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Modeling, Simulation and Control of Flow Tank System

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Abstract: ***Process control refers the methods that are used to control and manipulating of processes variable in manufacturing a product. It has manyimportances in industrial processes. In this paper, a brief introduction about several aspects of process control has been provided on flow tank. Basic types of controller and their tuning methodshave been discussed here. This paper mainly concern with modelling and designing of controller on real time single tank system and simulation. Here theoretical and practical processes model for single tank system are identified and compare on real time work. From those best model used for calculating parameters of PID Controller based on servo and regulatory system. Here three controller tuning methods such as Relay- Auto-Tunning, Ziegler-Nichols, and Tyreus-Luyben are used to design controller for flow tank and compare their output response in real tome flow tank system and on simulation.***

Keywords: SISO, MIMO, P-controller, PI-controller AND PID-controller

1. Introduction

Chemical industry constitute of a very vast, complex and sensitive processes. It is very difficult to handle such a complex network of chemical processes without interrupting the goal (i.e. a quality product with economically optimum operation). Every process has operating conditions which are to be maintained during working time. The violation of operating conditions may be hazardous and even may cause human death. Hence for the purpose of harnessing such destructive processes and for the well-being operation it is very necessary to design a device which will satisfy all its needs. And this has become the origin of controller.

1.1 Definition of Process Control and the Problem

Process control is a mixture between the statistics and engineering discipline that deals with the mechanism, architectures, and algorithms for controlling a process. Some examples of controlled processes are:

- Controlling the temperature of a water stream by controlling the amount of steam added to the shell of a heat exchanger.
- Maintaining a set ratio of reactants to be added to a reactor by controlling their flow rates.
- Controlling the height of fluid in a tank to ensure that it does not overflow.

The central problem in control is to find a technically feasible way to act on a given process so that the process behaves, as closely as possible, to some desired behaviour. Furthermore, this approximate behaviour should be achieved in the face of uncertainty of the process and in the presence of uncontrollable external disturbances acting on the process [1].

There are three general classes of needs that a control system is called on to satisfy:

1. Suppressing the influence of external disturbances,
2. Ensuring the stability of a chemical process, and
3. Optimizing the performance of a chemical process.

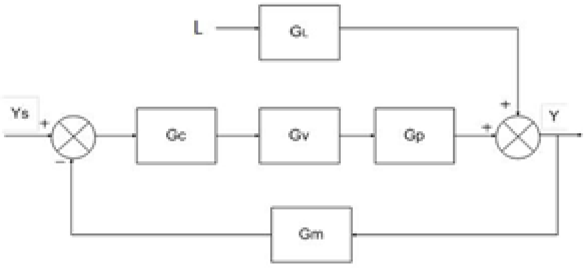

Figure 1.1: Block Diagram of Control System

Consider the basic feedback structure as shown in figure1-1. Where *Gp* represents the consolidated transfer function of the process and *Gc, Gv ,Gm* represents transfer functions of controller, final control element and measurement device respectively. In this case the closed loop transfer function is given as [8]

$$\frac{Y}{Y_s} = \frac{G_c G_p G_v}{1+G_c G_p G_v G_m}$$

1.2 Control Configurations

Depending on how many controlled outputs and manipulated inputs we have in a chemical process, we can distinguish the control configurations as:

1) SISO (Single Input Single Output) Systems.
2) MIMO (Multiple Input Multiple Output) Systems.

The steps involved in both the problems for the design of controller are similar rather they are different in mechanism and model formulations.

1.2.1 SISO Systems:

When a process has only one input variable to be used in controlling one output variable, then that system is called as SISO system. Consider a very simple process as shown in fig. below in which the control objective is to maintain constant temperature of the system. In this case temperature of the system is measured by using thermocouple and it is fed back to regulate the manipulated stream i.e. steam flow rate here so as to maintain constant temperature [1].

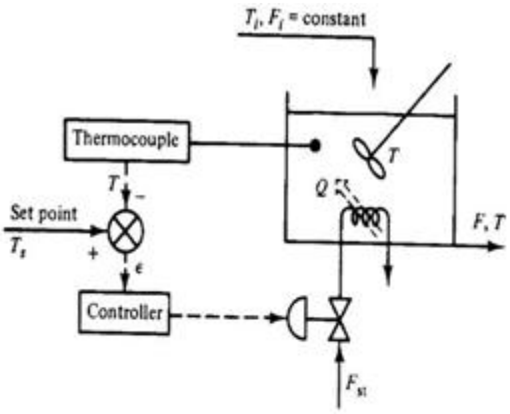

Figure 1.2: Maintain constant temperature

1.2.2 MIMO Systems:

A multivariable process is one with multiple inputs, u_1, u_2, u_3, …, u_m, and multiple outputs, y_1, y_2, y_3, …, y_n where m is not necessarily equal to n; it could be a single process, such as the stirred mixing tank, or it could be an aggregate of many process units constituting part of an entire plant, or it could be the entire plant itself. Consider the stirred mixing tank shown in Figure 1.3 has two input variables, the cold stream flow rate, and the hot stream flow rate, to be used in controlling two output variables, the temperature of the liquid in the tank, and the liquid level. [1]

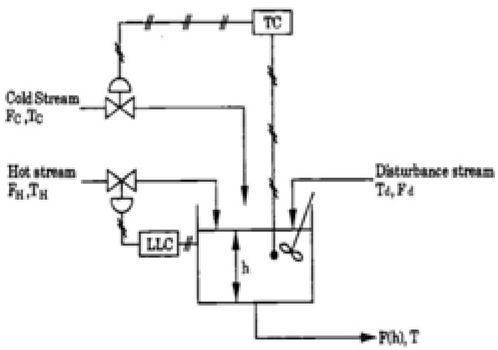

Figure 1.3: Maintain constant temperature and liquid level in the tank.

The MIMO systems are different from SISO due to more number of variables as its name itself indicates but rather important is interaction between input and output variables. This leads to the following problems.

Input / Output Pairing

The interaction between input and output variables leads to the difficulty in choosing best Input/output pairing i.e. Which input variable should be used in controlling which output variable?

There are two possible configurations (consider an example of Figure 1.3):

1. Use hot stream flow rate to control liquid level, and use cold stream to control liquid temperature.
2. Use cold stream flow rate to control liquid level, and use hot stream to control liquid
3. Temperature.

One of the consequences of having several input and output variables is that such a control system can be configured several different ways depending on, which input variable is paired with which output variable. Each u_i to y_i "pairing" constitutes a control configuration, and for a two-input, two-output system (often abbreviated as a 2 x 2 system) we have two such configurations.

Configuration1	Configuration2
u1-y1	u1-y2
And	
u2-y2	u2-y1

It can be shown that for a 3x3 system, there are six such configurations; for a 4 x 4 system there are twenty-four; and in general, for an *n* x *n* system, there are *n!*Possible input-output pairing configurations. We know, of course, that our controllers can be set up according to *only one* of these configurations. Furthermore, we would intuitively expect one of the configurations to yield "better overall control system performance" than the others: how, then, to choose among these possibilities?

At the simplest level, therefore, the first problem in the analysis and design of multivariable control systems is that of deciding on what input variable to pair with what output variable; and the problem is by no means trivial. [1]

1.3 Types of Feedback Controller:

In this section, we present the transfer functions for the controllers frequently used in industrial processes. Relay based PID controller is designed for the Feedback Coupled tanks system. So here we will see brief introduction about P, I and D controller modes [10].

Proportional Controller

The simplest type of controller is the proportional controller. (The ON/OFF control is really the simplest, but it is a special case of the proportional controller as we'll see shortly.) Our goal is to reduce the error between the process output and the set point. The proportional controller, can reduce the error, but cannot eliminate it. If we can accept some residual error, proportional control may be the proper choice for the situation.

