A Representation of all Tolerances on a Set by Certain Type of Covers

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Abstract: ***Most of the correspondences, especially those in Computer Science, and Mathematics which are not a Galois correspondence are actually Lagois correspondences introduced by Austin Melton. In this paper we represent the poset of all tolerances on any set, including the infinite ones, by certain type of covers on the same via a Lagois correspondence in such a way that it is a natural extension of the complete isomorphism between the set of all partitions of a non empty set and the set of all equivalence relations on the same set.***

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1. Introduction

A tolerance relation or simply a tolerance on a set is a binary relation which is reflexive and symmetric but not necessarily transitive. Introduced by Poincare, Carnap and independently by Zeeman, tolerances provide a general tool for studying indiscernible phenomena. Tolerances have found applications in rough set theory, digital mathematics and other areas. Also, in Computer Science, they frequently appear under such other names as compatibility relations, similarity relations and proximity relations, depending on the field of study.

Bartol-Pioro-Rossello[3], gave a characterization of those coverings of a finite set that are families of classes of some tolerance. However, in this paper we aim to represent the poset of all tolerances on a set, *not* necessarily *finite*, by certain type of covers on the same via a Lagois connection. Further, we realize that this connection is a natural generalization of the complete isomorphism between the set of all partitions of a non empty set and the set of all equivalence relations on the same.

Let us recall that a special, namely, transitive tolerances are the equivalence relations and that there is a Galois connection which is in fact a complete isomorphism between the complete lattice of all partitions of a set and the complete lattice of all equivalence relations on the same. Now, in a Galois connection the composite maps behave differently with respect to order relations in posets, while in case of Lagois connection Austin Melton[1,2] their behavior is same (either increasing or decreasing).

In this paper we introduce the notion of an *i-cover* of a set and there by establish a *Lagois connection* between the set of all i-covers of a set and the set of all tolerance relations on the same.

2. Preliminaries

In this section, some notions and results from Lattice Theory that are useful in proving some important Theorems in the coming sections are mentioned. Further, some standard notions and results from Bartol-Pioro-Rossello[3] are recalled.

2.1 Proposition

Let L be a meet complete semi lattice with the greatest element 1_L. For any subset $\phi \neq S \subseteq L$ define $\bar{\vee} S = \wedge\{\alpha \in L \mid \alpha \wedge \beta = \beta \ \forall \beta \in S\}$. Then, L is a complete lattice, where the join is given by $\bar{\vee}$.

2.2 Theorem

For any pair of posets L and M such that M is a complete lattice with 0, 1 as the least,the greatest elements respectively and for any pair of order preserving maps $\phi : L \rightarrow M$ and $\psi : M \rightarrow L$ such that $\phi \circ \psi = I_M$ and I_M is the identity map on M, $Image(\psi)$ is a complete lattice with $\psi(0)$ as the least element and $\psi(1)$ as the greatest element. In particular for any $(\beta_i)_{i\in I} \subseteq Image(\psi)$, (1) $\vee_{i\in I} \beta_i = \psi(\vee_{i\in I} \phi(\beta_i))$ and (2) $\wedge_{i\in I} \beta_i = \psi(\wedge_{i\in I} \phi(\beta_i))$.

Proof: It follows from $\phi \circ \psi = I_M$ that ϕ is onto and ψ is one-one. Now for any $(\beta_i)_{i\in I} \subseteq Image(\psi)$, $\exists\ (\alpha_i)_{i\in I} \subseteq M \ni \psi(\alpha_i) = \beta_i$. This implies $\phi(\psi(\alpha_i)) = \phi(\beta_i)$, which in turn gives $\alpha_i = \phi(\beta_i)$, $\forall i \in I$. Thus $\beta_j \leq \psi(\vee_{i\in I} \phi(\beta_i))\ \forall j \in I$. Thus $\psi(\vee_{i\in I} \phi(\beta_i))$ is an upper bound of $(\beta_i)_{i\in I} \subseteq Image(\psi)$. Further, it is easy to see that $\psi(0)$ and $\psi(1)$ are the least and the greatest elements respectively of $Image(\psi)$. Now, for $(\beta_i)_{i\in I} \subseteq Image(\psi)$, let $\beta \in Image(\psi)$ be $\ni \beta_i \leq \beta \forall i \in I$. Then it follows that $\psi(\vee_{i\in I} \phi(\beta_i)) \leq \beta$. Thus $\vee_{i\in I} \beta_i = \psi(\vee_{i\in I} \phi(\beta_i))$. Similarly, we can show that $\wedge_{i\in I} \beta_i = \psi(\wedge_{i\in I} \phi(\beta_i))$.

