On Moderate Analytic Hierarchy Process Pairwise Comparison Model

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Abstract: ***Decisions usually involve getting the best solution, selecting the suitable experiments, most appropriate judgments, taking the quality result etc., using some techniques. Every decision making can be considered as the choice from the set of alternatives based on a set of criteria. The Analytic Hierarchy Process is a Multi-Criteria Decision Making and is dealing with decision making problem through pair wise comparison and priority vectors, which was introduced by Saaty(1977). This paper concerns with an alternate method of finding the priority vectors for the original AHP decision matrix (Moderate AHP) that has the same rank as obtained in original AHP and ideal AHP decision problems.***

Keywords: Multi-criteria Decision making, Analytic Hierarchy Process, pairwise comparison, AHP calculation software by CGI, Ideal AHP

1. Introduction

Decision makers take decisions from the priorities on set of alternatives based on a set of criteria, called Multiple Criteria Decision Making (MCDM).It plays a important role in many real life problems. Each criterion induces a particular ordering of the alternatives and we need a procedure by which to construct one overall preference ordering. The number of criteria in MCDM is always assumed to be finite and we assume that the number of alternatives is also finite. A decision should also consider issues such as: cost, performance characteristics, availability of software, maintenance, expendability, etc. These may be some of the decision criteria for particular problems. In such problems we are interested in determining the best alternative. In some other situations, however, one may be interested in determining the relative importance of all the alternatives under consideration.

The AHP enforces the researchers, scientists, educationalists and industrialists for supporting tools, which can be used to solve complex decision problems. The AHP generates a weight for each evaluation criterion according to the decision maker's pairwise comparisons of the criteria. The higher the weight, the more important the corresponding criterion. Next, for a fixed criterion, the AHP assigns a score to each alternative according to the decision maker's pairwise comparisons of the alternative based on that criterion. The higher the score, the better the performance of the alternatives with respect to the considered criterion. Finally, the AHP combines the criteria weights and the alternatives scores, thus determining a global score for each alternative. The global score for a given alternative is a weighted sum of the scores it obtained with respect to all the criteria. Thus we have ranking for a set of objectives.

Data are collected from decision-makers corresponding to the hierarchical structure in the pairwise comparison of criteria and alternatives on a scale of relative importance(weight) as described below Table [1].

Table 1: Scale of Relative Importance (According to Saaty 1980)

Intensity of Importance	*Definition*	*Explanation*
1	Equal importance	Two activities contribute equally to the objective
3	Weak importance of one over another	Experience and judgement slightly favour one activity over another
5	Essential or strong importance	Experience and judgement strongly favor one activity over another
7	Demonstrated importance	An activity is strongly favored and its demonstrated in practice
9	Absolute importance	The evidence favoring one activity over another is of the highest possible order of affirmation
2, 4, 6, 8	Intermediate values between the two adjacent judgments	When compromise is needed
Reciprocals of above nonzero	If activity i has one of the above nonzero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i.	

In order to compute the weights for the different criteria, we start creating a parwise comparison matrix A. The matrix A is a n x n real matrix, where n is the number of criteria for considered problem. Each entry a_{ij} of the matrix A represents the importance of the ith criterion, relative to the jth criterion. If $a_{ij} > 1$, then the ith criterion is more important than the jth criterion, while if $a_{ij} < 1$, then the ith criterion is less important than the jth criterion. If two criteria have the same importance, then the entry a_{ij} is 1. The entries a_{ij} and a_{ji} satisfy the following constraint:

$a_{ij} \cdot a_{ji} = 1.$

Obviously $a_{ii} = 1$ for all i. The relative importance between two criteria is measured according to a numerical scale from 1 to 9, as shown in Table 1 above, where it is assumed that the ith criterion is equally or more important than the jth criterion. The phrases in the "Interpretation" column of Table 1 are only suggestive, and may be used to translate the decision makers, qualitative evaluations of the relative importance between two criteria into numbers. It is also possible to assign intermediate values which do not correspond to a precise interpretation. The values in the matrix A are by construction pairwise consistent.

