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Spherically Symmetrical Cosmological Model in Brans Dicke Theory of Gravitation with Anisotropic Dark Energy

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Abstract: ***Spherically symmetrical cosmological models are examined in the presence of Brans Dicke theory of gravity with anisotropy dark energy. We use the power law relation between scalar field and scalar factor to find the solution. Exact solutions of Einstein's field equations are obtained by assuming a special law of variation for the Hubble parameter, which yields a constant value of the deceleration parameter. Geometric and kinematic properties of the models and the behaviour of the dark energy have been carried out.***

Keywords: Dark Energy and Brans Dicke theory of gravity.

1. Introduction

One of the most successful attempt to resolve the problems of standard Big Bang cosmology such as homogeneity, isotropy, and flatness of the universe is the inflationary paradigm, characterised by an epoch of accelerated expansion, "inflation", in the very early universe [1–4]. During the inflationary epoch, quantum fluctuations are highly amplified, their wavelengths are stretched to outside the Hubble horizon and inevitably, superhorizon fluctuations are generated. These fluctuations become classical after crossing the event horizon and are coherent on what appear to be superhorizon scales at decoupling [5–14]. After the end of inflation, they re-enter the horizon, and seed the matter and the radiation fluctuations observed in the universe. These primordial fluctuations are Gaussian, adiabatic and nearly scale-invariant in the majority of inflation models and even a slight deviation from these properties can strongly constrain the assumptions in cosmological models [13,14].

A spatially ellipsoidal geometry of the universe can be described with Bianchi type metrics. However, Bianchi type I, V, VII models isotropize at late times even for ordinary matter and the possible anisotropy of the Bianchi metrics necessarily dies away during the inflationary era [15,16]. In fact this isotropization of the Bianchi metrics is due to the implicit assumption that the Dark energy (DE) is isotropic in nature. If the implicit assumption that the pressure of the DE is direction independent is relaxed, the isotropization of the Bianchi metrics can be fine tuned to generate arbitrary ellipsoidality (eccentricity). Therefore, the cosmic microwave background (CMB) anisotropy can also be fine tuned, since the Bianchi universe anisotropies determine the CMB anisotropies. The price of this property of DE is a violation of the null energy condition (NEC) since the DE crosses the Phantom Divide Line (PDL), in particular depending on the direction [17].

Recently, Rodrigues [17] and Koivisto & Mota [18,19] have investigated cosmological models with anisotropic equation of state (EoS). Rodrigues has constructed a Bianchi type-I ^-dominated cold dark matter (^-CDM) cosmological model with a DE component which is non dynamical but yields anisotropic vacuum pressure in two ways: (i) by implementing of anisotropic vacuum pressure consistent with energy-momentum tensor conservation; (ii) by implementing a Poisson structure deformation between canonical momenta such that rescaling of the scale factor is not violated [17]. He suggests to fine tune the DE so as to not wipe out the anisotropic imprints in the inflationary epoch. On the other hand, Koivisto & Mota have proposed a different approach to resolve CMB anisotropy problem; even if the CMB formed isotropically at early time, it could be distorted by the direction dependent acceleration of the later universe in such a way that it appears to us anomalous at the largest scales. They have investigated a cosmological model containing a DE component which has a non-dynamical anisotropic EoS and interacts with the perfect fluid component. They have also suggested that cosmological models with anisotropic EoS can explain the quadrupole problem and can be tested by SNIa data [18,19].

Recently, Mota et al. [20] have concluded that even though a perfect fluid representation might ultimately turn out to be a phenomenologically sufficient description of all the observational consequences of DE, imperfectness in DE cannot be excluded . Although there is compelling evidence that the expansion of universe is speeding up, we are far from understanding of the nature of the DE which is thought be the reason for this behaviour [21-23]. Hence, we should examine models with anisotropic dark energy, in order to determine what possible new physical consequences they might give rise to, and if for no other reason than to rule such models out.

The experimental data [24-26] about late time acceleration expansion of the universe has attracted much attention in the recent years. Cosmic acceleration can be well explained from high red- shift supernova experiments. The results from cosmic microwave background fluctuation [27] and large scale structure [28] suggest the expansion of universe. Dark energy is the most popular way to explain the observation that the universe is expanding at an acceleration rate. The exact nature of the dark energy is a matter of speculation. It is known to interact through any of the fundamental forces other than gravity.. Since it is not very dense, roughly 10-29 grams per cubic centimetre, so it is not difficult task to detect it in the laboratory. It is thought that dark energy have a strong negative pressure in order to explain the observed acceleration in the expansion rate of the universe.