The proportional controller has only one adjustable parameter, the controller gain. The proportional controller produces an output signal (pressure in the case of a pneumatic controller,current, or voltage for an electronic controller) that is proportional to the error ε. This action may be expressed as

Proportional Controller,
$P= K_c\varepsilon+ p_s$

Where,
p – Output signal from controller, psig or

K_c–proportional gain or sensitivity

ε –Error (set point)–(measured variable)

p_s–a constant, the steady-state output from the controller or the bias value

Equation in deviation form, $P(t) = K_c\varepsilon(t)$

$$\frac{P(t)}{\varepsilon(t)} = K_c$$

Proportional-Integral (PI) Controller:

If we cannot tolerate any residual error, we will have to introduce an additional control mode: integral control. If we add integral control to our proportional controller, we have what is termed PI, or proportional-integral control. The integral mode ultimately drives the error to zero. This controller has two adjustable parameters for which we select values, the gain and the integral time. Thus it is a bit more complicated than a proportional controller, but in exchange for the additional complexity, we reap the advantage of no error at steady state. PI control is described by the relationship,
Proportional-Integral controller,

$$p = K_c\varepsilon + \frac{Kc}{\tau}\int_0^t \varepsilon dt + p$$

Where,
K_c – proportional gain
τ –Integral time, min
p – Constant (the bias value)

Transfer Function,

$$\frac{P(s)}{\varepsilon(s)} = Kc(1 + \frac{1}{\tau/_s})$$

Proportional-Integral-Derivative (PID) Controller:

Derivative control is another mode that can be added to our proportional or proportional-integral controllers. It acts upon the derivative of the error, so it is most active when the error is *changing* rapidly. It serves to reduce process oscillations. This mode of control is a combination of the previous modes and is given by the expression,

Proportional-Integral-Derivative controller,
$$p = K_c\varepsilon + \frac{Kc}{\tau}\int_0^t \varepsilon dt + (Kc)\tau\frac{d\varepsilon}{dt} + p$$

Where,
τ – Derivative time, min
In this case, all three values K_c, τ_I and τ_D can be adjusted in the controller.
Transfer function,
$$\frac{P(s)}{\varepsilon(s)} = Kc(1 + \frac{1}{\tau/_s})$$

1.4 Tuning of PID Controller

1.4.1 Ziegler – Nichols PID Tuning Method

Ziegler and Nichols published in 1942 a paper [10] where they described two methods for tuning the parameters of P-, PI- and PID controllers. These two methods are the Ziegler-Nichols" closed loop method (also called as Ultimate gain method), and the Ziegler-Nichols" open loop method (also called as Process reaction curve method). Of these methods the closed-loop method is a trial and error tuning method based on sustained oscillations. This method is probably the most known and the most widely used method for tuning of PID controllers and it is described below. A ¼ decay ratio has considered as design criterion for this method. The advantage of Z-N method is that it does not require the process model. The open-loop method is not described here.

Tuning Procedure

1. Bring the process to (or as close to as possible) the specified operating point of the control system to ensure that the controller during the tuning is "feeling" representative process dynamic and to minimize the chance that variables during the tuning reach limits.
2. Turn the PID controller into a P controller by setting set $\tau_I = \infty$ and $\tau_D = 0$. Initially set gain Kp = 0. Close the control loop by setting the controller in automatic mode.
3. Increase Kp until there are sustained oscillations in the signals in the control system, e.g. in the process measurement, after an excitation of the system. (The sustained oscillations correspond to the system being on the stability limit.) This Kp value is denoted the ultimate (or critical) gain, Kpu.
4. Measure the ultimate (or critical) period Pu of the sustained oscillations.

Calculate the controller parameter values according to Table 1, and use these parameter values in the controller.

Table 1.1: Ziegler-Nichols optimum parameter settings

Controller Type	*Proportional Gain,*	*Integral time*	*Derivative Time*
	Kc	τ_I	τ_D
P	0.5Ku	------	------
PI	0.45Ku	Pu/1.2	------
PID	0.6Ku	Pu/2	Pu/8

1.4.2 Tyreus – Luyben Method:

The Tyreus-Luyben [11] procedure is quite similar to the Ziegler–Nichols method but the final controller settings are different. Also this method only proposes settings for PI and PID controllers. These settings that are based on ultimate gain and period are given in Table 2. Like Z-N method this method is time consuming and forces the system to margin if instability.

Table 1.2: Tyreus-Luyben controller parameter settings

Controller Type	*Proportional Gain, Kc*	*Integral time, τ_I*	*Derivative Time, τ_D*
PI	Ku/3.22	2.2Pu	------
PID	Ku/2.2	2.2Pu	Pu/6.3

1.4.3 Relay Auto-tuning

The majority of the controllers used in industry are of the PID type. A large industrial process may have hundreds of these controllers. They have to be tuned individually to match the process dynamics in order to provide good and robust control performance [13]. The tuning procedure, if done manually, is very tedious and time consuming; the resultant system performance mainly depends on the experience and the process knowledge the engineers have. It is recognized that in practice, many industrial control loops are poorly tuned. Automatic tuning techniques thus draw more and more attention of the researchers and practicing engineers. By automatic tuning (or auto-tuning), we mean a method which enables the controller to be tuned automatically on demand from an operator or an external signal [14]. Typically, the user will either push a button or send a command to the controller. Industrial experience has clearly indicated that this is a highly desirable and useful feature. Earlier authors [14] proposed different auto-tuning methods which have great practical values. However, they all suffer from some major limitations. The Cohen–Coon method requires an open-loop test on the process and is thus inconvenient to apply. The disadvantage of the Yuwana and Seborg method and the Bristol method is the need of large setpoint change to trigger the tuning which may drive the process away from the operating point. Self-tuning controllers based on minimum variance, pole placement or LQG design methods may also be configured to give PID control. These controllers have the disadvantage that a priori information about the time scale of the process dynamics must be provided to determine the suitable sampling intervals and filtering. Besides, a conventional self-tuning controller based on the recursive estimation of a parametric model requires a computer code of few kilobytes. Relay auto-tuning method does not have these shortcomings.

Relay was mainly used as an amplifier in the fifties and the relay feedback was applied to adaptive control in the sixties. Astrom and co-workers successfully applied the relay feedback technique to the auto-tuning of PID controllers for a class of common industrial processes [14]. The relay feedback auto-tuning technique has several attractive features. Firstly, it facilitates simple push-button tuning since the scheme automatically extracts the process frequency response at an important frequency and the information is usually sufficient to tune the PID controller for many processes. The method is time-saving and easy to use [15]. Secondly, the relay feedback auto-tuning test is carried out under closed-loop control so that with an appropriate choice of the relay parameters, the process can be kept close to the set point. This keeps the process in the linear region where the frequency response is of interest, which is precisely why the method works well on highly nonlinear processes [14]. Thirdly, unlike other auto-tuning methods, the technique eliminates the need for a careful choice of the sampling rate from the a priori knowledge of the process. This is very useful in initializing a more sophisticated adaptive controller. Fourthly, the relay feedback auto-tuning can be modified to cope effectively with disturbances and perturbations to the process.

The critical point, i.e. the process frequency response at the phase lag of π, has been employed to set the PID parameters for many years since the advent of the Ziegler–Nichols (Z–N) rule [9]. The point is traditionally described in terms of the ultimate gain K_u and the ultimate period P_u. The relay auto-tuning is based on the observation that a system with a phase lag of at least π at high frequency may oscillate with the period P_u under relay control. To determine the critical point, the system is connected in a feedback loop as shown in Figure 4.1.

Since the describing function of the relay is the negative real axis, the output y (t) is then a periodic signal with the period P_u and the ultimate gain K_u is approximately given by [12]

$$Ku = \frac{4d}{\pi a} \tag{1.1}$$

Where,
d is the relay amplitude and *a* is amplitude of the process output.