2.3 Definitions and Statements

(a) For any nonempty set U, a family $\mathcal{A}$ of nonempty subsets of U is called a *cover* of U, if $\bigcup_{A\in\mathcal{A}} A = U$. The set of all covers of U is denoted by $\sum(U)$.

(b) For any pair of sets U and V, a *relation* from U to V is any subset of $U \times V$. We write some times U^2 in place of $U \times U$. If R is a relation from a set U to itself, then R is a *binary relation* on U. For any nonempty set U, the relation $\{(u,u) \mid u \in U\}$ is the *identity relation* on U and is denoted by Δ_U. The set of all binary relations on U denoted by $R^2(U)$, is a meet complete semi lattice where the $\wedge$ is given by *the usual set intersection* and the underlying partial order is given by the set inclusion. The least element is ϕ and the greatest element is $U \times U$. In fact, for any set U, $R^2(U)$ is a complete lattice with the infimum and the supremum given by $\inf\mathcal{R} = \cap_{j\in J} R_j$, $\sup\mathcal{R} = \cup_{j\in J} R_j$, where $\mathcal{R} = (R_j)_{j\in J}$ is any subset of $R^2(U)$.

(c) For any nonempty set U and for any binary relation R on U: (1) R is *reflexive* iff for each $u \in U$, $(u,u) \in R$ (2) R is *symmetric* iff for each $u, v \in U$, $(u,v) \in R$ implies $(v,u) \in R$ (3) R is *transitive* iff (u,v), $(v,w) \in R$ implies $(u,w) \in R$.

(d) A binary relation R on the nonempty set U is called a *tolerance relation* or simply a *tolerance* if it is reflexive and symmetric. The set of all tolerance relations on U is denoted by $R^{tl}(U)$.

(e) For any nonempty set U, $(R^{tl}(U), \subseteq)$ is a poset with Δ_U as the least element and $U \times U$ as the greatest element. In fact $(R^{tl}(U), \cap, \cup)$ is a complete lattice with Δ_U as the least element and $U \times U$ as the greatest element. In what follows some notions and results from Bartol-Pioro-Rossello[3] are recalled.

(f) For any nonempty set U, for any tolerance relation R on U, a nonempty subset A of U is called a *T-preblock* (or simply *preblock*) of R if $A \times A \subseteq R$.

(g) For any nonempty set U, for any tolerance relation R on U, a T-preblock A of R is called a *T-block* (or simply *block*) of R if $A \subseteq B$ such that $B \times B \subseteq R$, then we must have $B = A$. A T-block of the tolerance relation R is usually denoted by RB.

(h) For any nonempty set U, for any tolerance relation R on U, a subset A of U is a T-preblock of R if and only if it is contained in some T-block of R.

3. Main Results

In this section, the notion of *i-cover* of a non empty set is introduced and a Lagois Connection between the set of all i-covers and the set of all tolerance relations is established.

3.1 Definitions and Statements

(a) For any non empty set U, a cover $\mathcal{A}$ of U is said to be an *i-cover* of U, if $A, B \in \mathcal{A}$ such that $A \neq B$ implies $A \cap B \subset B$ (or equivalently $A \subseteq B$ implies $A = B$). The set of all i-covers of U is denoted by $\sum_i(U)$.

