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Applications of Soft Sets in BH-algebra

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Abstract: ***In this paper, the concept of soft set BH-algebra is introduced and in the meantime, some of their properties and structural characteristics are discussed and studied. The bi-intersection, extended intersection, restricted union, ⋎-union, ⋏-intersection and cartesian product of the family of soft BH-algebra and soft BH-subalgebra are established. Also, the theorems of homomorphic image and homomorphic pre-image of soft sets are given. Moreover, the notion of soft BH- homomorphism is introduced and its basic properties are studied.***

Keywords: Soft sets, BH-algebra, Soft BH-algebra, Soft BH-subalgebra, Cartesian product of soft sets.

1. Introduction

To solve complicated problem in economics, engineering, and environment,we can't successfully use classical methods because of various uncertainties typical for those problems. There are three theories: theory of probability, theory of fuzzy sets, and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. Uncertainties can't be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as probability theory, theory of (intuitionistic) fuzzy sets, theory of vague sets, theory of interval mathematics, and theory of rough sets. However, all of these theories have their own difficulties which were pointed out in Molodtsov suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties In 1999,Molodtsov introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly[2]. In 2002,Maji et al. described the application of soft set theory to a decision making problem [5]. In 2003, Maji et al. studied several operations on the theory of soft sets[6]. In 2005,Chen et al. presented a new definition of soft set parametrization reduction, and compared this definition to the related concept of attributes reduction in rough set theory. The algebraic structure of set theories dealing with uncertainties has been studied by some authors[1].In 1998 ,Y. B. Jun, E. H. Roh and H .S. Kim introduced the notion of BH-algebra[8].In this paper we apply the notion of soft sets introduced by Molodtsov to the theory of BH-algebras. Soft BH-subalgebras and homomorphisms in soft BH-algebras arediscussed.

2. Preliminaries

In this section, we give some basic concepts about a BH-algebra, Soft sets, Soft BH-subalgebra, Cartesian product, intersection and unionof soft sets

Definition 1. (see[8])A*BH-algebra*is a nonempty set X with a constant 0 and a binary operation"*" satisfying the following conditions:

i. $x*x=0$, for all $x\in X$.
ii. $x*y=0$ and $y*x=0$ imply $x = y$, for all $x, y\in X$.
iii. $x*0 =x$, for all $x\in X$.

Definition 2. (see[8]) Let X a BH-algebra and $S \subseteq X$. Then S is called a ***subalgebra***of X if $x*y\in S$ for all $x,y\in S$.

Remark 1. (see[7]) Let X and Y be BH-algebras. A mapping f:X→Y is called a homomorphism if $f(x*y)=f(x)*f(y)$ for all $x,y\in X$. A homomorphism f is called a monomorphism (resp., epimorphism) if it is injective (resp., surjective). A bijective homomorphism is called an isomorphism. Two BH-algebras X and Y are said to be isomorphic, written $X \cong Y$, if there exists an isomorphism f:X→Y. For all homomorphism f:X →Y, the set $\{x\in X:f(x)=0\}$ is called the kernel of f, denoted by Ker(f), and the set $\{f(x):x\in X\}$ is called the image of f, denoted by Im(f). Notice that $f(0)= 0'$, for all homomorphism f,and$f^{-1}(Y) =\{x\in X:f(x) = y$, for some $y\in Y\}$

Definition 3. (see[2]) The notion of a soft set defined in the following way: Let U be an initial universe set and E a set of parameters. The power set of U is denoted by $\mathcal{P}(U)$ and A is a subset of E. A pair (F,A) is called a soft set over U, where F is a mapping $F : A \rightarrow \mathcal{P}(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $x\in A$, F(x) may be considered as the set of x-approximate elements of the soft set (F,A). Clearly, a soft set is not just a subset of U.

Definition 4. (see[2]) Let (F, A), (G,B) be soft sets over a common universe U.

(i) (F,A) is said to be a soft subset of (G,B), denoted by $(F,A)\tilde{\subseteq}(G,B)$, if $A \subseteq B$ and $F(a) \subseteq G(a)$ for all $a \in A$,

(ii) (F,A) and (G, B) are said to be soft equal, denoted by (F, A) = (G, B), if $(F,A)\tilde{\subseteq}(G,B)$ and $(G,B)\tilde{\subseteq}(F,A)$.