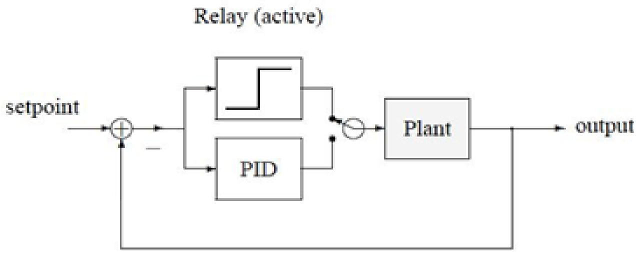

Figure 1.5: Relay feedback system

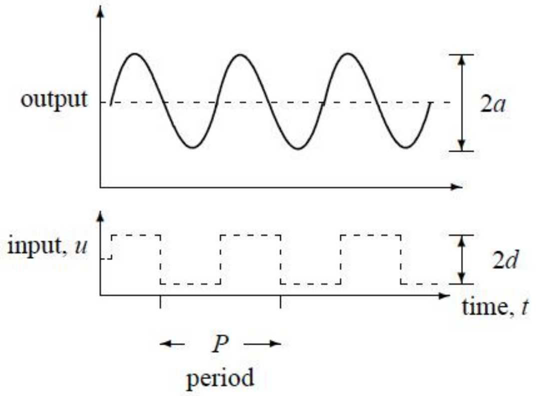

Figure 1.5: Plant Input and Output oscillatory signals using Relay

After getting Pu and K_u, we can calculate controller parameter Kc,τ_I and τ_D by using Z-N table (table1.1)

2. Experimental Setup and Description

2.1 Theoretical Modelling for Single Tank System

Typical chemical plants are tightly integrated processes which exhibit nonlinear behaviour and complex dynamic properties. For decades, the chemical process industry has relied on single-loop linear controllers to regulate such systems. In many cases, the tuning of these controllers can be described as “heuristic". The performance of such a control design in an uncertain process environment is difficult to predict. Although a nonlinear process model of a system may be available, linear process models are usually developed for controller synthesis and system analysis. Using a linear model, an uncertainty characterization can be used to mathematically describe the model error and other variations in the system.

2.1.1 Feedback Coupled Tanks System

The four tanks system has attracted recent attention as it exhibits characteristics of interest in both control research and education [2, 3, and 4]. The four tanks system exhibits elegantly complex dynamics which emerge from a simple cascade of tanks. Such dynamic characteristics include interactions and transmission zero location that are tunable in operation. With appropriate "tuning", this system exhibits non-minimum phase characteristics that arise completely from the multivariable nature of the problem. The four tanks system has been used to illustrate both traditional and advanced multivariable control strategies [4, 5, and 6] and has been utilized as an educational tool in teaching advanced multivariable control techniques [2].

2.1.2 Description

The experimental work carried out in the study is described here. The Coupled Tanks setup is a model of a chemical plant fragment. Very often tanks are coupled through pipes and the reactant level and flow has to be controlled. The Coupled Tanks experiment offers a possibility of system configuration. The couplings between the tanks can be modified to change the dynamics of the system imposing the use of different controllers. The Coupled Tanks unit allows for the design of different controllers and tests in real-time using MATLAB and SIMULINK environment.

As shown in Figure 1.3, the Coupled Tanks unit consists of 4 tanks placed on a rig. Fifth reservoir tank is placed at the bottom. In the reservoir two submersible pumps are placed, which pump the water on command to the tanks. The water flows freely to the bottom tanks through the configurable orifice. The way the water flows through the setup can be configured in many ways with manual valves labelled (MVA, MVB, MVC, MVD, MVE, MVF, MVG, MV1, MV2…MV4). Figure 2.1 represents the line diagram of the experimental set up.

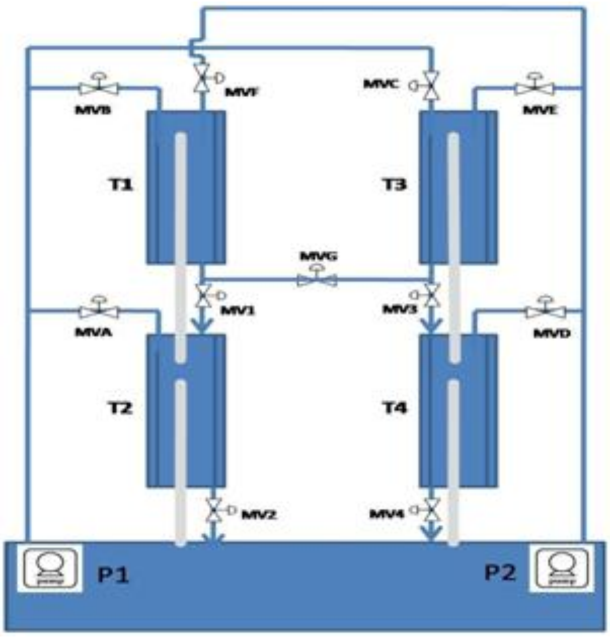

Figure 2.1: Line Diagram for Coupled Tanks system

Configuration with valves allows for dynamics couplings introduction and step disturbances generation giving vast possibilities of control. Apart from the mechanical parts, the Coupled Tanks system is equipped with Power Supply Unit and Power Amplifier (PSUPA), the Universal Control Interface (UCI) presented in Figure 2.2 and Advantech PCI1711 card (not shown in Fig.). The UCI serves as an interface between the PC and the PSUPA. Furthermore, it can be used as a controller. The PSUPA unit amplifies the water pressure-level signals and passes them as analogue signals to the UCI and PC. The pumps control signal can be sent from the PC through the UCI and PSUPA or from the UCI to the PSUPA assuming proper UCI controller is chosen. PCI1711 (Peripheral Communication Interface) card has to be installed in any free PCI slot provided in UPS after installing Advantech software. This card is used as a SCSI connector for UCI with PC. After installing the Advantech software and PCI1711 card the system is ready to install the Coupled Tanks software [7].

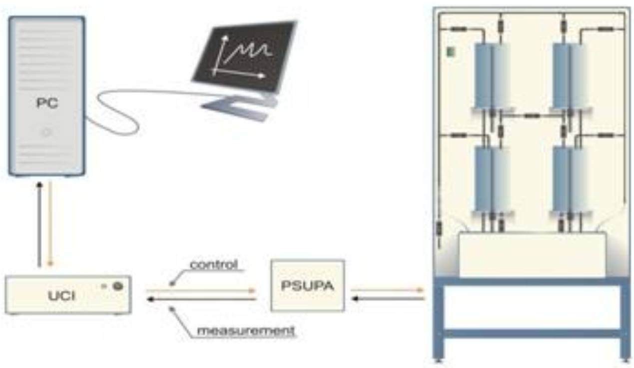

Figure 2.2: Control system schematic

The PC with Advantech card and Matlab and Simulink environment serve as the main control unit. The control signals, which are voltages between 0 V and 5 V, are transferred to the Power Amplifier where they are transformed into 24 V PWM signals driving the pumps. The water level in the tanks is measured using pressure sensors. The water level information is transferred to the PC via the PSUPA.

2.2 Mathematical Modelling

2.2.1 Single Tanks Model

Every control project starts with plant modelling, so as much information as possible is given about the process itself. Firstly a single tank setup for the modelling task is considered. The time constant of the electrical circuit is significantly smaller from the time constant of the tanks, thus the electrical circuits driving the pump can be treated as an amplification gain in the model. Figure 2.4 presents the single tank system with its description.

Theoretical Approach

Usually, phenomenological models are nonlinear, that means at least one of the states (i – pump driving current, h– water level) is an argument of a nonlinear function. In order to present such a model as a transfer function (a form of linear plant dynamics representation used in control engineering), it has to be linearized.

