

fundamental forces other than gravity.. Since it is not very dense, roughly 10-29 grams per cubic centimetre, so it is not difficult task to detect it in the laboratory. It is thought that dark energy have a strong negative pressure in order to explain the observed acceleration in the expansion rate of the universe.

Dark energy models have significant importance now as far as theoretical study of the universe is concerned. It would be interesting to study the variable equation of state (EoS), i.e. $P = \rho\omega(t)$, where P is the pressure and ρ is the energy density of universe. Usually EoS parameter is assumed to be a constant with the values $-1, 0, -\frac{1}{3}$ and $+1$ for vacuum, dust, radiation and stiff matter dominated universe, respectively. However, it is a function of time or redshift [29] in general.

The isotropic models are considered to be most suitable to study large scale structure of the universe. However, it is believed that the early universe may not have been exactly uniform. This prediction motivates us to describe the early stages of the universe with models having anisotropic background. Thus, it would be worthwhile to explore anisotropic dark energy models in the context of modified theories of gravity. Among the various modifications of general relativity (GR), Brans- Dicke (BD) theory of gravity [30] is a well known example of a scalar tensor theory in which the gravitational interaction involves a scalar field and the metric tensor. One extra parameter ϖ is used in this theory which satisfies the equation given by

$$\Delta\phi = \frac{8\pi T}{3 + 2\varpi},$$

Where ϕ is known as BD scalar field while T is the trace of the matter energy- momentum tensor. It is mentioned here that the general relativity is recovered in the limiting case $\varpi \rightarrow \infty$. Thus we can compare our results with experimental tests for significantly large value of ϖ .

Recently Prandha and Amirhashchi [31] investigated anisotropic dark energy Bianchi type III model with variable EoS parameter in GR. Kumar and Singh [32] explored perfect fluid solution using Bianchi type I space-time in scalar- tensor theory.

In this paper, we have discussed the solutions of anisotropic dark energy spherically symmetric cosmological model in the presence of Brans- Dicke theory of gravitation. We find the solutions using the assumption of constant deceleration parameter and law relation between ϕ and R .

2. Models and Field Equations

The line element for the spatially homogenous and anisotropic Spherically symmetric space time is given by

$$ds^2 = -dt^2 + a^2 dr^2 + b^2 d\Omega^2 \quad (1)$$

where a and b are function of t only. The energy momentum tensor for anisotropic dark energy is given by

$$T^i_j = \text{diag}[-p, p_x, p_y, p_z] = \text{diag}[-1, \omega_x, \omega_y, \omega_z]\rho \quad (2)$$

where ρ is the energy density of the fluid while p_x, p_y, p_z are the pressures on the x, y, z respectively. Here ω is EoS parameter of the fluid with no deviation and $\omega_x, \omega_y, \omega_z$ are the EoS parameters in the directions of x, y and z axes respectively.

The energy momentum tensor can be parameterized as

$$T^i_j = \text{diag}[-1, \omega, (\omega + \delta), (\omega + \delta)]\rho \quad (3)$$

For the simplification, we choose $\omega_x = \omega$ and the skewness parameter δ is the deviations from ω on y and z axes respectively.

The Brans- Dicke field equations are

$$R^i_j - \frac{1}{2}Rg_{ij} - \frac{\varpi}{\phi^2}(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}) - \frac{1}{\phi}(\phi_{,ij} - g_{ij}\Delta\phi) = \frac{8\pi T^i_j}{\phi} \quad (4)$$

And

$$\Delta\phi = \phi_{,k}^{,k} = \frac{8\pi T}{3 + 2\varpi} \quad (5)$$

where ϖ is a dimensionless coupling constant. For the line element (1), the field equations (4) leads to the following system of equation

$$\frac{2\dot{a}\dot{b}}{ab} + \frac{1}{b^2} + \frac{\dot{b}^2}{b^2} - \frac{\varpi}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{\phi}}{\phi}\left(2\frac{\dot{b}}{b} + \frac{\dot{a}}{a}\right) = \frac{8\pi\rho}{\phi} \quad (6)$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} + \frac{\varpi}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right) = -\frac{8\pi(\omega + \delta)\rho}{\phi} \quad (7)$$

$$\frac{2\ddot{b}}{b} + \frac{1}{b^2} + \frac{\dot{b}^2}{b^2} + \frac{\ddot{\phi}}{\phi} \frac{\varpi}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + 2\frac{\dot{\phi}\ddot{b}}{\phi b} = -\frac{8\pi\omega\rho}{\phi} \quad (8)$$