Definition 5.**(see[4])**i. The bi(restricted) -intersection of two soft sets (F,A) and (G,B) over a commonuniverse U is defined as the soft set.

$(H,C) = (F,A)\tilde{\sqcap}(G,B)$, where $C=A \cap B\neq\emptyset$, and $H(c) = F(c)\cap G(c)$, for all $c\in C$.

(ii) The bi(restricted)-intersection of a nonempty family soft sets $\{(F_i,A_i) \mid i\in\alpha\}$ over a common universe U is defined as the soft set $(H,B) = \tilde{\sqcap}_{i\in\alpha} (F_i,A_i)$, where $B =\cap_{i\in\alpha} A_i\neq\emptyset$, and $H(x) = \cap_{i\in\alpha} F_i(x)$, for all $x \in B$.

Definition6. (see[4])(i) The extended intersection of two soft sets (F,A) and (G,B) over a common universe U is defined as the soft set.

(H,C) = (F,A)$\tilde{\cap}$(G,B), where C=A∪B, and for all c ∈C,

$$H(c) = \begin{cases} F(c)\ if\ c \in A\backslash B \\ G(c)\ if\ c \in B\backslash A \\ F(c) \cap G(c)\ if\ c \in A \cap B \end{cases}$$

(ii) The extended intersection of a non- empty family soft sets $\{(F_i,A_i) \mid i\in\alpha\}$ over a common universe U is defined as the soft set (H,B) =$\tilde{\cap}_{i\in\alpha}$ (F_i,A_i), where
B = $\cup_{i\in\alpha}A_i$, and H(x) = $\cap_{i\in\alpha(x)}$ $F_i(x)$, and$\alpha(x)=\{i \mid i\in A_i\}$, for all x ∈B.

Definition 7. (see[4]) The bi (restricted) union of two soft sets (F,A) and (G,B) over a commonuniverse U is defined as the soft set (H,C)= (F,A)$\tilde{\cup}$(G,B), where C =A∩B≠∅, and H(c) = F(c)∪G(c), for all c∈C.

Definition 8. (see[8]) The restricted union of a nonempty family soft sets$\{(F_i, A_i) \mid i\in\alpha\}$over a common universe U is defined as the soft set.

(H,B) =$\tilde{\cup}_{i\in\alpha}$ (F_i,A_i), where B =$\cap_{i\in\alpha}$ A_i≠∅, and H(x) = $\cup_{i\in\alpha}$ $F_i(x)$, for all x∈B.

Definition 9. (see[3])(i) The ∧-intersection of two soft sets (F,A) and (G,B) over a common universe U is defined as the soft set.

(H,C) = (F,A)$\tilde{\wedge}$ (G,B), where C=A×B, and H(a,b)=F(a)∩G(b), for all(a,b)∈A×B

(ii) The ∧-intersection of a nonempty family soft sets $\{(F_i, A_i) \mid i\in\alpha\}$over a common universe U is defined as the soft set (H,B) =$\tilde{\wedge}_{i\in\alpha}(F_i, A_i)$, where B =$\prod_{i\in\alpha}A_i$ and H(x) = $\cap_{i\in\alpha}$ $F_i(x_i)$, for all $x=(x_i)_{i\in\alpha}$∈B.

Definition 10. (see[3])(i) The ∨-union of two soft sets (F,A) and (G,B) over a common universe U is defined as the soft set (H,C) =(F,A) $\tilde{\vee}$ (G,B),

where C= A × B, and H(a,b)=F(a)∪G(b), for all (a, b) ∈A × B.

(ii) The ∨-union of a nonempty family soft sets $\{(F_i, A_i) \mid i\in\alpha\}$over a common universe U is defined as the soft set (H,B)= $\tilde{\vee}_{i\in\alpha}(F_i,A_i)$, where B=$\prod_{i\in\alpha}A_i$ and H(x) = $\cup_{i\in\alpha}$ $F_i(x_i)$, for all $x=(x_i)_{i\in\alpha}$∈B.

Definition11. (see[6]) Let (F,A) and (G,B) be two soft sets over U and V , respectively. The cartesian product of the two soft sets (F,A) and (G,B) is defined as the soft set.

(C,A×B) = (F,A) × (G,B), where C(x, y) = F(x)×G(y), for all (x, y)∈A×B.