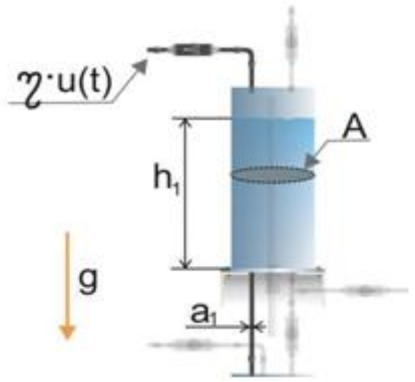

Figure 2.3: single Tank phenomenological model

a) Taking Mass Balance:

$$\frac{dV}{dt} = A\frac{dh}{dt} = q(in) - q(out) \quad (2.1)$$

Where,
V – Volume of tank,
q ($_{In)}$ – inlet,
q ($_{Out)}$ – outlet

b) Bernoulli's Law:

$$p + \frac{1}{2}\rho v^2 + \rho gh = const \quad (2.2)$$

Where,
P – Pressure,
v – Velocity,
h – Height,
ρ – Density

At the water surface of tank velocity, v=0 and at the bottom of tank height, h=0. Hence, it gives,

$$q(out) = a.v = a\sqrt{2gh} \quad (2.3)$$

c) Pump Generated Flow:

$$q(pump) = \eta.u \quad (2.4)$$

Substituting from equations (2.2) through (2.4) in equation (2.1), the simplest nonlinear model of the single tanks system relating the water level h_1 with the voltage u applied to the pump is the following:

$$\frac{dh_1(t)}{dt} = -\frac{a1}{A}\sqrt{2gh(t)} + \eta.u(t) \quad (2.5)$$

Where,

h_1 – water level in tank 1,

a_1–tank 1 outlet area, a_2–tank 2 outlet area,

A –Cross-sectional area of the tanks, g – gravitational constant,

η – Constant relating the control voltage with the water flow from the pump.

Equation (2.6) constitutes a nonlinear model, which has been assembled in Simulink. The bound for the control signal is set to [0 ... +5V].

When controlling the water level in the first tank or in the second tank the plant is a SISO plant – single input single output (Figure 2.3). Water level is the model output and pump control voltage is the control signal (input).For the initial exercise the user has been provided with the nonlinear Coupled Tanks model described by equation (2.6). The model can be opened in Simulink - *CT_model.mdl*. In order to present such a model as a transfer function (a form of linear plantdynamics representation used in control engineering), it has to be linearized.

2.2.2 Linearization

In mathematics linearization refers to finding the linear approximation to a function at a given point. In the study of dynamical systems, linearization is a method for assessing the local stability of an equilibrium point of a system of nonlinear differential equations or discrete dynamical systems.

Working points calculation:

At steady state point of h_{10}, h_{20} and u_0,

$$0 = -\frac{a1}{A}\sqrt{2gh_{10}(t)} + \eta.u_0(t) \quad (2.6)$$

$$0 = -\frac{a1}{A}\sqrt{2gh_{10}(t)} - \frac{a2}{A}\sqrt{2gh_{20}(t)} \quad (2.7)$$

$$-\frac{a1}{A}\sqrt{2gh_{10}(t)} = \eta.u(t)$$

$$\rightarrow h_{10} = \frac{1}{2g}\left(\frac{\eta.uA}{a1}\right)^2 \quad (2.8)$$

$$\frac{a1}{A}\sqrt{2gh_{10}(t)} = \frac{a2}{A}\sqrt{2gh_{20}(t)}$$

$$\rightarrow h_{20} = \left(\frac{a1}{a2}\right)^2 h_{10} \quad (2.9)$$

Linearization makes it possible to use tools for studying linear systems to analyse the behaviour of a nonlinear function near a given point. The linearization of a function is the first order term of its Taylor expansion around the point of interest. For a system defined by the equation,

$$\frac{dx}{dt} = F(x,t) \quad (2.10)$$

The linearized system can be written as

$$\frac{dx}{dt} = F(x_0,t) + DF(x_0,t).(x - x_0) \quad (2.11)$$

From equation (2.6),

$$\frac{dh(t)}{Dt} = -\frac{a}{A}\sqrt{2gh_{10}(t)} + \eta.u_0(t) + D\left(\frac{a1}{A}\sqrt{2gh_{10}(t)}\right)\Big|_{h=h_{10}}.(h_1(t) - h_{10}(t)) \quad (2.12)$$

Solving for second term,

$$D\left(\frac{a1}{A}\sqrt{2gh_{10}(t)}\right)\Big|_{h=h_{10}} = -\left(\frac{a2}{A}\right)\frac{1}{\sqrt{2gh_{10}}}2g$$

Substituting from equation (2.9),

$$= -\left(\frac{a2}{A}\right)\frac{1}{\eta\mu_0/(a1/A)}2g$$

$$= -\left(\frac{a1}{A}\right)^2\frac{g}{\eta u_0}$$

From above equations,

$$\frac{dh(t)}{Dt} = -\frac{a}{A}\sqrt{2gh_{10}(t)} - \left(\frac{a1}{A}\right)^2 \frac{g}{\eta u_0} +.\left(h_1(t) - h_{10}(t)\right) \quad (2.13)$$

At steady state,

$$0 = -\frac{a1}{A}\sqrt{2g\,h_{10}(t)} + \eta.u_0(t) \quad (2.14)$$

Subtracting equation (1.15) from (1.14) which gives in the form of deviation variable

$$\frac{d(h_1(t) - h_{10}(t))}{dt} = \eta\left(u(t) - u_0(t)\right) - \left(\frac{a1}{A}\right)^2 \frac{g}{\eta u_0} +.\left(h_1(t) - h_{10}(t)\right) \quad (2.15)$$

$$\Delta h_1(t) = \eta.\Delta u(t) - \left(\frac{a1}{A}\right)^2 \frac{g}{\eta u_0}.\Delta h_1(t) \quad (2.16)$$

The above equation (2.17) could also be obtained directly from (2.6) as,

$$\frac{d\Delta h(t)}{Dt} = \mathrm{D}\left(\frac{a1}{A}\sqrt{2g\,h_{10}(t)}\right)\Big|_{h=h_{10}} \Delta h_1(t) + \eta\Delta u(t) \quad (2.17)$$

$$d\Delta h_1 = \eta\Delta u(t) - \left(\frac{a1}{A}\right)^2 \frac{g}{\eta u_0} d\Delta h_1 \quad (2.18)$$

2.2.3 Laplace Transformation

Taking Laplace transforms of equations (2.17), it gives transfer function model for Tank 1

$$s\Delta H_1(s) = -\left(\frac{a1}{A}\right)^2 \frac{g}{\eta u_0}\Delta H_1(s) + \eta.U(s) \quad (2.20)$$

Consequently, the respective transfer functions are as follows:

$$\frac{\Delta H_1(s)}{\Delta U(s)} = \frac{\eta}{s + \left(\frac{a1}{A}\right)^2 \frac{g}{\eta u_0}} \quad (2.21)$$

Hence, the process transfer function for single tank system is,

$$\frac{\Delta H_1(s)}{\Delta U(s)} = G_p(s) = \frac{k_p}{\tau_p s + 1} \quad (2.22)$$

Where,

$$k_p = \frac{\eta}{\left(\frac{a1}{A}\right)^2 \frac{g}{\eta u_0}}$$

$$\tau_p = \frac{1}{\left(\frac{a1}{A}\right)^2 \frac{G}{\eta u_0}}$$

For a given system, Feedback Coupled tanks system, following are the values of constants,
$\eta = 2.4\text{x}10^{-3}$, $a_1 = 50.265\text{x}10^{-6}$ m^2,
$A = 0.01389$ m^2, $g = 9.81$ m^2/s, $u_0 = 3.17$ V
After substituting these values we get transfer function as,

$$G_p(s) = \frac{0.14213}{59.22s + 1} \quad (2.23)$$

This is a transfer function for given single tank system.

3. Experimental Work

In the previous chapters, Introduction to Feedback Coupled tank system, its modelling and formation of Transfer function is covered. This chapter includes the experimental procedure of Sensor Calibration, Static Characteristics and Real-time Model Identification.