Dark energy models have significant importance now as far as theoretical study of the universe is concerned. It would be interesting to study the variable equation of state (EoS), i.e. $P=\rho\omega(t)$, where *P* is the pressure and ρ is the energy density of universe. Usually EoS parameter is assumed to be a constant with the values $-1,\ 0,-\frac{1}{3}\ and\ +1$ for vacuum, dust, radiation and stiff matter dominated universe, respectively. However, it is a function of time or redshift [29] in general.

The isotropic models are considered to be most suitable to study large scale structure of the universe. However, it is believed that the early universe may not have been exactly uniform. This prediction motivates us to describe the early stages of the universe with models having anisotropic background. Thus, it would be worthwhile to explore anisotropic dark energy models in the context of modified theories of gravity. Among the various modifications of general relativity (GR), Brans- Dicke (BD) theory of gravity [30] is a well known example of a scalar tensor theory in which the gravitational interaction involves a scalar field and the metric tensor. One extra parameter ϖ is used in this theory which satisfies the equation given by

$$\Delta\phi=\frac{8\pi T}{3+2\varpi},$$

Where ϕ is known as BD scalar field while T is the trace of the matter energy- momentum tensor. It is mentioned here that the general relativity is recovered in the limiting case $\varpi\to\infty$. Thus we can compare our results with experimental tests for significantly large value of ϖ.

Recently Prandha and Amirhashchi [31] investigated anisotropic dark energy Bianchi type III model with variable EoS parameter in GR. Kumar and Singh [32] explored perfect fluid solution using Bianchi type I space-time in scalar- tensor theory.

In this paper, we have discussed the solutions of anisotropic dark energy spherically symmetric cosmological model in the presence of Brans- Dicke theory of gravitation. We find the solutions using the assumption of constant deceleration parameter and law relation between ϕ and *R*.

2. Models and Field Equations

The line element for the spatially homogenous and anisotropic Spherically symmetric space time is given by

$$ds^2=-dt^2+a^2dr^2+b^2d\Omega^2 \tag{1}$$

where *a and* b are function of *t* only. The energy momentum tensor for anisotropic dark energy is given by

$$T^i{}_j=dia[-p,p_x,p_y,p_z]=diag[-1,\omega_x,\omega_y,\omega_z]\rho \tag{2}$$

where ρ is the energy density of the fluid while p_x , p_y, p_z are the pressures on the $x,\ y,\ z$ respectively. Here ω is EoS parameter of the fluid with no deviation and $\omega_x,\ \omega_y,\ \omega_z$ are the EoS parameters in the directions of *x*,*y* and *z* axes respectively.

The energy momentum tensor can be parameterized as

$$T^i{}_j=diag[-1,\omega,(\omega+\delta),(\omega+\delta)]\rho \tag{3}$$

For the simplification, we choose $\omega_x=\omega$ and the skewness parameter δ is the deviations from ω on *y and z* axes respectively.

The Brans- Dicke field equations are

$$R^i{}_j-\frac{1}{2}Rg_{ij}-\frac{\varpi}{\phi^2}(\phi,_i\phi,_j-\frac{1}{2}g_{ij}\phi,_k\phi^{,k})-\frac{1}{\phi}(\phi,_{ij}-g_{ij}\Delta\phi)=\frac{8\pi T_{ij}}{\phi} \tag{4}$$

And

$$\Delta\phi=\phi_{;k}{}^{,k}=\frac{8\pi T}{3+2\varpi} \tag{5}$$

where ϖ is a dimensionless coupling constant.

For the line element (1), the field equations (4) leads to the following system of equation

$$\frac{2\dot{a}\dot{b}}{ab}+\frac{1}{b^2}+\frac{\dot{b}^2}{b^2}-\frac{\varpi}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+\frac{\dot{\phi}}{\phi}\left(2\frac{\dot{b}}{b}+\frac{\dot{a}}{a}\right)=\frac{8\pi\rho}{\phi} \tag{6}$$

$$\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}+\frac{\dot{a}\dot{b}}{ab}+\frac{\varpi}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{a}}{a}+\frac{\dot{b}}{b}\right)=-\frac{8\pi(\omega+\delta)\rho}{\phi} \tag{7}$$

$$\frac{2\ddot{b}}{b}+\frac{1}{b^2}+\frac{\dot{b}^2}{b^2}+\frac{\ddot{\phi}}{\phi}\frac{\varpi}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+2\frac{\dot{\phi}\dot{b}}{\phi b}=-\frac{8\pi\omega\rho}{\phi} \tag{8}$$