Definition 12.(see[8]) Let$\{(F_i, A_i) \mid i\in\alpha\}$ be a nonempty family of soft sets over U_i I ∈α. The cartesian product of the non-
empty family of soft sets $\{(F_i,A_i) \mid i\in\alpha\}$ over the universes U_i is defined as the soft set(H,B) = $\tilde{\prod}_{i\in\alpha}(F_i,A_i)$, where B =$\prod_{i\in\alpha}A_i$ and H(x) =$\prod_{i\in\alpha}F_i(x_i)$, for all $x=(x_i)_{i\in\alpha}$∈B.

3. The Main Results

In this section, we introduce the concepts ofsoft set of BH-algebras and soft BH-algebra. Also we state and prove some theorems and examples about theseconcepts.

Definition 13. If X is a BH-algebra and A a nonempty set, a set-valued function

F : A→ $\mathcal{P}$(X) can be defined by : F(x) ={y ∈X | (x, y)∈R}, x∈A, where R is an arbitrary binary relation from A to X, that is a subset of A×X. The pair (F,A) is then a soft set over X. The soft sets in the examples that follow are obtained by making an appropriate choice for the relation R. For a soft set (F,A). The set Supp (F,A) ={x∈A| F(x)≠∅} is called the support of the soft set (F,A), and the soft set (F,A) is called a non-null if Supp (F,A)≠∅.

Definition 14. Let (F,A) be a non-null soft set over X a BH-algebra. Then (F,A) is called a soft BH-algebra over X if F(x) is a BH-subalgebra of X, for all x∈Supp(F,A).

Remark 2. The order of x, denoted by *o(x)*, as *o(x)* = min{n ∈N | $0*x^n$ = 0}.
where $0*x^n$ =(· · · ((0 ∗x) ∗x) ∗ · · ·) ∗x in which x appears n-times.

Example 1: Consider the BH-algebra X={0,1,2,3,4} with binary operation"*" defined as follows:

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	1	1
2	2	2	0	2	2
3	3	3	3	0	3
4	4	4	4	4	0

Let (F,A) be a soft set over X, where A = X and F : A → $\mathcal{P}$(X) is the set-valued function defined by : F(x) ={y∈X| x R y ⟺y ∈x^{-1}I}, for all x∈A where I={0,1} and x^{-1}I={y ∈ X | y ∗ (y ∗ x)∈I}. Then F(0)=F(1)=X, F(2)={0,1,3,4}, F(3)={0, 1,2,4} and F(4)={0,1,2,3} are BH-subalgebras of X, for all x∈Supp (F,A). Therefore (F,A) is a soft BH-algebra over X.

Example 2: Consider the BH-algebra X={0,1,2,3,4,5,6,7} with binary operation"*" defined as follows

*	0	1	2	3	4	5	6	7
0	0	0	0	0	4	4	4	4
1	1	0	0	0	5	4	4	4
2	2	2	0	0	6	6	4	4
3	3	2	1	0	7	6	5	4
4	4	4	4	4	0	0	0	0
5	5	4	4	4	1	0	0	0
6	6	6	4	4	2	2	0	0
7	7	6	5	4	3	2	1	0

Let (F,A)be a soft set over X where A=X and F:A⟶$\mathcal{P}$(X) the set-valued function defined by F(x) = {0}∪{y∈X| xRy⟺*o(x)*=*o(y)*}, for all x∈ A. Then F(0) = F(1)=F(2)=F(3)=X is a soft BH-algebra, but F(4)= F(5)=F(6)=F(7) ={0,4,5,6,7} is not a BH-subalgebras of X. Hence (F, A) is not a soft BH-algebra over X. If we take B ={1, 2, 3}⊂ X and define a set-valued function G:B⟶$\mathcal{P}$(X) by G(x) ={y∈X | xRy⇔$o(x)$=o(y)},for all x∈B, then(G, B) is a soft BH-algebra over X, since G(1) = G(2) = G(3) ={0,1,2,3} is a BH-subalgebra of X.

Proposition1. Let{(F_i, A_i) | i∈α}be a nonempty family of soft BH-algebras over X. Then the bi-intersection$\widetilde{\sqcap}_{i\in\alpha}$ (F_i,A_i) is a soft BH-algebra over X if it is non-null.