After complete installation of Feedback Coupled tanks system software, which includes Coupled tanks Simulation models and Coupled tanks Real-time models (as shown in Figure 3.1), open the Coupled tanks Real-time models menu.

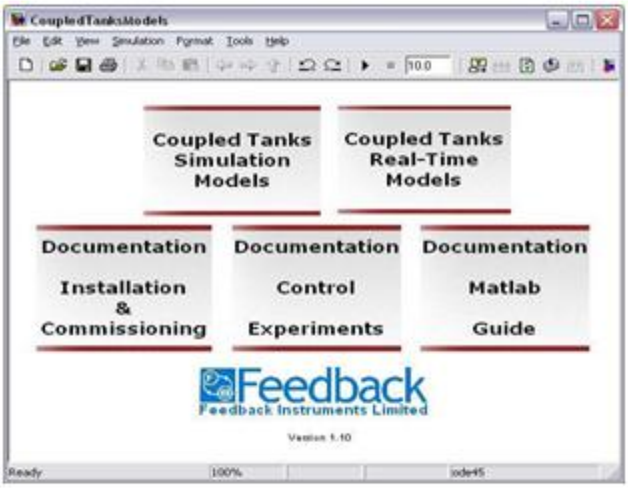

Figure 3.1: Simulink model menu

3.1 Sensor Calibration

Each sensor has to be calibrated before any experiments are run. The calibration should be done periodically to ensure agreement between the real data and the measured signal. With time passing the water might change its density, which might influence the level reading.

Calibration Procedure

Follow the instructions below to calibrate the sensors. The sequence has to be repeated for each sensor and tank.

1. Shut the outflow valves of the tank, in which you are going to perform calibration.
2. Open the proper pump output valve (valve **MVB** for tank T1 calibration, valve **MVA** for tank T2 calibration, valve **MVE** for tank T3 calibration, valve **MVD** for tank T4 calibration). Keep the other valves closed, so the water flows directly to the tank, which is being calibrated.
3. Double click on the *CT calibration* block in the Real-Time Models menu (as shown in Figure 3.2). Follow the given instructions to perform calibration on all of the tanks.

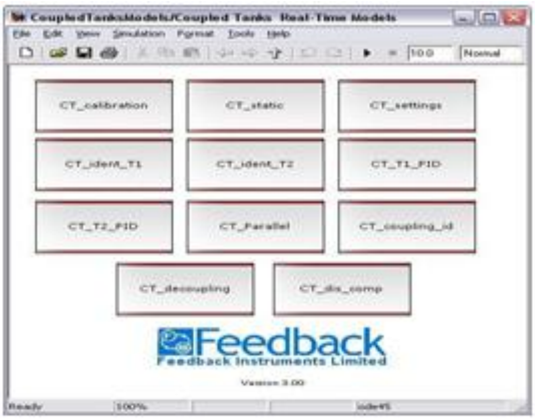

Figure3.2: Real-time models menu

4. Select the tank number for calibration [1, 2, 3, and 4].
5. Turn the proper gain potentiometer (lower potentiometer) counter-clockwise until you reach the boundary position or the potentiometer clicks.The gain will be driven to the minimal. Confirm if the gain is zeroed [y/n] (n – will cancel the calibration)
6. Set the corresponding offset potentiometer so the voltage display in CT calibration displays 1 [V] +/- 0.1 [V].

Confirm that the desired voltage is displayed [y/n] (n- will cancel the calibration)

7. Try to reach the 25 cm water as close as possible.

Adjust the gain potentiometer so the voltage displayed in CT calibration displays 3.5 [V] +/- 0.1 [V] (as shown in Figure 3.3). Confirm that the desired voltage is displayed [y/n] (n- will cancel the calibration).

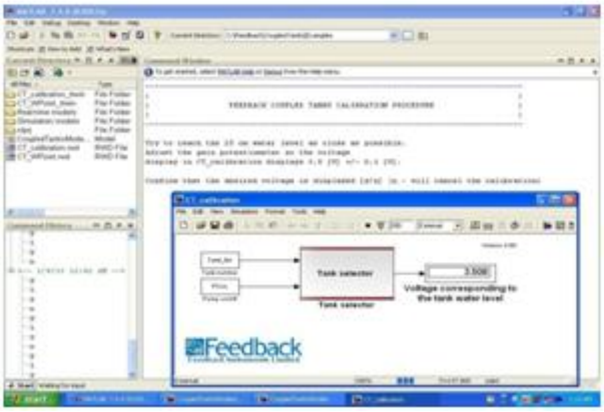

Figure 3.3: Calibration window

8. Empty the calibrated tank. Confirm that the tank is empty [y/n] (n- will cancel the calibration)
9. The files containing the calibration constants will be saved in the Real-Time models directory.

The calibration has to be performed before any exercises are run. Calibration constants will be saved in a file and automatically loaded when models are run. The calibration gives the relation between the voltage measured by potentiometer (level measurement device) and the corresponding height in the tank. The calibration coefficients linear relations between potentiometer voltage and height are saved in calibration text file in the Real-time models directory and automatically loaded when models are run.

3.2 Static Characteristic Identification

Before any experiments are run it is necessary to define the equilibrium for the Coupled Tanks system. For that purpose a static characteristic of the system has to be identified. The equilibrium can be determined from that static characteristic. Use the *CT_Static.m* to identify the static characteristic. You can use the same m-file to change the working point according to the previously identified static characteristic. Every algorithm will use the working point information for start-up and the control will be performed around the working point water level height and the corresponding control voltage. The simulation is performed automatically. The water height and the control voltage working point values are saved and each of the models will read the working point information.

Working point identification:

Make sure the valves are set-up correctly for this exercise. Open valves MVB and MVE. The rest of the valves should be kept closed. Make sure the unit sensors have been calibrated according to the instructions given in the earlier chapter "Sensor Calibration".

Task

The experiment lasts 2000 seconds. Make sure the pumps are properly submersed in the water. Compile and run the model. The working point water levels and control voltages are recorded for both columns. The static characteristic will be recorded. The default working point in terms of water height will be set to 20 cm in the top tank. The voltage will be adjusted according to that level depending on the identified static characteristic. You can change the working point afterwards by running the same CT_static.m file, but omitting the static characteristic identification procedure this time.

Static Characteristics results and comments:

The default working point is saved automatically. The static characteristic is saved and displayed for all 4 tanks in groups of 2. You can look the settings up by executing the CT_Settings.m, by typing CT Settings in the MATLAB command window.
The calibration, static characteristic and working point data is displayed as follows:

FEEDBACK COUPLED TANKS SETTINGS

Voltage to centimetres conversion:

Tank 1: h1 = 12.4163 * u1 -18.5007

Tank 2: h2 = 11.5173 * u2 -15.3525

Tank 3: h3 = 13.0679 * u3 -20.7614

Tank 4: h4 = 12.3216 * u4 -18.1442

3.3 Model Identification by Practical Approach:

In the previous section a phenomenological model was derived and then linearized. A model can be identified through an identification experiment, upon which controllers will be designed, which is described in the "*Coupled Tanks Control"* section.
Open valve MVB (leave MVE open – for safety). The rest of the valves should be kept closed. Make sure the unit sensors have been calibrated according to the instructions given in the earlier chapter "Sensor Calibration". The control real time simulations are carried out with a sampling time of *Ts* = 0.1 [s]. The model identification is carried out with the same sampling time. For the identification the Matlab System Identification Toolbox is used. The identification experiment is carried out using the model called *CT_ident_T1.mdl* and *CT_ident_T2.mdl*. This models use an
Excitation signal to vary the value of the control signal, which results in the changes of the water level in the tanks. The experiments last 300 and 1000 seconds and three signals are collected in the form of vectors and are available in the Workspace.

Task

1. Run the model *CT_ident_T1.mdl* in Simulink. Make sure the pumps are properly submersed in the water. The

water levels and the control signal are recorded.

