, v, w\}$, the i-cover $\mathcal{A} = \{\{u, v\}, \{v, w\}, \{w, u\}\}$. Now the associated tolerance relation $T(\mathcal{A})$ for the i-cover $\mathcal{A}$ is given by $U \times U$. Now the associated i-cover for $T(\mathcal{A})$, is given by $\mathcal{C}(T(\mathcal{A})) = \{U\}$.

Thus it is clear that $\mathcal{A} \neq \mathcal{C}(T(\mathcal{A}))$.

3.3 Remarks

Volume 4 Issue 2, February 2015
www.ijsr.net
Licensed Under Creative Commons Attribution CC BY
We recall that (1) a family \( \mathcal{A} \) of non empty subsets of a non empty set \( U \), is called a partition for \( U \), if (a) \( \bigcup_{A \in \mathcal{A}} A = U \) and (b) \( A, B \in \mathcal{A} \) such that \( A \neq B \) implies \( A \cap B = \emptyset \) or disjoint (2) a transitive tolerance on a non empty set \( U \) is an equivalence relation.

For an equivalence relation \( R \) on a non empty set \( U \), \( uR \) denotes an equivalence class containing \( u \) for \( R \). We know that any two equivalence classes for \( R \) on a nonempty set \( U \) are either same or disjoint and the set of all equivalence classes \( R \) constitutes a partition of \( U \).

For any non empty set \( U \), the set of all partitions of \( U \) is denoted by \( \Pi(U) \) and the set of all equivalence relations on \( U \) is denoted by \( R^{eq}(U) \).

For any non empty set \( U \), the set of all partitions \( \Pi(U) \) is a sub poset (cf. 3.1 (b)) of \( \sum \Pi(U) \) and the set of all equivalence relations \( R^{eq}(U) \) is a complete sub lattice (cf. 2.3 (e)) of \( R^{eq}(U) \).

It is easy to see that (1) every partition of a nonempty set \( U \) is an \( \mathcal{Z} \)-cover of \( U \) and (2) for any non empty set \( U \), for any partition \( \mathcal{A} \) of \( U \), the associated equivalence relation \( T(\mathcal{A}) \) (for the partition \( \mathcal{A} \) is an equivalence relation on \( U \) (3) for any non empty set \( U \), for any transitive tolerance \( R \), the associated \( \mathcal{Z} \)-cover \( \mathcal{C}_{\mathcal{Z}}(R) \) is a partition of \( U \) and there is a bijective correspondence (by 3.1 (i)) between the set of all \( \mathcal{Z} \)-blocks \( \mathcal{C}_{\mathcal{Z}}(R) \) and the set of all \( \mathcal{Z} \)-classes \( \mathcal{C}(R) \) of \( R \).

Now it is easy to see that, in 3.2, when \( \phi \) and \( \psi \) are restricted to \( \Pi(U) \) and \( R^{eq}(U) \) respectively, conclusion 3 becomes an equality (by 3.1(i)), and as a consequence Image(\( \phi \)) is realized as \( R^{eq}(U) \) and further Image(\( \psi \)) is any way \( \Pi(U) \).

4. Conclusions

In this paper we represented the poset of all tolerances on a set by certain type of covers on the same via a Lagois connection. Further, we also realized that this connection is the natural generalization of the complete isomorphism between the set of all equivalence relations on a non empty set and the set of all partitions of the same set.

References


Author Profile

Nistala V.E.S. Murthy received his M.S. in Mathematics from University of Hyderabad, Hyderabad, A.P. State, India, in 1981, Ph.D. in Mathematics from the University of Toledo, Toledo, OH-43606, U.S.A. in 1989 and an M.S. in Software Systems from B.I.T.S., Pilani Rajasthan State, India, in 2000; taught a wide range of courses both in Mathematics and Computer Science in various levels and is currently professor in the department of Mathematics, AUCST, Andhra University, Vizag-53003, A.P. State, India. His areas of publications include Various Fuzzy Set Theories and Their Applications in Mathematics (Set Theory, Algebra and Topology) and Computer Science (Data Security/Warehousing/Mining/Hiding) and Natural Language Modeling (Reprints/Preprints are Available on Request at drnvesmurthy@rediffmail.com or at http://andhrauniversity.academia.edu/NistalaVES_Murthy). In his little own way, he (1) developed f-Set Theory generalizing L-fuzzy set Theory of Goguen which generalized the [0,1]-fuzzy set theory of Zadeh, the Father of Fuzzy Set Theory (2) imposed and studied algebraic/topological structures on f-sets (3) proved Representation Theorems for f-Algebraic and f-Topological objects in general.

Pusuluri V.N.H. Ravi received his M.S. and M. Phil. in Mathematics from the University of Hyderabad, Hyderabad, India in 1990 and 1992 respectively. He received his Ph.D. in Mathematics from Andhra University, Visakhapatnam, India in 2012. He qualified National Eligibility Test conducted jointly by CSIR and UGC, India and received research fellowship from CSIR. He also qualified Graduate Aptitude Test in Engineering, India. He has more than 20 years of teaching experience, taught a wide range of courses both in Mathematics and Computer Science for both under graduate and post graduate programs in Arts, Science and Engineering. Currently he is working as an Associate Professor of Mathematics in the Department of Basic Sciences and Humanities at Sri Vasavi Engineering College, Pedatadepalli, A.P. State, India.