Proof. Let{(F_i,A_i) | i∈α} be a nonempty family of soft BH-algebras over X. By definition 5 (ii), we can write $\widetilde{\sqcap}_{i\in\alpha}$($F_i$,$A_i$)=(H,B), where B=$\cap_{i\in\alpha}A_i$, and H(x)=$\cap_{i\in\alpha}$ F_i(x), for all x∈B. Let x∈Supp(H,B). Then $\cap_{i\in\alpha}$ F_i(x) ≠ ∅, and so we have F_i(x) ≠∅,for all i ∈α. Since {(F_i, A_i) | i ∈α }is a nonempty family of soft BH-algebras over X, it follows that F_i(x) is a BH-subalgebra of X, for all i∈α, and its intersection is also a BH-subalgebra of X, that is, H(x)=$\cap_{i\in\alpha}F_i$(x) is a BH-subalgebra of X, for all x∈Supp(H,B). Hence (H,B) = $\widetilde{\sqcap}_{i\in\alpha}$ (F_i,A_i) is a soft BH-algebra over X.

Proposition 2. Let {(F_i,A) | i∈α}be a nonempty family of soft BH-algebras over X. Then the bi-intersection $\widetilde{\sqcap}_{i\in\alpha}$ (F_i,A) is a soft BH-algebra over X if it is non-null.

Proof. Straightforward.

Proposition 3. Let {(F_i,A_i) | i∈α} be a nonempty family of soft BH-algebras over X. Then the extended intersection $\widetilde{\cap}_{i\in\alpha}$($F_i$,$A_i$) is a soft BH-algebras over X.

Proof. Assume that {(F_i, A_i) | i∈α } is a nonempty family of soft BH-algebras over X. By definition 6 (ii), we can write $\widetilde{\cap}_{i\in\alpha}$($F_i$,$A_i$) = (H,B), where B =$\cup_{i\in\alpha}$ A_i, and H(x) = $\cap_{i\in\alpha(x)}F_i$(x), for all x∈B. Let x ∈Supp (H,B). Then$\cap_{i\in\alpha(x)}F_i$(x)≠ ∅,and so we have F_i(x) ≠∅,for all i∈α(x). Since (F_i,A_i) is a soft BH-algebras over X, for all i∈α, we deduce that the nonempty set F_i(x) is a BH-algebras of X, for all i ∈α. It follows that H(x) = $\cap_{i\in\alpha(x)}F_i$(x) is a BH-subalgebra of X, for all x ∈Supp (H,B). Hence, the extended intersection $\widetilde{\cap}_{i\in\alpha}$($F_i$,$A_i$) is a soft BH-algebra over X.

Proposition4. Let {(F_i, A_i) | i∈α} be a nonempty family of soft BH-algebras over X. If $F_i(x_i)$ ⊆$F_j(x_j)$ or $F_j(x_j)$⊆$F_i(x_i)$ for all i,j∈α, x_i∈A_i, then the restricted union $\widetilde{\cup}_{i\in\alpha}$ (F_i,A_i) is a soft BH-algebra over X.

Proof. Assume that {(F_i, A_i) | i∈α} is a nonempty family of soft BH-algebra over X. By definition 8, we can write $\widetilde{\cup}_{i\in\alpha}$ (F_i,A_i) = (H,B), where B = $\cap_{i\in\alpha}A_i$, and H(x) = $\cup_{i\in\alpha}$ F_i(x), for all x ∈B. Let x ∈Supp (H,B). Since Supp (H,B) = $\cup_{i\in\alpha}$Supp(F_i,A_i) ≠∅,we have Fi_o(x) ≠∅, for some i_o∈α. By assumption, $\cup_{i\in\alpha}$ $F_i(x_i)$ is a BH-subalgebra of X, for all x ∈Supp (H,B). Hence the restricted union $\widetilde{\cup}_{i\in\alpha}$ (F_i,A_i) is a soft BH-algebra over X.

Proposition5.Let{(F_i,A_i) |i∈α}be a nonempty family of soft BH-algebras over X. Then the ∧-intersection $\widetilde{\wedge}_{i\in\alpha}$($F_i$, A_i) is a soft BH-algebra over X if it is non-null.