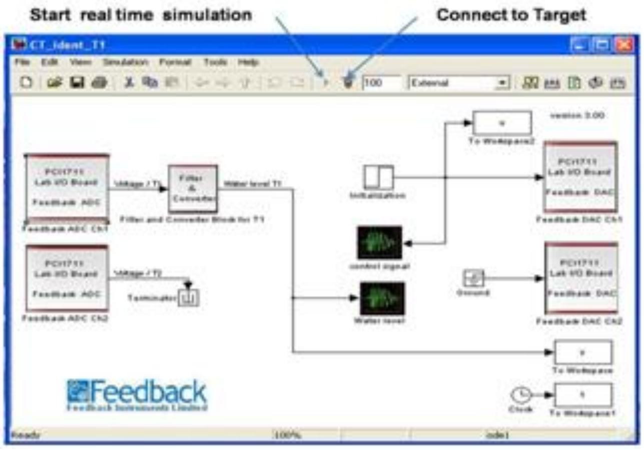

Figure 3.4: Single tank identification window

2. Compile the model by pressing „Ctrl+B". Wait for the compilation to finish. The "successful completion of Real-time workshop build procedure for model: <Model Name> " statement should appear in the command window when the compilation has finished.
3. Press the „Connect to target"  button. You may turn the device power ON.
4. Press the „Start simulation"  button. The identification experiment will be carried out.
5. When the experiment finishes, due to the fact that the „To Workspace" block has been used some of the necessary signals will be available in Workspace. Then assign these signals to some variables so that these signals can be used for model identification.
6. Type „indent" at the Matlab command line. The identification interface will open.
7.

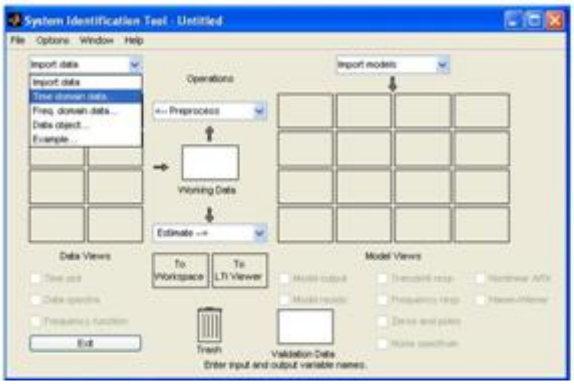

Figure 3.5: Model identification window

7. Select the Import Data drop-down tab then Time Domain Data. Fill in the dialogue with proper input-output variable names and select Import. Remember to specify the proper sampling time.
8. Now select Estimate→ then Parametric Model. Select the OE structure and enter the order specified in the proper exercise of the Control Experiments.
9. The estimated model will appear in the list of imported models.

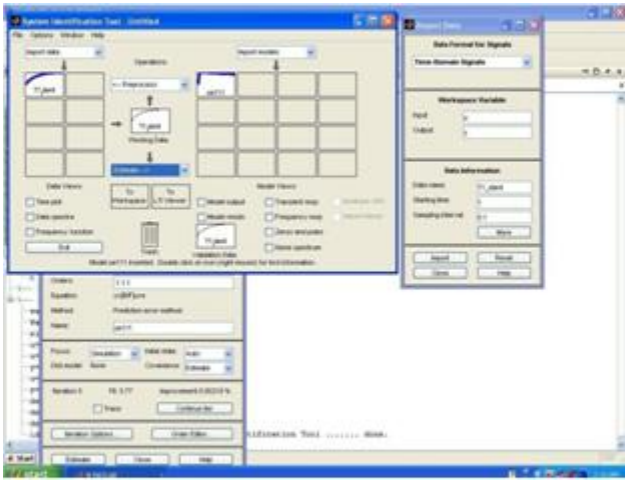

Figure 3.6: Model created and ready for analysis

I have identified the model using the methods described in the mentioned above.
A FOPTD model for Tank 1 is obtained as follows:

Tank 1:

$$Gp(s) = \frac{0.67}{8.45s + 1}$$

(FOPTD Practically Identified Model)

Theoretical model is identified in previous chapter is:

$$Gp(s) = \frac{0.14213}{59.22s + 1}$$

(FOPTD Theoretical Identified Model)

3.4 Control the Height of Tank1 by PID Controller in Real Time System

We have identified the transfer function of the single tank1 system in previous section. Now we have to calculate controller parameter Kc by using Relay-tuning method or any other method as discussed before. Here we are using **Relay-Tuning method.** By using the value of controller parameters, we have to check whether parameters controlling the tank or not for Servo and Regulatory problem.

Task

1. We have to create simulation block diagram first using relay for first order time delay system, which is already created.
2. Go to desktop and open the file „relay-tuning". The window will open as;

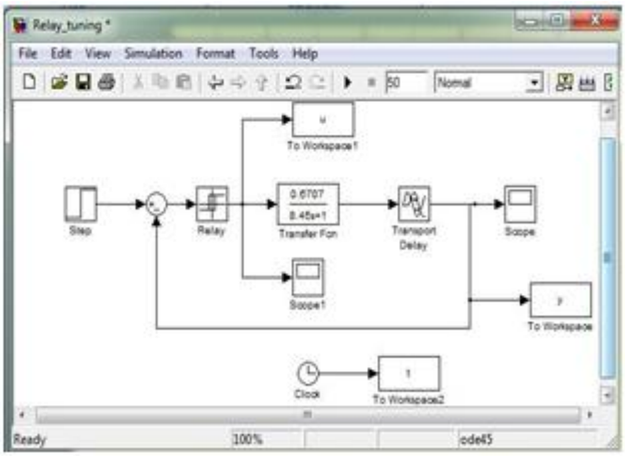

Figure 3.7: Simulation block diagram of relay system

3. Now inserting the transfer function which is already identified in the transfer function block.
4. Giving the limit for relay is [-1, 1]
5. Inserting the identified time delay in transportation delay block which is 0.01.
6. Save and run the model.
7. We get output graph from scope as;

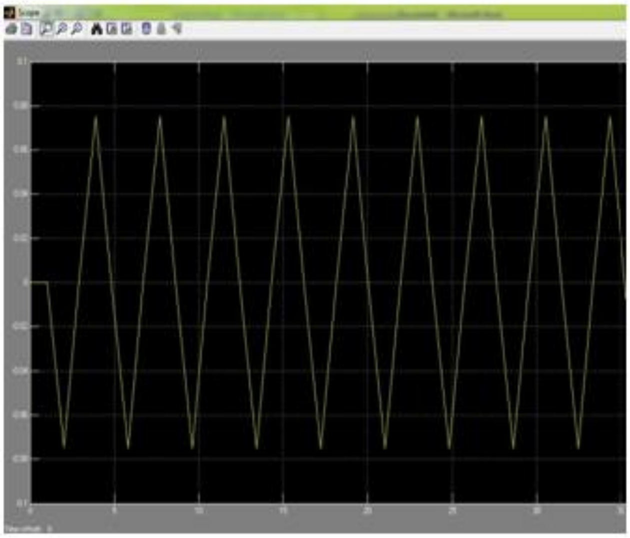

Figure 3.8: Output response of relay system

9. By relay method, we have:

$$Ku = \frac{4d}{\pi a} = 16.93$$

Pu=4

Where

h=limit of relay taken,

A=amplitude of sustained curve obtained.

Ku=ultimate value of proportional gain,

Pu=ultimate period of oscillation = difference between two successive peaks time, after getting sustained oscillations.

10. By Z-N method Table:

Kc = 9.95,

τ_I = 1.9,

τ_d = 0.475

3.4.1 Controlling by Servo System:

PID parameter values obtained above are then applied on Real-time Coupled Tanks tank T1 model. In this Real-time model step input change in set point, water level, is given. The purpose behind giving step input change is that there will be continuous variation in set point called Servo system and from the corresponding PID controller response one can say that these are the results for highest variation in input change.