Using equation (5), we get

$$\frac{\ddot{\phi}}{\phi}+\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}=\frac{8\pi(1-3\omega-2\delta)\rho}{\phi(3+2\varpi)} \tag{9}$$

The physical quantities that are of importance in cosmology are expansion scalar θ and shear scalar σ^2 and have the following expression for the metric

$$\theta=\frac{\dot{a}}{a}+2\frac{\dot{b}}{b} \tag{10}$$

$$\sigma^2=\frac{2}{3}\left(\frac{\dot{b}}{b}-\frac{\dot{a}}{a}\right)^2 \tag{11}$$

The average scale factor R and the volume scale factor V are defined

$$R=\sqrt[3]{ab^2},\qquad V=R^3=ab^2 \tag{12}$$

The generalized mean Hubble parameter H is given in the form

$$H=\frac{\dot{R}}{R}=\frac{1}{3}\frac{\dot{V}}{V}=\frac{1}{3}(H_1+H_2+H_3) \tag{13}$$

The directional Hubble parameters in the directions of *x*, *y* and *z* respectively may be defined as

$$H_1 = \frac{\dot{a}}{a};\ H_2 = H_3 = \frac{\dot{b}}{b} \tag{14}$$

3. Solution of the Field Equations

Integration after subtracting equation (8) and (7) , we get

$$\left(\frac{\dot{b}}{b} - \frac{\dot{a}}{a}\right) b^2 a\phi = \exp\left[\int \left(\frac{8\pi\delta\rho}{\phi} - \frac{1}{b^2}\right)\left(\frac{\dot{b}}{b} - \frac{\dot{a}}{a}\right)^{-1} dt + \log\lambda\right] \tag{15}$$

where λ is an integration constant. The integral term in above equation vanishes for

$$\delta = \frac{\phi}{8\pi\rho b^2} \tag{16}$$

Using equation (16) in equation (15), it follows that

$$\frac{b}{a} = c_2 \exp\left[\int \frac{\lambda}{R^3\phi} dt\right] \tag{17}$$

Where $R^3 = ab^2$ and c_2 is an integration constant. Here we use the power law assumption to solve the integral part in the above equations . The power law relation between scale factor *R*

And scalar field ϕ has already been used by Johri and Desikan [33] in the context of Robertson Walker Brans-Dicke models. Thus the power law relation between ϕ *and* R is $\phi \propto R^m$, where *m* is any integer, implies that

$$\phi = BR^m \tag{18}$$

where *B* is the constant of proportionality. The deceleration parameter *q* in cosmology is the measure of the cosmic acceleration of the universe expansion and is defined as

$$q = -\frac{R\ddot{R}}{\dot{R}^2} \tag{19}$$

It is mentioned here that *q* was supposed to be positive initially but recent observations from the supernova experiments suggest that it is negative. Thus the behaviour of the universe models depend upon the sign of *q* . The positive deceleration parameter corresponds to a decelerating model while the negative value provides inflation. We also use a well – known relation[34] between the average Hubble parameter *H* and average scale factor *R* given as

$$H = lR^{-n} \tag{20}$$

where $l > 0$ *and* $n \geq 0$. This is an important relation because it gives the constant value of the deceleration parameter. From equation (13) and (20), we get

$$\dot{R} = lR^{-n+1} \tag{21}$$

Using this value, we find that deceleration parameter is constant i.e. $q = n - 1$. Integrating equation (21), it follows that

$$R = (nlt + k_1)^{\frac{1}{n}},\ n \neq 0 \tag{22}$$

$$\text{And } R = k_2 \exp(lt) \quad , \quad n = 0 \tag{23}$$

where k_1 *and* k_2 are constants of integration. Here we obtain two values of the average scale factor that correspond to two different models of the universe.