Proof. By definition 9 (ii), we can write$\widetilde{\wedge}_{i\in\alpha}$($F_i$, A_i)= (H,B), where B =$\prod_{i\in\alpha}A_i$,and H(x) =$\cap_{i\in\alpha}F_i(x_i)$, for all x=$(x_i)_{i\in\alpha}$∈B. Suppose that the soft set (H,B) is non-null. If x = $(x_i)_{i\in\alpha}$∈Supp (H,B), then H(x) =$\cap_{i\in\alpha}$ $F_i(x_i)$ ≠∅. Since (F_i,A_i) is a soft BH-algebra over X, for all i∈α, we deduce that the nonempty set $F_i(x_i)$ is a BH-subalgebra of X, for all i∈α. It follows that H(x) =$\cap_{i\in\alpha}F_i(x_i)$ is a BH-subalgebra of X, for all x=$(x_i)_{i\in\alpha}$∈Supp(H,B). Hence, the ∧-intersection$\widetilde{\wedge}_{i\in\alpha}$($F_i$,$A_i$) is a soft BH-algebra over X.

Proposition6. Let {(F_i,A_i) | i∈α} be a nonempty family of soft BH-algebras over X. If $F_i(x_i)$ ⊆$F_j(x_j)$ or $F_j(x_j)$⊆$F_i(x_i)$, for all i,j∈α, x_i∈A_i, then the ∨-union $\widetilde{\vee}_{i\in\alpha}$ (F_i,A_i) is a soft BH-algebra over X.

Proof. Assume that {(F_i, A_i) | i ∈α } is a nonempty family of soft BH-algebra over X. By definition 10(ii) we can write $\widetilde{\vee}_{i\in\alpha}$ (F_i,A_i) =(H,B), where B=$\prod_{i\in\alpha}A_i$ and H(x) =$\cup_{i\in\alpha}F_i(x_i)$, for all x = $(x_i)_{i\in\alpha}$∈ B. Let x = $(x_i)_{i\in\alpha}$∈Supp (H,B). Then H(x) =$\cup_{i\in\alpha}F_i(x_i)$ ≠∅, and so we have$F i_o(x_{i_o}) \neq \emptyset$, for some i_o∈α. By assumption, $\cup_{i\in\alpha}$ $F_i(x_i)$ is a BH-subalgebra of X, forall x = $(x_i)_{i\in\alpha}$∈Supp (H,B). Hence the ∨-union $\widetilde{\vee}_{i\in\alpha}$ (F_i, A_i) is a soft BH-algebraover X.

Proposition7. Let {(F_i,A_i) | i∈α}be a non-empty family of soft BH-algebras over X_i. Then the cartesian product$\widetilde{\prod}_{i\in\alpha}$($F_i$, A_i) is a soft BH-algebra over $\prod_{i\in\alpha}X_i$.

Proof. By definition 12, we can write $\widetilde{\prod}_{i\in\alpha}$($F_i$, A_i) = (H,B), where B=$\prod_{i\in\alpha}A_i$ andH(x)=$\prod_{i\in\alpha}F_i(x_i)$, for all x=$(x_i)_{i\in\alpha}$∈B. Let x = $(x_i)_{i\in\alpha}$∈Supp (H,B). Then H(x) =$\prod_{i\in\alpha}F_i(x_i)$ ≠∅, and so we have$F_i(x_i)$ ≠∅,for all i∈α. Since {(F_i,A_i) | i∈α}is a soft BH-algebras over X_i forall i∈α, we have that $F_i(x_i)$ is a BH-subalgebra of X_i, so $\prod_{i\in\alpha}F_i(x_i)$ is a BH-subalgebra of $\prod_{i\in\alpha}X_i$ for all x = $(x_i)_{i\in\alpha}$∈Supp (H,B). Hence, the cartesian product$\widetilde{\prod}_{i\in\alpha}$($F_i$,$A_i$) is a soft BH-algebra over $\prod_{i\in\alpha}X_i$.

Definition 15. Let (F,A) be a soft BH-algebra over X.
(i) (F,A) is called the trivial soft BH-algebra over X, if F(x)={0},for all x∈A.
(ii) (F,A) is called the whole soft BH-algebra over X, if F(x) = X, for all x∈A.