Using equation (5), we get

$$\frac{\ddot{\phi}}{\phi} + \frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} = \frac{8\pi(1 - 3\omega - 2\delta)\rho}{\phi(3 + 2\varpi)} \quad (9)$$

The physical quantities that are of importance in cosmology are expansion scalar θ and shear scalar σ^2 and have the following expression for the metric

$$\theta = \frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} \quad (10)$$

$$\sigma^2 = \frac{2}{3}\left(\frac{\dot{b}}{b} - \frac{\dot{a}}{a}\right)^2 \quad (11)$$

The average scale factor R and the volume scale factor V are defined

$$R = \sqrt[3]{ab^2}, \quad V = R^3 = ab^2 \quad (12)$$

The generalized mean Hubble parameter H is given in the form

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}(H_1 + H_2 + H_3) \quad (13)$$

The directional Hubble parameters in the directions of x , y and z respectively may be defined as

$$H_1 = \frac{\dot{a}}{a}; H_2 = H_3 = \frac{\dot{b}}{b} \quad (14)$$

3. Solution of the Field Equations

Integration after subtracting equation (8) and (7), we get

$$\left(\frac{\dot{b}}{b} - \frac{\dot{a}}{a}\right) b^2 a \phi = \exp \left[\int \left(\frac{8\pi\delta\rho}{\phi} - \frac{1}{b^2} \right) \left(\frac{\dot{b}}{b} - \frac{\dot{a}}{a} \right)^{-1} dt + \log \lambda \right] \quad (15)$$

where λ is an integration constant. The integral term in above equation vanishes for

$$\delta = \frac{\phi}{8\pi\rho b^2} \quad (16)$$

Using equation (16) in equation (15), it follows that

$$\frac{b}{a} = c_2 \exp \left[\int \frac{\lambda}{R^3 \phi} dt \right] \quad (17)$$

Where $R^3 = ab^2$ and c_2 is an integration constant. Here we use the power law assumption to solve the integral part in the above equations. The power law relation between scale factor R

And scalar field ϕ has already been used by Johri and Desikan [33] in the context of Robertson Walker Brans-Dicke models. Thus the power law relation between ϕ and R is $\phi \propto R^m$, where m is any integer, implies that

$$\phi = BR^m \quad (18)$$

where B is the constant of proportionality. The deceleration parameter q in cosmology is the measure of the cosmic acceleration of the universe expansion and is defined as

$$q = -\frac{R\ddot{R}}{\dot{R}^2} \quad (19)$$

It is mentioned here that q was supposed to be positive initially but recent observations from the supernova experiments suggest that it is negative. Thus the behaviour of the universe models depend upon the sign of q . The positive deceleration parameter corresponds to a decelerating model while the negative value provides inflation. We also use a well-known relation [34] between the average Hubble parameter H and average scale factor R given as

$$H = lR^{-n} \quad (20)$$

where $l > 0$ and $n \geq 0$. This is an important relation because it gives the constant value of the deceleration parameter. From equation (13) and (20), we get

$$\dot{R} = lR^{-n+1} \quad (21)$$

Using this value, we find that deceleration parameter is constant i.e. $q = n - 1$. Integrating equation (21), it follows that

$$R = (nlt + k_1)^{\frac{1}{n}}, \quad n \neq 0 \quad (22)$$

$$\text{And } R = k_2 \exp(lt), \quad n = 0 \quad (23)$$

where k_1 and k_2 are constants of integration. Here we obtain two values of the average scale factor that correspond to two different models of the universe.

D) Dark Energy Model of the Universe when $n \neq 0$ and $m = -2$

Now we discuss the model of universe when $n \neq 0$ i. e.