Task

1. Go to control model of tank-1 which is stored at desktop by name “CT_T1_PID2” and open it.
2. We get the window;

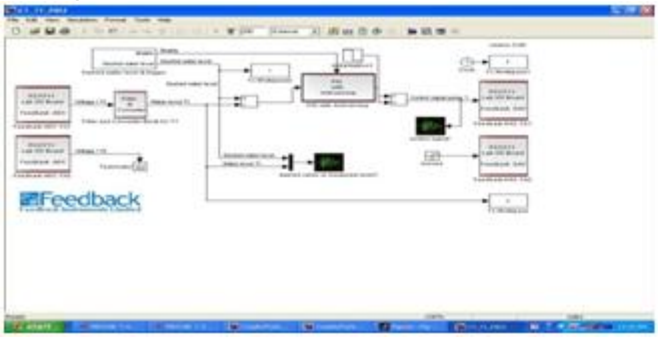

Figure 3.9: Tank1 PID model

3. Insert values of tuning parameters found in PID with anti-windup block.
4. Again adjust the step change and sample time.
5. Connect to target with build (ctrl+B). Run the simulation.
6. Observe the graph.

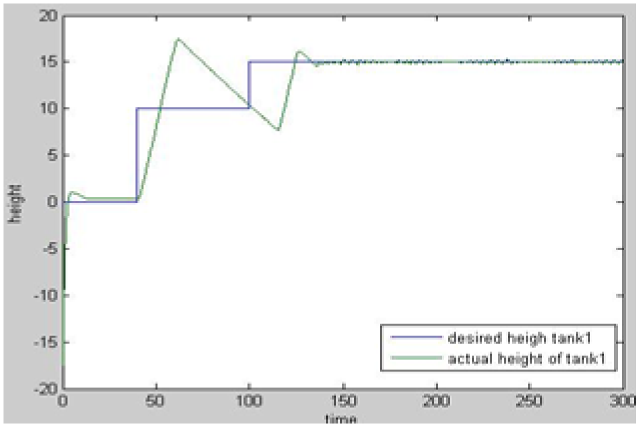

Figure 3.10: Output response of tank1 by servo system

7. Compare the desired values vs. Actual water level of Tank1 in Graph
8. Deciding whether parameters are controlling the tank or not.

Same procedure had done for Theoretical Model and calculated values of parameters by Relay-Tuning method and checked out whether parameters are controlling the tank or not. Calculated values of parameters for PID controller using theoretical model are:

Kc=156, $\tau I = 2 , \tau d = 0.5$

3.4.2 Controlling by Regulatory System

In this Real-time model step input is fixed, i.e. water level is fixed some value and change the input variable (input flow rate). After inserting the value of tuning parameter, model connects to target and run the model. By changing the position of valve of input flow of the water, the system getting disturbed called Regulatory system. After disturbing the system, observe the output response. Output response for regulatory is given below:

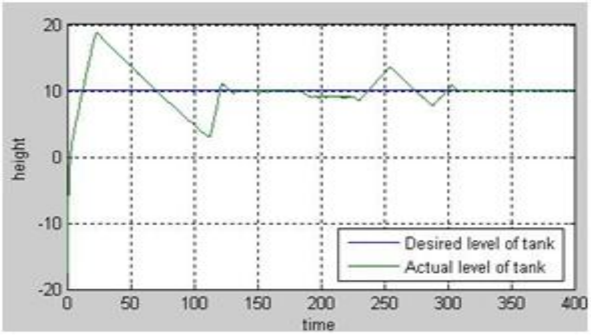

Figure 3.11: Output response of tank1 by Regulatory system

4. Results and Discussion

In this project three different methods for controller design are considered. These are Relay-Auto tuning Method, Ziegler-Nichols Method, Tyreus-Luyben Method. These methods are based on based on ultimate parameters i.e. ultimate gain and ultimate period for a real time system. These ultimate parameters are found by using Relay. Theoretical model also consider for controller design and comparison. Controlling the height of tank by PID controller with the controller parameters are calculated by different methods in real time model and checked in simulation also.

4.1 Comparison of Theoretical and Practical Model

Estimation of K_u and P_u using Relay

In this session ultimate gain, K_u and ultimate period, P_u for the theoretically and practically obtained models are found out by using Relay auto-tuning method. The simulation model and Relay responses for respective process model are shown below.

Simulink Block Diagram for Theoretical Model:

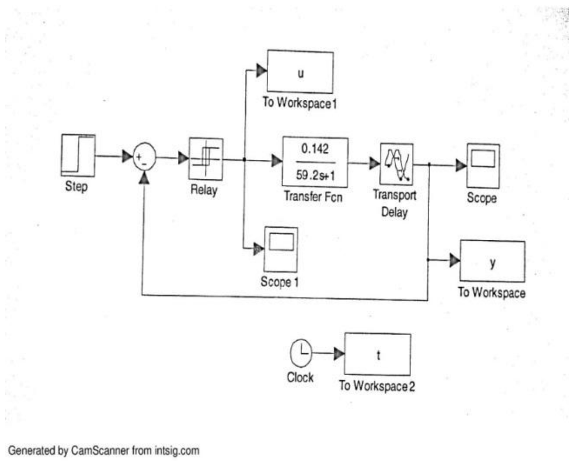

Figure 4.1: Simulink model for Theoretical transfer function

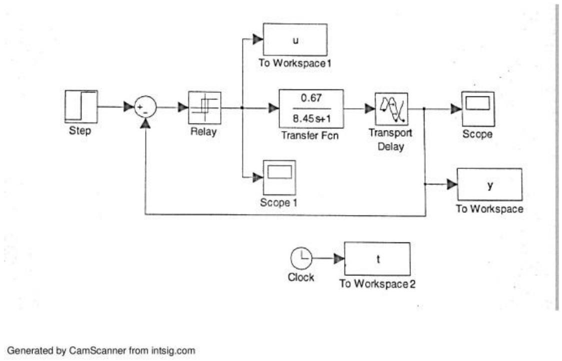

Figure 4.2: Simulink model for Practical transfer function

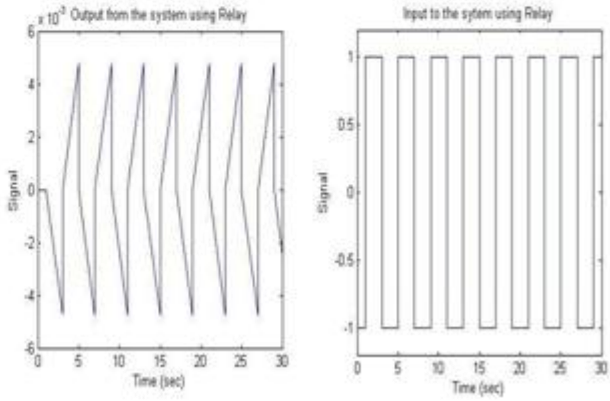

Figure 4.3: Input and Output to Theoretical model using Relay

From the Relay auto-tune responses (Figure 4.3 and Figure 4.4) for theoretically and practically obtained models, the values of ultimate gain (Equation 1.1), ultimate period (Figure1.2) and PID parameters using Ziegler-Nichols settings (Table 1.1) are found to be as follows (shown in Table 4.1),

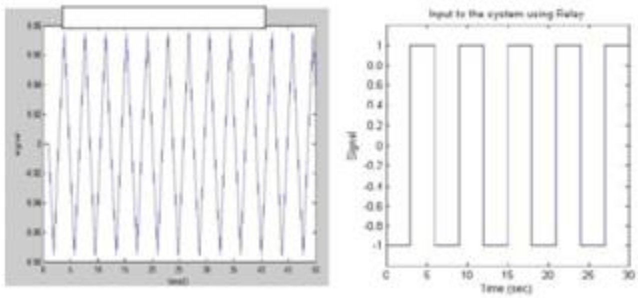

Figure 4.4: Input and Output to Practical model using Relay

Table 4.1: PID values for theoretical and practical model

Model	*Ultimate Gain,*	*Ultimate Period,*	*PID Controller*		
	K_u	P_u (sec)	K_c	τI	τD
Theoretical Model	265.5	4	156	2	0.5
Practical Model	8.72	4	5.11	2	0.5

Comparison of PID controller for Theoretical and Practical Model

These values of PID controllers are used on Coupled Tanks tank 1 Real-time PID model and we have got the following results. In this task a step input change is given to the set point (water level) so as to provide continuous change in input. This gives desired water level in the form of sinusoidal wave. The water level is measured using sensor in the form of voltage and is compared with desired water level.