(b) For any nonempty set U, $(\sum_i(U), \lesssim)$ is a poset with the i-cover $(u)_{u\in U}$ as the least element and the i-cover consisting of (U) alone as the greatest element, where $\lesssim$ is given by: $\mathcal{A}$, $\mathcal{B} \in \sum_i(U)$, $\mathcal{A} \lesssim \mathcal{B}$ if and only if for every $A \in \mathcal{A}$ there exists a $B \in \mathcal{B}$ such that $A \subseteq B$.

Proof: Let $\mathcal{A}_1$, $\mathcal{A}_2$ and $\mathcal{A}_3$ be any three i-covers of the L-set U.

To show that $\lesssim$ is reflexive. We know that $A \subseteq A \forall A \in \mathcal{A}_1$. Thus, $\mathcal{A}_1 \lesssim \mathcal{A}_1$. Thus, $\lesssim$ is reflexive.

To show that $\lesssim$ is antisymmetric. Let $\mathcal{A}_1 \lesssim \mathcal{A}_2$ and $\mathcal{A}_2 \lesssim \mathcal{A}_1$. Now for $A_1 \in \mathcal{A}_1$, we have $A_1 \subseteq A_2$ for some $A_2 \in \mathcal{A}_2$ because $\mathcal{A}_1 \lesssim \mathcal{A}_2$. Now for this A_2, we have $A_2 \subseteq A_{1_0}$ for some $A_{1_0} \in \mathcal{A}_1$ because $\mathcal{A}_2 \lesssim \mathcal{A}_1$. This implies $A_1 \subseteq A_{1_0}$. This implies $1 = 1_0$, otherwise $A_1 = A_1 \cap A_{1_0}$ which is a contradiction. This implies $A_2 = A_1$. Thus, $\mathcal{A}_1 \subseteq \mathcal{A}_2$. Similarly we can show that $\mathcal{A}_2 \subseteq \mathcal{A}_1$. Thus, $\mathcal{A}_1 = \mathcal{A}_2$. Thus, $\lesssim$ is antisymmetric.
To show that $\lesssim$ is transitive. Let $\mathcal{A}_1 \lesssim \mathcal{A}_2$ and $\mathcal{A}_2 \lesssim \mathcal{A}_3$. Now for $A_1 \in \mathcal{A}_1$, we have $A_1 \subseteq A_2$ for some $A_2 \in \mathcal{A}_2$ because $\mathcal{A}_1 \lesssim \mathcal{A}_2$. Now for this $A_2 \in \mathcal{A}_2$, we have $A_2 \subseteq A_3$ for some $A_3 \in \mathcal{A}_3$ because $\mathcal{A}_2 \lesssim \mathcal{A}_3$. This implies for $A_1 \in \mathcal{A}_1$, we have $A_1 \subseteq A_3$ for some $A_3 \in \mathcal{A}_3$. This implies $\mathcal{A}_1 \lesssim \mathcal{A}_3$. Thus, $\lesssim$ is transitive.

It is clear that the i-cover $(u)_{u\in U}$ is the least element the i-cover consisting of U alone is the greatest element of $\sum_i(U)$. Thus, $(\sum_i(U), \lesssim)$ is a poset with the i-cover $(u)_{u\in U}$ as the least element and with the i-cover consisting of U alone, namely (U) as the greatest element.

The above relation *need not* be a partial order when it is defined on $\sum(U)$ as can be seen in the following example.

Example: For the set $U = \{u, v\}$ and for the covers $\mathcal{A} = \{\{u\}, U\}$, $\mathcal{B} = \{U\}$ of U, we have $\mathcal{A} \lesssim \mathcal{B}$ and $\mathcal{B} \lesssim \mathcal{A}$ but $\mathcal{A} \neq \mathcal{B}$. Further observe that $\mathcal{B}$ is an i-cover while $\mathcal{A}$ is a mere cover but not an i-cover.