Example 3: Consider the BH-algebra X={0,1,2,3} with binary operation"*" defined as follows:

*	0	1	2	3
0	0	0	0	0
1	1	0	0	3
2	2	3	0	3
3	3	0	0	0

Let (F,A) be a soft set over X, where A={1,2,3} and F : A → $\mathcal{P}$(X) the set-valued function defined by F(x)={y∈X| x R y ⇔y *x∈{0,3}} for all x∈A. Then F(x) = X for all x∈A, so (F,A) is a whole soft BH-algebra over X Let (G,A) be a soft set over Xand G: A→ $\mathcal{P}$(X) the set-valued function defined by G(x)={y∈X| x R y ⇔x*y = x}, for all x∈A. Then G(x) = {0}, for all x ∈A, so (G,A) is a trivial soft BH-algebra over X.

Definition16. Let X, Y be two BH-algebras and f : X →Y a mapping of BH-algebras. If (F,A) and (G,B) are soft sets over X and Y respectively, then (f(F),A) is a soft set over Y, where

f(F) : A $\to\mathcal{P}$(Y) is defined by f(F)(x) = f(F(x)), for all x ∈A and (f^{-1}(G),B) is a soft set over X, where f^{-1}(G) : B$\to\mathcal{P}$(X) is defined by f^{-1}(G)(y) =f^{-1}(G(y)), for all y ∈B.

Theorem 1.Let f:X→Ybe an onto homo-morphism of BH-algebras.
(i) If (F,A) is a soft BH-algebra over X, then (f(F),A) is a soft BH-algebra over Y
(ii) If (G,B) is a soft BH-algebra over Y , then(f^{-1}(G),B) is a soft BH-algebra over X if it is non-null.

Proof. i.Since (F,A) is a soft BH-algebra over X, it is clear that (f(F),A) is a non-null soft set over Y. For every x∈Supp(f(F),A),we have f(F)(x) = f(F(x)) ≠ ∅. Since the nonempty set F(x) is a BH-subalgebra of X, its onto homomorphic
image f(F(x)) is a BH-subalgebra of Y. Hence f(F(x)) is a BH-subalgebra of Y, for all x∈Supp(f(F),A). That is (f(F),A) is a soft BH-algebra over Y .
(ii) It is easy to see that Supp (f^{-1}(G),B) ⊆Supp (G,B). Let y ∈Supp (f^{-1}(G),B). Then G(y) ≠∅. Since the nonempty set G(y) is a BH-subalgebra of Y, its homo- morphicinverse image f^{-1}(G(y)) is also a BH-subalgebra of X. Hence f^{-1}(G(y)) is a BH-subalgebra of Y, for all y∈Supp(f^{-1}(G),B). That is, (f^{-1}(G),B) is a soft BH-algebra over X.

Theorem 2. Let f : X → Y be a homo- morphism of a BH-algebras. Let (F,A) and (G,B) be two soft BH-algebras over X and Y respectively.
(i) If F(x) = ker(f), for all x ∈A, then (f(F),A) is the trivial soft BH-algebra over Y .
(ii) If $f$is onto and (F,A) is whole, then (f(F),A) is the whole soft BH-algebra over Y.
(iii) If G(y) = f(X), for all y ∈B, then (f^{-1}(G),B) is the whole soft BH-algebra over X.
(iv) If $f$is injective and (G,B) is trivial, then (f^{-1}(G),B) is the trivial soft BH-algebra over X.

Proof. (i) Assume that F(x) = ker(f), for all x∈A. Then, f(F)(x) = f(F(x)) ={0_Y},for all x∈A. Hence (f(F),A) is soft BH-algebra over Y.
By theorem 1 and definition 15 (i).
(ii) Suppose that $f$is onto and that (F,A) is whole. Then, F(x)=X, for all x∈A, and so f(F)(x)=f(F(x))=f(X)=Y for all x∈A. It follows from theorem 1 and definition 15(ii) that (f(F),A) is the whole soft BH- algebra over Y .
(iii) Assume that G(y)=f(X), for all y∈B Thenf^{-1}(G)(y) =f^{-1}(G(y)) = f^{-1}(f(X)) = X , for all y∈B. Hence (f^{-1}(G), B) is the whole soft BH-algebra over X. By theorem 1 and definition 15(ii).
(iv) Suppose that f is injective and (G,B) is trivial. Then, G(y) ={0},for all y ∈B, and so f^{-1}(G)(y) = f^{-1}(G(y)) = f^{-1}({0}) = ker(f) = {0_X},for all y∈B. It follows from theorem 1 and definition 15(i)that (f^{-1}(G),B) is the trivial soft BH-algebra over X.