I) Dark Energy Model of the Universe when $n \neq 0$ and $m = -2$

Now we discuss the model of universe when $n \neq 0$ i. e. $R = (nlt + k_1)^{\frac{1}{n}}$. For this model, ϕ becomes

$$\phi = B(nlt + k_1)^{\frac{-2}{n}} \tag{24}$$

Using this value of ϕ in equation (17) ,we get

$$b = c_2^{\frac{1}{3}} (nlt + k_1)^{\frac{1}{n}} \exp\left[\frac{\lambda (nlt + k_1)^{\frac{n-1}{n}}}{3lB(n-1)}\right] \quad , \quad n \neq 1 \tag{25}$$

$$a = c_2^{-\frac{2}{3}} (nlt + k_1)^{\frac{1}{n}} \exp\left[\frac{-2\lambda (nlt + k_1)^{\frac{n-1}{n}}}{3lB(n-1)}\right] \quad , \quad n \neq 1 \tag{26}$$

The directional Hubble parameter H_i $(i = 1,2,3)$ take the form

$$H_1 = \frac{l}{nlt + k_1} - \frac{2}{3}\frac{\lambda}{B(nlt + k_1)^{\frac{1}{n}}} \tag{27}$$

$$H_2 = H_3 = \frac{l}{nlt + k_1} + \frac{1}{3}\frac{\lambda}{B(nlt + k_1)^{\frac{1}{n}}} \tag{28}$$

The mean generalized Hubble parameter becomes

$$H = \frac{l}{nlt + k_1} \tag{29}$$

While the volume scale factor turns out to be

$$V = (nlt + k_1)^{\frac{3}{n}} \tag{30}$$

The expansion scalar θ and shear scalar σ take the form

$$\theta = \frac{3l}{nlt + k_1} \tag{31}$$

$$\sigma^2 = \frac{2\lambda^2}{3B^2 (nlt + k_1)^{\frac{2}{n}}} \tag{32}$$

The anisotropy parameter of the expansion Δ is defined as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{33}$$

where H_i $(i = 1,2,3)$ represents the directional Hubble parameters in the directions of *x, y and z* respectively. By using (27) (28) (29) in (33) we get

$$\Delta = \frac{2\lambda^2}{9B^2}(nlt + k_1)^{2-\frac{2}{n}} \tag{34}$$

Using equations(7) to (9) , we obtained

$$\frac{8\pi\rho}{\phi} = -l^2(3+2\omega)(nlt+k_1)^{-2} - \left\{\frac{\lambda^2}{3B^2} - \frac{1}{c_2^{2/3}}\exp\left(-\frac{2\lambda(nlt+k_1)^{\frac{n-1}{n}}}{3B(n-1)l}\right)\right\}(nlt+k_1)^{\frac{-2}{n}} \quad (35)$$

$$\frac{-8\pi(\omega+\delta)\rho}{\phi} = l^2\left[(3+2n+3\omega)(nlt+k_1)^{-2} + \frac{\lambda^2}{3B^2}(nlt+k_1)^{-2/n}\right] \quad (36)$$

$$\frac{-8\pi\omega\rho}{\phi} = l^2(3+2n+2\varpi)(nlt+k_1)^{-2} + \left[\frac{\lambda^2}{3B^2} + \frac{1}{c_2^{2/3}}\exp\left(\frac{-2\lambda(nlt+k_1)^{n-1/n}}{3B(n-1)}\right)\right](nlt+k_1)^{-\frac{2}{n}} \quad (37)$$

In this case the EoS parameter ω becomes

$$\omega = \frac{l^2(3+2n+2\varpi)(nlt+k_1)^{-2} + \left[\frac{\lambda^2}{3B^2} + \frac{1}{c_2^{2/3}}\exp\left(\frac{-2\lambda(nlt+k_1)^{n-1/n}}{3B(n-1)}\right)\right](nlt+k_1)^{\frac{-2}{n}}}{l^2(3+2\varpi)(nlt+k_1)^{-2} + \left[\frac{\lambda^2 l^2}{3B^2} - \frac{1}{c_2^{\frac{2}{3}}}\exp\left(\frac{-2\lambda(nlt+k_1)^{\frac{n-1}{n}}}{3B(n-1)}\right)\right](nlt+k_1)^{\frac{-2}{n}}} \quad (38)$$

II) Dark Energy Model of the Universe when $n = 0$ and $m = -2$

The average scale factor for this model of the universe is $R = k_2 e^{lt}$ and hence ϕ takes the form

$$\phi = \frac{B}{k_2^{\,2}}\exp(-2lt) \quad (39)$$

Using this value of ϕ in equation (17), we get the following exact expressions for the scale factors

$$b = c_2^{\frac{1}{3}} k_2 \exp(lt)\exp\left[\frac{-\lambda\exp(-lt)}{3Blk_2}\right] \quad (40)$$

$$a = c_2^{-\frac{2}{3}} k_2 \exp(lt)\exp\left[\frac{2\lambda\exp(-lt)}{3Blk_2}\right] \quad (41)$$