(c) For any nonempty set U, an i-cover $\mathcal{A}$ of U is said to be an *s-cover* of U, if $B \subset U$ such that every pair of elements of B belongs to some member of $\mathcal{A}$ then we must have $B \subseteq A$ for some $A \in \mathcal{A}$. The set of all s-covers of U is denoted by $\sum_s(U)$. However an i-cover *need not* be an s-cover as can be seen in the following example.

Example: For the set $U = \{u, v, w, x\}$, the cover $\mathcal{A} = \{\{u, v\}, \{v, w\}, \{u, w\}, \{x\}\}$ is an i-cover but not an s-cover.

(d) For any nonempty set U, for any tolerance relation R on U, the set of all T-preblocks (T-blocks) of R, denoted by $\mathcal{C}(R)$ ($\mathcal{C}_i(R)$), is a cover (an i-cover) of U.

Proof: First notice that both $\cup_{B\in\mathcal{C}(R)} B$ and $\cup_{B\in\mathcal{C}_i(R)} B$ are subsets of U always. For $u \in U$,we have $(u,u) \in R$ because R is reflexive. Thus $\{u\}$ is a T-preblock for R. This implies $u \in\cup_{B\in\mathcal{C}(R)} B$. Now by 2.3.(h), there exists a T-block RB_1 for R such that $\{u\} \subseteq RB_1$. Thus $u \in\cup_{B\in\mathcal{C}(R)} B$ and $u \in\cup_{RB\in\mathcal{C}_i(R)} RB$. Therefore, we have $U = \cup_{B\in\mathcal{C}(R)} B = \cup_{RB\in\mathcal{C}_i(R)} RB$. Further, it is easy to see that RB_1, $RB_2 \in \mathcal{C}_i(R) \ni RB_1 \subseteq RB_2$ implies $RB_1 = RB_2$. Thus $\mathcal{C}(R)$ and $\mathcal{C}_i(R)$ are cover and i-cover respectively for U.

(e) For any tolerance relation R on U, $\mathcal{C}(R)$ $(\mathcal{C}_i(R))$, is called the *associated* cover (i-cover) for R.

(f) For any nonempty set U, for any tolerance relation R on U, we have $(u,v) \in R$ if and only if $u,\ v \in A$ for some $A \in \mathcal{C}_i(R)$. In fact $\mathcal{C}_i(R)$ is an s-cover of U.

(g) For any nonempty set U, for any tolerance relation R on U and for any $u \in U$, the *T-class* of u for R, denoted by uR is defined by $uR = \{v \in U | (u,v) \in R\}$. For any nonempty set U, for any tolerance relation R on U, the family of T-classes for R is denoted by $\mathcal{T}(R)$.

(h) For any nonempty set U, for any tolerance relation R on U, the family of T-classes $\mathcal{T}(R)$ is a cover. How ever, the family of T-classes $\mathcal{T}(R)$ for a tolerance relation R on a nonempty set U *need not* be an i-cover as can be seen in the following example:

Example: For the set $U = \{u,v,w,x\}$, for the tolerance relation R on U given by: $\begin{array}{c} \\ u \\ v \\ w \\ x \end{array}\left[\begin{array}{cccc} u & v & w & x \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right]$, we have $\mathcal{T}(R) = \{U, \{u,v,w\}, \{u,x\}\}$, which is clearly not an i-cover. Further $uR = U$ and $U \times U \supset R$. Thus a T-class of a tolerance relation *need not* be even a T-preblock.

(i) For any nonempty set U, for any tolerance relation R on U, the following are true:

- $uR = \cup_{B\in\mathcal{C}_i(R)\ni u\in B} B$, where $u \in U$
- $B = \cap_{u\in B} uR$, where $B \in \mathcal{C}_i(R)$

(j) For any cover (i-cover) $\mathcal{A}$ of a nonempty set U, $\cup_{A\in\mathcal{A}} A \times A$ is a tolerance relation on U and is denoted by $T(\mathcal{A})$.

(k) For any cover (i-cover) $\mathcal{A}$ of a nonempty set U, $T(\mathcal{A})$ is called the *associated* tolerance relation for $\mathcal{A}$.