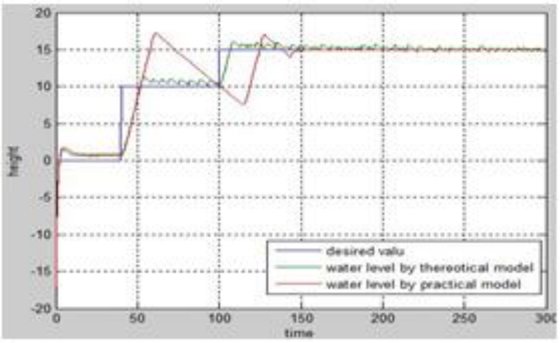

Figure 4.5: Real time response of tank1 by using theoretical and practical model

Figure 4.5 shows the Real-time response of PID controller tuned for tank T1 by using theoretical model and practical model. From this response it is observed that the water level in tank T1 is closely maintained to that of desired water level.
Hence it is concluded that the theoretical model gives best results for single tank system and that can be used for single tank PID controller design. The practical model obtained here is not giving desired results and this model cannot be used for PID controller design and needs to improve the model identification method.

4.2 Comparison of Relay-Tuning, Z-N, T-L methods by Real Time system

Relay-Tuning, Ziegler-Nichols (ZN) and Tyreus-Luyben (TL) methods are dependent on frequency response of process model. Theoretically designed model for single tank system is considered for further study. The values of critical data, ultimate gain and ultimate period, are found from Relay based frequency response (Figure 4.3) of single tank system. These values are then used for above three methods and controller parameters are calculated by using respective parameter setting (Table 1.1, 1.2). Controller parameters obtained for these three methods are shown in following table

Table 4.2: PID parameters for Relay-tuning, Z-N and T-L methods,

PID	*Relay-Tuning*	*Zeigler-Nichols*	*Tyreus-Luyben*
k_c	9.95	8.97	6.931
τ_I	1.9	1	4.4
τ_D	0.475	0.25	0.31

PID parameter values obtained above (shown in Table 4.2) are then applied on Real-time Coupled Tanks tank T1 model. In this Real-time model sinusoidal input change in set point, water level, is given. The purpose behind giving sinusoidal input change is that there will be continuous variation in set point and from the corresponding PID controller response one can say that these are the results for highest variation in input change. Single tank PID controller responses for Relay-Tuning, ZN and TL methods are superimposed on single plot and are shown below (Figure 4.6).

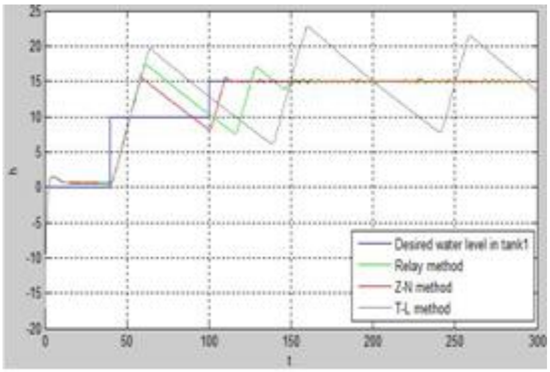

Figure 4.6: Comparison of PID response for Relay, ZN and TL, by real time system

Above Figure 4.10 shows the PID controller responses for single tank system using Relay-Tuning, Ziegler-Nichols, Tyreus-Luyben, Methods. These PID controller settings are applied on Feedback Coupled Tanks‟ Real-time tank T1 PID model and we have observed above results. From the above figure it is very clear that all PID controllers are giving better results i.e. all the controllers are maintaining the desired water level in the tank. It is also visual that all controllers are giving approximately similar responses.

4.2.1 Integral Square Error (ISE) of response for these three methods:

Table 4.3: PID parameters for Relay-tuning, Z-N and T-L methods on real time,

	Relay-Tunning	Zeigler-Nichols	Tyreus-Luyben
ISE	0.5	2.65	9.4

From above table 4.3, it is observed that the ISE of Relay-Tuning method is less than other two methods. We know that the controller which has lowest value of ISE parameter is preferred.

4.3 Comparison of Relay-Tuning, Z-N, T-L methods by Simulation:

PID parameter values obtained above (shown in Table 4.2) are also applied on simulation of Coupled Tanks tank T1 model. In this simulation, sinusoidal input change in set point, water level is given. The purpose behind giving sinusoidal input change is that there will be continuous variation in set point and from the corresponding PID controller response one can say that these are the results for highest variation in input change. Single tank PID controller responses for Relay-Tuning, ZN and TL methods by simulation are superimposed on single plot and are shown below (Figure 4.7).

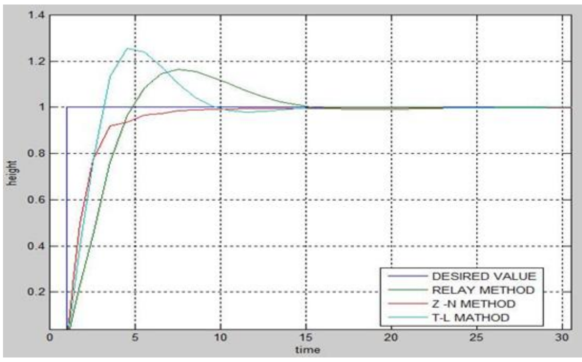

Figure 4.7: Comparison of PID response for Relay, ZN and TL, by simulation.

4.3.1 ISE of response for these three methods:

ISE decide whether which controller parametric value is better from their output responses.

Table 4.4: PID parameters for Relay-tuning, Z-N and T-L methods by simulation,

	Relay-Tuning	*Zeigler-Nichols*	*Tyreus-Luyben*
ISE	0.707	0.8621	0.9023

Above table 4.4 is shows the values of ISE for different controller parametric values calculated by different three methods. Here also ISE for Relay-Tuning method is less than other two methods. Hence it is finally concluded that, even if these methods gives different responses for other systems (found in literature), these methods are giving similar but better controller responses. So these methods can be used more preferably for the design of single tank water level system.

5. Conclusion and Future Scope

Feedback Coupled Tanks System is used for the experimental verification of designed controllers. The system is found to be very simple to operate and it is a digital control system which works on MATLAB / Simulink platform. The system can be used with different configurations which would have industrial applications. Two approaches, theoretical and practical, are used for the modelling of system. It is found that theoretically designed model used for controller design is giving best performance. So it can be used for controller design of single tank system and it is needed to improve the experimental model identification method.

Controller design on the basis of critical frequency data i.e. ultimate gain and ultimate period of systems are studied by using Relay auto-tuner. It is observed that Relay is giving best response for this system. In this project work, Relay-Auto-tuning, Ziegler-Nichols and Tyreus-Luyben, Methods of PID controller design are studied. After Real-time verification and calculated ISE value for these three methods, it is observed that these all methods are giving best controller performances. Hence it is concluded that all these methods can be used for controller design of single tank system but out these three methods Relay is giving best response for controller design of single tank system.

The Coupled Tanks System can be studied with different configurations. This is a digital control system. There are four tanks coupled with each other and two pumps inlet are provided. So there is a lot of scope to work on this system using other methods of controller design and by changing configurations.

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