The values of the pairwise comparisons in the AHP are determined according to the scale introduced by Saaty (1980). According to this scale, the available values for the pairwise comparisons are members of the set: {9,8,7,6,5,4,3,2,1 , 1/2 , 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, 1/9 }.

After constructing pairwise comparisons matrix, the corresponding maximum left eigenvector is approximated by using the geometric mean of each row. The is, the elements in each row are multiplied with each other and then the n-th root is taken (where n is the number of elements in the row). Next the numbers are normalized by dividing them with their sum. Hence obtaining the corresponding final priority vector.

The procedure for obtaining the following values of the criterion is as follows:

In the AHP the pairwise comparison matrix is considered to be adequately consistent if the corresponding consistency ratio (CR) is less than 10% (Saaty, 1980). The CR coefficient is calculated as follows. First the consistency index (CI) needs to be estimated. This is done by adding the columns in the pairwise comparison matrix of the criterion and multiply the resulting vector by the vector priorities obtained. This yields an approximation of the maximum eigenvalue, denoted byλ. Then, the CI value is calculated by using the formula: $CI = (\lambda - n)/(n-1)$. Next the consistency ratio CR is obtained by dividing the CI value by the Random Consistency index (RCI) as given in table [2].

ie. CR = $\frac{CI}{RCI}$, If CR > 0.10, we must re-evaluate the pairwise comparison for the criterion.

Table 2: RCI Values For Different Values of N

N	1	2	3	4	5	6	7	8	9
RCI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

Similar procedure mentioned above should be followed for all the n-alternatives

After the alternatives are compared with each other in term of each one of the decision criteria and the individual priority vectors are derived, the synthesis step is taken. The priority vectors become the columns of the decision matrix. The weights of importance of the criteria are also determined by using pairwise comparisons. Therefore, if a problem has m alternatives and n criteria, then the decision maker is required to construct n judgment matrices (one for each criterion) of order mxm and one pairwise comparison matrix of order nxn (for the n criteria). Finally, given a decision matrix the final priorities, denoted, by A^{i}_{AHP}, of the alternatives in terms of all the criteria combined are determined according to the following formula below.

$$A^{i}_{AHP} = \sum_{j-1}^{n} a_{ij} C'_{j}, \text{ for i = 1,2, , 3, , ...M . ...}$$

Recall that in the case of the ideal mode AHP the columns of the decision matrix are normalized by dividing by the largest entry in each column.

Some of the industrial engineering applications of the AHP include its use in integrated manufacturing (Putrus, 1990), in the evaluation of technology investment decision (Boucher and McStravic, 1991), in flexible manufacturing system (Wabalickis,1988), layout design (Cambron and Evans, 1991), in other engineering problems (Wang and Raz, 1991) and the selection of preference in the field of academic activities.

2. Geometrical Interpretation

The structure of the typical decision problem can consists of a number of criteria, say n and number of alternatives, say m. Each alternative can be evaluated in terms of the decision criteria and the relative importance (or weight) of each criterion can be estimated as well. Let a $_{ij}$ (i=1,2,3,…,m, and j=1,2,3,…n) denote the performance value of the i-th alternative (i.e.,A_i) In terms of the j-th criterion (i.e.,Cj). Also denote as PC_j the periorty value.

Criterion

C_1 C_2 C_3 … C_n

Alt. C'_1 C'_2 C'_3 … C'_n

A_1 a_{11} a_{12} a_{13} … a_{1n}
A_2 a_{21} a_{22} a_{23} … a_{2n}
A_3 a_{31} a_{32} a_{33} … a_{3n}
. . . . . .
. . . . . .
. . . . . .
A_m a_{m1} a_{m2} a_{m3} … a_{mn}

3. Model of the Problem

We define the 4 criteria (C_1, C_2, C_3, C_4) and 4 alternatives (A_1, A_2, A_3, A_4) in order to obtain the best alternative and criterion.