3.2 Lagois Connection between The set of all i-covers and The set of all Tolerance Relations:

For any nonempty set U, define ϕ: $\sum_i(U) \to R^{tl}(U)$ by $\phi(\mathcal{A}) = T(\mathcal{A})$, the associated tolerance relation for $\mathcal{A}$ and ψ: $R^{tl}(U) \to \sum_i(U)$ by $\psi(R) = \mathcal{C}_i(R)$, the associated i-cover for R. Then, the following are true:

1) $\mathcal{A}_1 \leq \mathcal{A}_2 \Rightarrow \phi(\mathcal{A}_1) \subseteq \phi(\mathcal{A}_2)$
2) $R_1 \subseteq R_2 \Rightarrow \psi(R_1) \leq \psi(R_2)$
3) $I_{\Sigma_i(U)} \leq \psi \circ \phi$
4) $I_{R^{tl}(U)} = \phi \circ \psi$
5) Image$(\psi) = \sum_s(U)$
6) $\sum_s(U)$ is a complete lattice with $\phi(\Delta_U)$ as the least element and $\phi(U \times U)$ as the greatest element.
Further for any family of s-covers $(\mathcal{A}_i)_{i\in I}$ of U we have, $\wedge_{i\in I} \mathcal{A}_i = \psi(\cap_{i\in I} \phi\mathcal{A}_i)$ and $\vee_{i\in I} \mathcal{A}_i = \psi(\cup_{i\in I} \phi\mathcal{A}_i)$.
7) $\phi \circ \psi \circ \phi = \phi$
8) $\psi \circ \phi \circ \psi = \psi$

Proof: (1): Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be any two i-covers for U such that $\mathcal{A}_1 \leq \mathcal{A}_2$. Let $R_1 = T(\mathcal{A}_1)$ and $R_2 = T(\mathcal{A}_2)$. Of course both R_1 and R_2 are tolerance relations by 3.1(j) above. Now for $(u,v) \in R_1$, we have $(u,v) \in A \times A$ for some $A \in \mathcal{A}_1$. This implies $\exists\ B \in \mathcal{A}_2$ such that $A \subseteq B$ because $\mathcal{A}_1 \leq \mathcal{A}_2$. Thus $(u,v) \in B \times B$. Now it follows that $R_1 \subseteq R_2$.

(2): let R_1 and R_2 be any two tolerance relations on U. Let $\mathcal{A}_1 = \mathcal{C}_i(R_1)$ and $\mathcal{A}_2 = \mathcal{C}_i(R_2)$. Of course both $\mathcal{A}_1$ and $\mathcal{A}_2$ are both i-covers for U by 3.1(d) above. Now let R_1B be a T-block of R_1. Clearly $R_1B \in \mathcal{A}_1$. Clearly $R_1B \times R_1B \subseteq R_1$ by definition of $\mathcal{C}_i(R_1)$. But $R_1B \times R_1B \subseteq R_2$ because $R_1 \subseteq R_2$. Thus R_1B is a T-preblock of R_2. Now $\exists\ R_2B \in \mathcal{A}_2$ such that $R_1B \subseteq R_2B$ by 2.3(h) above. Thus $\mathcal{A}_1 \leq \mathcal{A}_2$.

(3): Let $\mathcal{A}$ be an i-cover for U. Let $R = T(\mathcal{A})$. Clearly R is a tolerance relation by 3.1(j) above and A is a T-preblock of R $\forall A \in \mathcal{A}$. Fix $A \in \mathcal{A}$. Now $\exists\ B \in \mathcal{C}_i(R)$ such that $A \subseteq B$ by 2.3(h) above. Thus $\mathcal{A} \leq \mathcal{C}_i(R)$.

(4): It is enough to show that $R = T(\mathcal{C}_i(R))$ for any tolerance relation R on U. Let $(u,v) \in R$. This implies $\{u,v\} \subseteq RB$ for some T-block RB of R by 2.3(h) above. This implies $(u,v) \in RB \times RB$. Thus $(u,v) \in T(\mathcal{C}_i(R))$.