Definition 17. Let (F,A) and (G,B) be two soft BH-algebras over X. Then(G,B) called is a soft BH-subalgebra of (F,A), denoted by (G,B)$\lessapprox_s$(F,A), if it satisfies thefollowing conditions:
(i) B ⊆A,
(ii) G(x) is a BH-subalgebra of F(x), for all x ∈Supp (G,B).

From the above definition, one easily deduces that if (G,B) is a soft BH-subalgebra of (F,A), then
Supp (G,B) ⊂Supp (F,A).

Example 4: Consider the BH-algebra X={0,1,2,3,4} with binary operation"*" defined as follows:

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	1
2	2	2	0	2	0
3	3	1	3	0	3
4	4	4	2	4	0

Let (F,A) be a soft set over X, where A={0,1,2} and F : A → $\mathcal{P}$(X) is the set-valued function defined by F(x)={y∈X| x R y ⇔ y*(y *x)∈{0,1}}, for all x∈A. Then (F,A) is a soft BH-algebra over X, for all x ∈Supp(F,A).
Let (G,B) be a soft set over X, where B={0,2} and G: B → $\mathcal{P}$(X) is the set-valued function defined by G(x)={y∈ X | x R y ⇔ y*(y *x)∈{0,4}}, for all x∈B. Then (G,A) is a soft BH-algebra over X, for all x∈Supp (G,B) and G(0) = X = F(0), G(2)={0,1, 3} = F(2). Hence (G,B) is a soft BH-subalgebra of (F,A).

Proposition8. Let (F,A) and (G,B) be two soft BH-algebras over X and(G,B)⫅̃(F,A).Then (G,B)$\lessapprox_s$ (F,A)

Proof. Straight forward.

Proposition9. Let (F,A) be a of soft BH-algebra over X and {(H_i,A_i) | i∈α}a nonempty family of soft BH-subalgebras of (F,A). Then the bi-intersection $\tilde{\sqcap}_{i\in\alpha}$ (H_i,A_i) is a soft BH-subalgebra over X if it is non-null.

Proof. Similar to the proof of proposition 1.

Proposition10. Let (F, A) be a of soft BH-algebras over X and {(H_i,A_i) |i∈α} a nonempty family of soft BH-subalgebras of (F,A). Then the bi-intersection $\tilde{\sqcap}_{i\in\alpha}$ (H_i,A) is a soft BH-subalgebra of (F,A) if it is non-null.

Proof. Straightforward.

Proposition11. Let (F, A) be a of soft BH-algebras over X and {(H_i,A_i) | i∈α}a nonempty family of soft BH-subalgebras of (F,A). Then the extended intersection $\tilde{\cap}_{i\in\alpha}$ (H_i,A_i) is a soft BH-subalgebra of (F,A).

Proof.Similar to the proof of proposition 3.

Proposition12. Let (F, A) be a of soft BH-algebra over X and {(H_i, A_i) | i∈α}a nonempty family of soft BH-subalgebras of (F,A). If $H_i(x_i)$ ⊆$H_j(x_j)$ or $H_j(x_j)$⊆$H_i(x_i)$, for all i,j∈α, x_i∈A_i, then the restricted union $\tilde{\cup}_{i\in\alpha}$ (H_i,A_i) is a soft BH-subalgebra of (F,A).

Proof. Assume that{(H_i, A_i) | i∈α} is a nonempty family of soft BH-subalgebra of (F,A). By Definition 8, we can write $\tilde{\cup}_{i\in\alpha}$ (H_i,A_i) = (H,B), where B = $\cap_{i\in\alpha}A_i$, andH(x) = $\cup_{i\in\alpha}$ H_i(x), for all x ∈B. Let x ∈Supp (H,B). Then H(x) = $\cup_{i\in\alpha}$ H_i(x) ≠∅,and so we have $H_{i_o}(x_{i_o}) \neq \emptyset$, for some i_o∈α. Since $H_i(x_i)$⊆$H_j(x_j)$ or $H_j(x_j)$⊆$H_i(x_i)$, for all i,j∈α, x_i∈A_i, clearly

$\bigcup_{i\in\alpha} H_i(x_i)$ is a BH-subalgebra of F(x),for all x ∈Supp (H,B). Hence the restricted union $\tilde{\bigcup}_{i\in\alpha}$ (H_i,A_i) is a soft BH-algebra of (F,A).

