$\bigcup_{i\in\alpha} H_i(x_i)$ is a BH-subalgebra of F(x),for all x ∈Supp (H,B). Hence the restricted union $\tilde{\bigcup}_{i\in\alpha}$ (H_i,A_i) is a soft BH-algebra of (F,A).

Proposition13. Let (F,A) be a of soft BH-algebra over X and $\{(H_i,A_i) \mid i\in\alpha\}$a nonempty family of soft BH-subalgebras of (F,A). Then the ∧-intersection $\tilde{\bigwedge}_{i\in\alpha}(H_i, A_i)$ is a soft BH-subalgebra of $\tilde{\bigwedge}_{i\in\alpha}(F, A)$.
Proof. Similar to the proof of proposition 5.

Proposition14. Let (F,A) be a of soft BH-algebra over X and $\{(H_i,A_i) \mid i\in\alpha\}$a nonempty family of soft BH-subalgebras of (F,A). If $H_i(x_i) \subseteq H_j(x_j)$ or $H_j(x_j)\subseteq H_i(x_i)$, for all i,j∈α, $x_i\in A_i$,, then the ∨-union $\tilde{\bigvee}_{i\in\alpha}$ (H_i,A_i) is a soft BH-subalgebra of $\tilde{\bigvee}_{i\in\alpha}(F,A)$.
Proof.Similar to the proof of proposition 6.

Theorem 3. Let (F,A) be a of soft BH-algebra over X and $\{(H_i,A_i) \mid i\in\alpha\}$ a nonempty family of soft BH-subalgebras of (F,A). Then the cartesian product of the family $\tilde{\prod}_{i\in\alpha}(H_i, A_i)$ is a soft BH-algebra over $\tilde{\prod}_{i\in\alpha}(F,A)$.

Proof. By definition 12, we can write $\tilde{\prod}_{i\in\alpha}(H_i, A_i) = (H,B)$, where $B = \prod_{i\in\alpha}A_i$ and$H(x)=\prod_{i\in\alpha}H_i(x_i)$, for all $x=(x_i)_{i\in\alpha}\in B$. Let $x = (x_i)_{i\in\alpha}\in$Supp (H,B). Then $H(x) =\prod_{i\in\alpha}H_i(x_i) \neq\emptyset$, and so we have$H_i(x_i) \neq\emptyset$,for all i∈α. Since $\{(H_i,A_i) \mid i\in\alpha\}$is a soft BH-subalgebras of (F,A),we have that $H_i(x_i)$ is a BH-subalgebra of $F(x_i)$,from which obtain that$\prod_{i\in\alpha}H_i(x_i)$ is a BH-subalgebra of $\prod_{i\in\alpha}F(x_i)$, for all $x = (x_i)_{i\in\alpha}\in$Supp(H,B). Hence, the cartesian product of the family $\tilde{\prod}_{i\in\alpha}(F_i,A_i)$ is a soft BH-algebra over $\tilde{\prod}_{i\in\alpha}(F,A)$.

Proposition15. Let $f : X \to Y$ be a homomorphism of BH-algebras and (F,A), (G,B) two soft BH-algebras over X. If $(G,B)\tilde{<}_s(F,A)$.
Then $(f(G),B)\tilde{<}_s$ $(f(F),A)$.

Proof. Assume that $(G,B)\tilde{<}_s$ (F,A). Let x∈Supp(G,B). Then x ∈Supp (F,A). Bydefinition 17, A ⊆B and G(x) is a BH-subalgebra of F(x) for all x∈Supp (G,B). Since f is a homomorphism, $f(G)(x)=f(G(x))$ is a BH-subalgebraof $f(F(x)) = f(F)(x)$.Therefore $(f(G),B)\tilde{<}_s$ $(f(F),A)$.

Theorem 4. Let $f : X \to Y$ be a homomorphism of BH-algebras and (F,A), (G,B) two soft BH-algebras over Y. If $(G,B)\tilde{<}_s$ (F,A).
Then $(f^{-1}(G),B)\tilde{<}_s$ $(f^{-1}(F),A)$.

Proof. Assume that $(G,B)\tilde{<}_s$ (F,A). Let y∈Supp$(f^{-1}(G),B)$.By definition17 B⊆A and G(y) is a BH-subalgebra of F(y), for all y∈B. Since f is a homomorphism, $f^{-1}(G)(y) =f^{-1}(G(y))$ is a BH-subalgebra of $f^{-1}(F(y)) = f^{-1}(F)(y)$, for all y ∈Supp $(f^{-1}(G),B)$ .Hence , $(f^{-1}(G),B)\tilde{<}_s(f^{-1}(F),A)$

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