$R = (nlt + k_1)^{\frac{1}{n}}$. For this model, ϕ becomes

$$\phi = B(nlt + k_1)^{\frac{-2}{n}} \quad (24)$$

Using this value of ϕ in equation (17), we get

$$b = c_2^{\frac{1}{3}} (nlt + k_1)^{\frac{1}{n}} \exp \left[\frac{\lambda (nlt + k_1)^{\frac{n-1}{n}}}{3lB(n-1)} \right], \quad n \neq 1 \quad (25)$$

$$a = c_2^{\frac{2}{3}} (nlt + k_1)^{\frac{1}{n}} \exp \left[\frac{-2\lambda (nlt + k_1)^{\frac{n-1}{n}}}{3lB(n-1)} \right], \quad n \neq 1 \quad (26)$$

The directional Hubble parameter H_i ($i = 1, 2, 3$) take the form

$$H_1 = \frac{l}{nlt + k_1} - \frac{2}{3} \frac{\lambda}{B(nlt + k_1)^{\frac{1}{n}}} \quad (27)$$

$$H_2 = H_3 = \frac{l}{nlt + k_1} + \frac{1}{3} \frac{\lambda}{B(nlt + k_1)^{\frac{1}{n}}} \quad (28)$$

The mean generalized Hubble parameter becomes

$$H = \frac{l}{nlt + k_1} \quad (29)$$

While the volume scale factor turns out to be

$$V = (nlt + k_1)^{\frac{3}{n}} \quad (30)$$

The expansion scalar θ and shear scalar σ take the form

$$\theta = \frac{3l}{nlt + k_1} \quad (31)$$

$$\sigma^2 = \frac{2\lambda^2}{3B^2 (nlt + k_1)^{\frac{2}{n}}} \quad (32)$$

The anisotropy parameter of the expansion Δ is defined as

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2 \quad (33)$$

where H_i ($i = 1, 2, 3$) represents the directional Hubble parameters in the directions of x , y and z respectively. By using (27) (28) (29) in (33) we get

$$\Delta = \frac{2\lambda^2}{9B^2} (nlt + k_1)^{2-\frac{2}{n}} \quad (34)$$

Using equations (7) to (9), we obtained

$$\frac{8\pi\rho}{\phi} = -l^2(3+2\omega)(nlt+k_1)^{-2} - \left\{ \frac{\lambda^2}{3B^2} - \frac{1}{c_2^{2/3}} \exp\left(-\frac{2\lambda(nlt+k_1)^{\frac{n-1}{n}}}{3B(n-1)l}\right) \right\} (nlt+k_1)^{\frac{-2}{n}} \quad (35)$$

$$\frac{-8\pi(\omega+\delta)\rho}{\phi} = l^2 \left[(3+2n+3\omega)(nlt+k_1)^{-2} + \frac{\lambda^2}{3B^2} (nlt+k_1)^{\frac{-2}{n}} \right] \quad (36)$$

$$\frac{-8\pi\omega\rho}{\phi} = l^2(3+2n+2\varpi)(nlt+k_1)^{-2} + \left[\frac{\lambda^2}{3B^2} + \frac{1}{c_2^{2/3}} \exp\left(\frac{-2\lambda(nlt+k_1)^{\frac{n-1}{n}}}{3B(n-1)}\right) \right] (nlt+k_1)^{\frac{-2}{n}} \quad (37)$$

In this case the EoS parameter ω becomes

$$\omega = \frac{l^2(3+2n+2\varpi)(nlt+k_1)^{-2} + \left[\frac{\lambda^2}{3B^2} + \frac{1}{c_2^{2/3}} \exp\left(\frac{-2\lambda(nlt+k_1)^{\frac{n-1}{n}}}{3B(n-1)}\right) \right] (nlt+k_1)^{\frac{-2}{n}}}{l^2(3+2\varpi)(nlt+k_1)^{-2} + \left[\frac{\lambda^2 l^2}{3B^2} - \frac{1}{c_2^{2/3}} \exp\left(\frac{-2\lambda(nlt+k_1)^{\frac{n-1}{n}}}{3B(n-1)}\right) \right] (nlt+k_1)^{\frac{-2}{n}}} \quad (38)$$