Proposition13. Let (F,A) be a of soft BH-algebra over X and $\{(H_i,A_i) \mid i\in\alpha\}$a nonempty family of soft BH-subalgebras of (F,A). Then the ∧-intersection $\tilde{\bigwedge}_{i\in\alpha}(H_i, A_i)$ is a soft BH-subalgebra of $\tilde{\bigwedge}_{i\in\alpha}(F, A)$.
Proof. Similar to the proof of proposition 5.

Proposition14. Let (F,A) be a of soft BH-algebra over X and $\{(H_i,A_i) \mid i\in\alpha\}$a nonempty family of soft BH-subalgebras of (F,A). If $H_i(x_i) \subseteq H_j(x_j)$ or $H_j(x_j)\subseteq H_i(x_i)$, for all i,j∈α, $x_i\in A_i$,, then the ∨-union $\tilde{\bigvee}_{i\in\alpha}$ (H_i,A_i) is a soft BH-subalgebra of $\tilde{\bigvee}_{i\in\alpha}(F,A)$.
Proof.Similar to the proof of proposition 6.

Theorem 3. Let (F,A) be a of soft BH-algebra over X and $\{(H_i,A_i) \mid i\in\alpha\}$ a nonempty family of soft BH-subalgebras of (F,A). Then the cartesian product of the family $\tilde{\prod}_{i\in\alpha}(H_i, A_i)$ is a soft BH-algebra over $\tilde{\prod}_{i\in\alpha}(F,A)$.

Proof. By definition 12, we can write $\tilde{\prod}_{i\in\alpha}(H_i, A_i) = (H,B)$, where $B = \prod_{i\in\alpha}A_i$ and$H(x)=\prod_{i\in\alpha}H_i(x_i)$, for all $x=(x_i)_{i\in\alpha}\in B$. Let $x = (x_i)_{i\in\alpha}\in$Supp (H,B). Then $H(x) =\prod_{i\in\alpha}H_i(x_i) \neq\emptyset$, and so we have$H_i(x_i) \neq\emptyset$,for all i∈α. Since $\{(H_i,A_i) \mid i\in\alpha\}$is a soft BH-subalgebras of (F,A),we have that $H_i(x_i)$ is a BH-subalgebra of $F(x_i)$,from which obtain that$\prod_{i\in\alpha}H_i(x_i)$ is a BH-subalgebra of $\prod_{i\in\alpha}F(x_i)$, for all $x = (x_i)_{i\in\alpha}\in$Supp(H,B). Hence, the cartesian product of the family $\tilde{\prod}_{i\in\alpha}(F_i,A_i)$ is a soft BH-algebra over $\tilde{\prod}_{i\in\alpha}(F,A)$.

Proposition15. Let f : X → Y be a homomorphism of BH-algebras and (F,A), (G,B) two soft BH-algebras over X. If $(G,B)\lesssim_s(F,A)$.
Then $(f(G),B)\lesssim_s (f(F),A)$.

Proof. Assume that $(G,B)\lesssim_s (F,A)$. Let x∈Supp(G,B). Then x ∈Supp (F,A). Bydefinition 17, A ⊆B and G(x) is a BH-subalgebra of F(x) for all x∈Supp (G,B). Since f is a homomorphism, $f(G)(x)=f(G(x))$ is a BH-subalgebraof $f(F(x)) = f(F)(x)$.Therefore $(f(G),B)\lesssim_s (f(F),A)$.

Theorem 4. Let f : X → Y be a homomorphism of BH-algebras and (F,A), (G,B) two soft BH-algebras over Y. If $(G,B)\lesssim_s (F,A)$.
Then $(f^{-1}(G),B)\lesssim_s (f^{-1}(F),A)$.

Proof. Assume that $(G,B)\lesssim_s (F,A)$. Let y∈Supp$(f^{-1}(G),B)$.By definition17 B⊆A and G(y) is a BH-subalgebra of F(y), for all y∈B. Since f is a homomorphism, $f^{-1}(G)(y) =f^{-1}(G(y))$ is a BH-subalgebra of $f^{-1}(F(y)) = f^{-1}(F)(y)$, for all y ∈Supp $(f^{-1}(G),B)$ .Hence , $(f^{-1}(G),B)\lesssim_s(f^{-1}(F),A)$

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