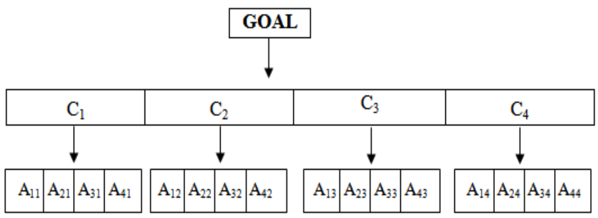

Figure 1: Selection of Best

Construct the Pair wise comparison matrix for each criterion C_{ij} represents the relative weight of the criterion C_i compared to C_j for the above model.

We get C'_j 's are priority vectors for the corresponding criteria, obtained from Saaty (1980) and λ , CI and CR are evaluated.

Construct the paire wise comparison matrix for the alternatives A_{11}, A_{21}, A_{31}, A_{41}, with respect to the criterion C_1 and obtain the priority vectors corresponding to A_{11}, A_{21}, A_{31}, A_{41} respectively as $A'_{11}, A'_{21}, A'_{31}, A'_{41}$ by Saaty (1980) and λ , CI and CR are evaluated.

Construct the paire wise comparison matrix for the alternatives A_{12}, A_{22}, A_{32}, A_{42}, with respect to the criterion C_2 and obtain the priority vectors corresponding to A_{12}, A_{22}, A_{32}, A_{42} respectively as $A'_{12}, A'_{22}, A'_{32}, A'_{42}$ by Saaty (1980) and λ , CI and CR are evaluated.

Construct the paire wise comparison matrix for the alternatives A_{13}, A_{23}, A_{33}, A_{43}, with respect to the criterion C_3 and obtain the priority vectors corresponding to A_{13}, A_{23}, A_{33}, A_{43} respectively as $A'_{13}, A'_{23}, A'_{33}, A'_{43}$ by Saaty (1980) and λ , CI and CR are evaluated.

Construct the paire wise comparison matrix for the alternatives A_{14}, A_{24}, A_{34}, A_{44}, with respect to the criterion C_4 and obtain the priority vectors corresponding to A_{14}, A_{24}, A_{34}, A_{44} respectively as $A'_{14}, A'_{24}, A'_{34}, A'_{44}$ by Saaty (1980) and λ , CI and CR are evaluated.

Table 3: Thus original AHP decision matrix is obtained as follows

Alternative/ Criterion	C_1	C_2	C_3	C_4	Final Priority vector
Criterion Priority	C'_1	C'_2	C'_3	C'_4	A^i_{AHP}
A_1	A'_{11}	A'_{12}	A'_{13}	A'_{14}	$A^1{}_{AHP}$
A_2	A'_{21}	A'_{22}	A'_{23}	A'_{24}	$A^2{}_{AHP}$
A_3	A'_{31}	A'_{32}	A'_{33}	A'_{34}	$A^3{}_{AHP}$
A_4	A'_{41}	A'_{42}	A'_{43}	A'_{44}	$A^4{}_{AHP}$

Table 4: Ideal AHP decision matrix and their priority vectors

Alternative/ criterion	C_1	C_2	C_3	C_4	Final Priority Vector IA_i
Criterion Priority	C'_1	C'_2	C'_3	C'_4	
A_1	I A'_{11}	I A'_{12}	I A'_{13}	I A'_{14}	IA_1
A_2	I A'_{21}	I A'_{22}	I A'_{23}	I A'_{24}	IA_2
A_3	I A'_{31}	I A'_{32}	I A'_{33}	I A'_{34}	IA_3
A_4	I A'_{41}	I A'_{42}	I A'_{43}	I A'_{44}	IA_4

It can also be extended to find the final alternative priority vectors for each alternative from the original AHP decision matrix. It can be obtained from the following formula

$MA_i = \sum_{j=1}^{4} C'_j(C'_j + A'_{ij})$ for all

Alternative indces i = 1, 2,3,4.

Fix i=1, we get 1st final alternative priority vector

$MA_1 = C'_1(C'_i + A'_{11}) + C'_2(C'_2 + A'_{12}) + C'_3(C'_3 + A'_{13}) + C'_4(C'_4 + A'_{14})$

For i=2, we get MA_2

i=3 we get MA_3

i=4 we get MA_4

Table 5: Thus the moderate decision matrix is obtained as follows.