II) Dark Energy Model of the Universe when $n=0$ and $m=-2$

The average scale factor for this model of the universe is $R = k_2 e^{lt}$ and hence ϕ takes the form

$$\phi = \frac{B}{k_2^2} \exp(-2lt) \quad (39)$$

Using this value of ϕ in equation (17), we get the following exact expressions for the scale factors

$$b = c_2^{\frac{1}{3}} k_2 \exp(lt) \exp\left[\frac{-\lambda \exp(-lt)}{3Blk_2}\right] \quad (40)$$

$$a = c_2^{\frac{2}{3}} k_2 \exp(lt) \exp\left[\frac{2\lambda \exp(-lt)}{3Blk_2}\right] \quad (41)$$

The directional Hubble parameters as defined in (14) are found as

$$\frac{8\pi\rho}{\phi} = -l^2(3+2\varpi) - \frac{\lambda^2 \exp(-2lt)}{3B^2 k_2^2} + \frac{1}{c_2^{\frac{2}{3}} k_2^2} \exp\left(-2lt + \frac{2\lambda \exp(-lt)}{3Blk_2}\right) \quad (47)$$

$$-\frac{8\pi(\omega+\delta)\rho}{\phi} = l^2(3+2\varpi) + \frac{\lambda^2 \exp(-2lt)}{3B^2 k_2^2} \quad (48)$$

$$-\frac{8\pi\omega\rho}{\phi} = l^2(3+2\varpi) + \frac{\lambda^2 \exp(-2lt)}{3B^2 k_2^2} + \frac{1}{c_2^{\frac{2}{3}} k_2^2} \exp\left(-2lt + \frac{2\lambda \exp(-2lt)}{3Blk_2}\right) \quad (49)$$

In this case the EoS parameter ω becomes

$$\omega = \frac{l^2(3+2\varpi) + \frac{\lambda^2 \exp(-2lt)}{3B^2 k_2^2} + \frac{1}{c_2^{\frac{2}{3}} k_2^2} \exp\left(-2lt + \frac{2\lambda \exp(-2lt)}{3Blk_2}\right)}{l^2(3+2\varpi) + \frac{\lambda^2 \exp(-2lt)}{3B^2 k_2^2} - \frac{1}{c_2^{\frac{2}{3}} k_2^2} \exp\left(-2lt + \frac{2\lambda \exp(-lt)}{3Blk_2}\right)} \quad (50)$$

$$H_1 = l - \frac{2\lambda \exp(-lt)}{3Bk_2} \quad (42)$$

$$H_2 = H_3 = l + \frac{\lambda \exp(-lt)}{3Blk_2} \quad (43)$$

From the equations (12) and (13), the mean generalised Hubble parameter and the volume scale factor becomes

$$H = l \quad V = k_2^3 \exp(3lt) \quad (44)$$

The expansion scalar θ and shear scalar σ are found as

$$\theta = 3l \quad \sigma^2 = \frac{2\lambda^2 \exp(-2lt)}{3B^2 k_2^2} \quad (45)$$

The mean anisotropy parameter Δ for this model yields

$$\Delta = \frac{2\lambda^2 \exp(-2lt)}{9B^2 l^2 k_2^2} \quad (46)$$

For this model, equations(7) to (9) becomes

4. Conclusion

In this paper we have discussed the spherically symmetric cosmological models in Brans Dicke theory of gravitation in the background of anisotropic dark energy. The assumption of constant deceleration parameter leads to two models of universe .

First we discussed power law model of the universe. This model corresponds to $n \neq 0$ with average scale factor

$$R = (nlt + k_1)^{\frac{1}{n}}. \text{ It has a point singularity at } t = -\frac{k_1}{nl}.$$

The physical parameters H_1, H_2, H_3 and H are infinite at this point but here the spatial volume vanishes. The metric

function a , and b vanish at $t = -\frac{k_1}{nl}$ Thus , it is concluded

from these observations that the model starts its expansion

with zero volume at $t = -\frac{k_1}{nl}$ and it continues to expand

for $0 < n < 1$.

The exponential model of the universe corresponds to $n = 0$ with average scale factor $R = k_2 \exp(lt)$. It is non-singular because exponential function is never zero and hence there does not exist any physical singularity for this model. The Hubble parameters H_1, H_2, H_3 are all finite values of t . The mean generalized Hubble parameter H is constant while metric functions a, b do not vanish for this model. The volume scale factor increases exponentially with time which indicates that the universe starts its expansion with zero volume from infinite past

The isotropy condition, $\frac{\sigma^2}{\theta} \rightarrow 0$ as $t \rightarrow 0$ in both cases.