The directional Hubble parameters as defined in (14) are found as

$$H_1 = l - \frac{2\lambda\exp(-lt)}{3Bk_2} \quad (42)$$

$$H_2 = H_3 = l + \frac{\lambda\exp(-lt)}{3Blk_2} \quad (43)$$

From the equations (12) and (13), the mean generalised Hubble parameter and the volume scale factor becomes

$$H = l \qquad V = k_2^{\,3}\exp(3lt) \quad (44)$$

The expansion scalar θ and shear scalar σ are found as

$$\theta = 3l \qquad \sigma^2 = \frac{2\lambda^2\exp(-2lt)}{3B^2k_2^{\,2}} \quad (45)$$

The mean anisotropy parameter Δ for this model yields

$$\Delta = \frac{2\lambda^2\exp(-2lt)}{9B^2l^2k_2^{\,2}} \quad (46)$$

For this model , equations(7) to (9) becomes

$$\frac{8\pi\rho}{\phi} = -l^2(3+2\varpi) - \frac{\lambda^2\exp(-2lt)}{3B^2k_2^{\,2}} + \frac{1}{c_2^{\frac{2}{3}}k_2}\exp\left(-2lt + \frac{2\lambda\exp(-lt)}{3Blk_2}\right) \quad (47)$$

$$-\frac{8\pi(\omega+\delta)\rho}{\phi} = l^2(3+2\varpi) + \frac{\lambda^2\exp(-2lt)}{3B^2k_2^{\,2}} \quad (48)$$

$$-\frac{8\pi\omega\rho}{\phi} = l^2(3+2\varpi) + \frac{\lambda^2\exp(-2lt)}{3B^2k_2^{\,2}} + \frac{1}{c_2^{\,2}k_2^{\,2}}\exp\left(-2lt + \frac{2\lambda\exp(-2lt)}{3Blk_2}\right) \quad (49)$$

In this case the EoS parameter ω becomes

$$\omega = \frac{l^2(3+2\varpi) + \frac{\lambda^2\exp(-2lt)}{3B^2k_2^{\,2}} + \frac{1}{c_2^{\,2}k_2^{\,2}}\exp\left(-2lt + \frac{2\lambda\exp(-2lt)}{3Blk_2}\right)}{l^2(3+2\varpi) + \frac{\lambda^2\exp(-2lt)}{3B^2k_2^{\,2}} - \frac{1}{c_2^{\frac{2}{3}}k_2^{\,2}}\exp(-2lt + \frac{2\lambda\exp(-lt)}{3Bk_2})} \quad (50)$$

4. Conclusion

In this paper we have discussed the spherically symmetric cosmological models in Brans Dicke theory of gravitation in the background of anisotropic dark energy. The assumption of constant deceleration parameter leads to two models of universe .

First we discussed power law model of the universe. This model corresponds to $n \neq 0$ with average scale factor $R = (nlt + k_1)^{\frac{1}{n}}$ .It has a point singularity at $t = -\frac{k_1}{nl}$. The physical parameters $H_{1,} H_2, H_3$ *and* H are infinite at this point but here the spatial volume vanishes. The metric function *a*, and *b* vanish at $t = -\frac{k_1}{nl}$ Thus , it is concluded from these observations that the model starts its expansion with zero volume at $t = -\frac{k_1}{nl}$ and it continues to expand for $0 < n < 1$.

The exponential model of the universe corresponds to $n = 0$ with average scale factor $R = k_2 \exp(lt)$. It is non-singular because exponential function is never zero and hence there does not exist any physical singularity for this model. The Hubble parameters $H_{1,} H_2, H_3$ are all finite values of *t*. The mean generalized Hubble parameter *H* is constant while metric functions *a,b* do not vanish for this model. The volume scale factor increases exponentially with time which indicates that the universe starts its expansion with zero volume from infinite past

The isotropy condition, $\frac{\sigma^2}{\theta} \rightarrow 0$ *as* $t \rightarrow 0$ in both cases.

The variable EoS parameter ω has a positive value at $t = 0$ which indicated that the universe was matter dominated in its early phase of its existence. At $t \rightarrow \infty$, $\omega \rightarrow 0$ which indicate that the pressure of the universe vanishes .

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