On the other hand $(u,v) \in T(\mathcal{C}_i(R))$ implies $(u,v) \in B \times B$ for some $B \in \mathcal{C}_i(R)$ by 3.1(f) above. This implies $u,v \in B$. Thus $(u,v) \in R$.

(5): Clear.

(6): Follows from Theorem 2.2.

(7): $\phi \circ \psi \circ \phi \supseteq \phi$ is obvious because of (3). Let $\mathcal{A}$ be an i-cover for U. It is enough to show that $T(\mathcal{C}_i(T(\mathcal{A}))) \subseteq T(\mathcal{A})$. Let $(u,v) \in T(\mathcal{C}_i(T(\mathcal{A})))$. This implies $\{u,v\} \subseteq B$ for some $B \in \mathcal{C}_i(T(\mathcal{A}))$ by 3.1(f) above. This $(u,v) \in T(\mathcal{A})$ by 3.1(f) above.

(8): Follows from (4).
Strict inequality in (3) of the above theorem can hold as shown in the following example:

Example: Let the set $U = \{u,v,w\}$, the i-cover $\mathcal{A} = \{\{u,v\},\{v,w\},\{w,u\}\}$. Now the associated tolerance relation $T(\mathcal{A})$ for the i-cover $\mathcal{A}$ is given by $U \times U$. Now the associated i-cover for $T(\mathcal{A})$, is given by $\mathcal{C}_i(T(\mathcal{A})) = \{U\}$. Thus it is clear that $\mathcal{A} < \mathcal{C}_i(T(\mathcal{A}))$.

3.3 Remarks

We recall that (1) a family $\mathcal{A}$ of non empty subsets of a non empty set U, is called a *partition* for U, if (a) $\bigcup_{A\in\mathcal{A}} A = U$ and (b) $A, B \in \mathcal{A}$ such that $A \neq B$ implies A, B are disjoint (2) a transitive tolerance on a non empty set U is an equivalence relation.

For an equivalence relation R on a non empty set U, uR denotes an equivalence class containing u for R. We know that any two equivalence classes for R on a nonempty set U are either same or disjoint and the set of all equivalence classes of R constitutes a partition of U

For any non empty set U, the set of all partitions of U is denoted by $\Pi(U)$ and the set of all equivalence relations on U is denoted by $R^{eq}(U)$.

For any non empty set U, the set of all partitions $\Pi(U)$ is a sub poset (cf. 3.1 (b)) of $\sum_i (U)$ and the set of all equivalence relations $R^{eq}(U)$ is a complete sub lattice (cf. 2.3 (e)) of $R^{tl}(U)$.

It is easy to see that (1) every partition of a nonempty set U is an s-cover of U and (2) for any non empty set U, for any partition $\mathcal{A}$ of U, the associated tolerance relation $T(\mathcal{A})$ (for the partition $\mathcal{A}$) is an equivalence relation on U (3) for any non empty set U, for any transitive tolerance R, the associated i-cover $\mathcal{C}_i(R)$ is a partition of U and there is a bijective correspondence (by 3.1 (i)) between the set of all T-blocks $\mathcal{C}_i(R)$ and the set of all T-classes $\mathcal{T}(R)$ of R.

Now it is easy to see that, in 3.2, when ϕ and ψ are restricted to $\Pi(U)$ and $R^{eq}(U)$ respectively, conclusion 3 becomes an equality (by 3.1(i)), and as a consequence Image(ϕ) is realized as $R^{eq}(U)$ and further Image(ψ) is any way $\Pi(U)$.

4. Conclusions

In this paper we represented the poset of all tolerances on a set by certain type of covers on the same via a Lagois connection. Further, we also realized that this connection is the natural generalization of the complete isomorphism between the set of all equivalence relations on a non empty set and the set of all partitions of the same set.

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