The variable EoS parameter ω has a positive value at $t = 0$ which indicated that the universe was matter dominated in its early phase of its existence. At $t \rightarrow \infty$, $\omega \rightarrow 0$ which indicate that the pressure of the universe vanishes .

References

[1] Guth, A.H.: Phys. Rev. D **23**, 347 (1981)
 [2] Sato, K.: Mon. Notices Roy. Astron. Soc. **195**, 467 (1981)
 [3] Linde, A.D.: Phys. Lett. B **108**, 389 (1982)
 [4] Albrecht, A., Steinhardt, P.J.: Phys. Rev. Lett. **48**, 1220 (1982)
 [5] Parker, L.: Phys. Rev. **183**, 1057 (1969)
 [6] Birrell, N.D., Davies, P.C.W.: Quantum Fields in Curved Space (1982)
 [7] Mukhanov, V.F., Chibisov, G.V.: Sov. Phys. J. Exper. Theor. Phys. Lett. **33**, 532–535 (1981)
 [8] Hawking, S.W.: Phys. Lett. B **115**, 295 (1982)
 [9] Guth, A.H., Pi, S.Y.: Phys. Rev. Lett. **49**, 1110 (1982)
 [10] Starobinsky, A.A.: Phys. Lett. B **117**, 175 (1982)
 [11] Bardeen, J.M., et al.: Phys. Rev. D **28**, 679 (1983)
 [12] Mukhanov, V.F., et al.: Phys. Rep. **215**, 203 (1992)

[13] Peiris, H.V., et al.: Astrophys. J. Suppl. Ser. **148**, 213–231 [arXiv:astro-ph/0302225v2] (2003)
 [14] Liddle, A.R., Lyth, D.H.: Cosmological Inflation and Large-Scale Structure(2000)
 [15] Ellis, G.F.R.: Gen. Relat. Gravit. **38**(6), 1003–1015 (2006)
 [16] Ellis, G.F.R.: Modern Cosmology. In Bonometto, et al. (ed) Institute of physics Publishing, Bristol, pp. **138-139** (2002)
 [17] Rodrigues, D.C.: Phys. Rev. D **77**, 023534 [arXiv:astro-ph/0708.1168v2] (2008)
 [18] Koivisto T., Mota, D.F.; arXiv:astro-ph/0801.367v2(2008)
 [19] Koivisto T., Mota, D.F.; Astrophys. J. **679**, 1 [arXiv:astro-ph/0707.0279v3] (2008)
 [20] Mota, D.F., et al.: Mon. Notices Roy. Astron. Soc. **382**, 793–800 [arXiv:astro-ph/0708.0830v2] (2007)
 [21] Carroll, S.M., Hoffman, M., Phys. Rev. D **68**, 023509 [arXiv:astro-ph/0301273] (2003)
 [22] Burdzyuzha, V., Vereshkov, G.: Astrophys. Space Sci. **305**, 235–239 (2006)
 [23] Turner, M., Huterer, D.: J. Phys. Soc. Jpn. **76**, 111015 [arXiv:astro-ph/0706.2186v2] (2007)
 [24] Bennett, C.L. et al.: Astrophys. J. Suppl. **148**, 1(2003)
 [25] Riess, A.G. et al.: Astrophys. J. **607**, 665(2004)
 [26] Riess, A.G. et al. (Supernova Search Team): Astron. J. **116**, 1009(1998)
 [27] Spergel, D.N. et al.: Astrophys. J. Suppl. **148**, 175(2003)
 [28] Tegmark, M. et al.: Phys. Rev. **D69**, 103501 (2004)
 [29] Jimenez, R.: New astron. Rev. **47**, 761 (2003)
 [30] Brans, C. and Dicke, R. H. : Phys. Rev. **124**, 925 (1961)
 [31] Pradhan, A. and Amirhashchi, H. : Astrophys. Space Sci. **332**, 441(2011)
 [32] Kumar, S. and Singh, C. P. : Int. J. Theor. Phys. **47**, 1722 (2008)
 [33] Johri, V.B. & Desikan, K.: Gen. Relativ. Grav. **26**, 1217 (1994)
 [34] Berman, M. S. : IL Nuovo Cim. **B74**, 183(1983)