Alternative / criterion	C_1	C_2	C_3	C_4	Final Priority Vector MA_i
Criterion Priority	C'_1	C'_2	C'_3	C'_4	
A_1	A'_{11}	A'_{12}	A'_{13}	A'_{14}	MA_1
A_2	A'_{21}	A'_{22}	A'_{23}	A'_{24}	MA_2
A_3	A'_{31}	A'_{32}	A'_{33}	A'_{34}	MA_3
A_4	A'_{41}	A'_{42}	A'_{43}	A'_{44}	MA_4

After normalization, we have ranked the alternatives. Finally we have the same ranking for original AHP decision matrix, Ideal AHP decision matrix and extension decision matrix, even though different the value of the final alternative priority vectors of respective alternatives.

4. Numerical Example

Suppose 4 alternatives subjects A, B, C and D are to be evaluated in terms of the 4 departments criteria C_1, C_2, C_3 and C_4 we can choose the best alternative from the best criterion from the following pairwise comparison matrix criteria and alternatives.

Pairwise comparison matrix for criteria

Alternative / criterion	C_1	C_2	C_3	C_4	Priority Vector
C_1	1	½	1/3	2	0.149
C_2	2	1	3	5	0.480
C_3	3	1/3	1	5	0.299
C_4	½	1/5	1/5	1	0.074

λ =4.230,C.I=0.0767, CR = 0.085

Pairwise comparison matrix for criterion C_1

C_1	A	B	C	D	Priority Vector
A	1	3	1/2	4	0.276
B	1/3	1	1/5	6	0.148
C	2	5	1	9	0.529
D	¼	1/6	1/9	1	0.047

λ =4.242 C.I = 0.081 CR = 0.09

Pairwise comparison matrix for criterion C_2

C_2	A	B	C	D	Priority Vector
A	1	3	2	1/3	0.240
B	1/3	1	3	¼	0.144
C	½	1/3	1	1/5	0.085
D	3	4	5	1	0.531

λ = 4.24 C.I = 0.08, CR = 0.088

Pairwise comparison matrix for criterion C_3

C_3	A	B	C	D	Priority Vector
A	1	2	6	½	0.294
B	½	1	2	¼	0.132
C	1/6	½	1	1/7	0.062
D	2	4	7	1	0.512

λ =4.029 , C.I = .0098, CR = 0.0108

Pairwise comparison matrix for criterion C_4

C_4	A	B	C	D	Priority Vector
A	1	½	4	1/2	0.191
B	2	1	3	1/4	0.216
C	1/4	1/3	1	1/8	0.058
D	2	4	8	1	0.535

λ = 4.196 C.I = 0.065, CR = 0.072

Original AHP decision matrix

Alternative / criterion	C_1	C_2	C_3	C_4	Final priority Vector	Ranking
Criterion Priority	0.149	0.480	0.299	0.074		
A	0.276	0.240	0.294	0.191	0.2583	2
B	0.148	0.144	0.132	0.216	0.1467	3
C	0.529	0.085	0.062	0.058	0.1424	4
D	0.047	0.531	0.512	0.535	0.4546	1

Ideal mode AHP decision matrix

Alternative / criterion	C_1	C_2	C_3	C_4	Final Priority Vector	After Normalization	Ranking
Criterion Priority	0.149	0.480	0.299	0.074			
A	0.522	0.452	0.574	0.357	0.4928	0.2583	2
B	0.280	0.271	0.258	0.404	0.2788	0.1461	3
C	1.000	0.160	0.121	0.108	0.2700	0.1415	4
D	0.089	1.000	1.000	1.000	0.8663	0.4541	1

Moderate AHP decision matrix

Alternative/ criterion	C_1	C_2	C_3	C_4	**Final Priority Vector**	**After Normalization**	**Ranking**
Criterion Priority	0.149	0.480	0.299	0.074			
A	0.0633	0.3456	0.1773	0.0196	0.6058	0.2533	2
B	0.0443	0.2995	0.1289	0.0215	0.4942	0.2066	3
C	0.1010	0.2712	0.1079	0.0098	0.4899	0.2048	4
D	0.0292	0.4853	0.2425	0.0451	0.8021	0.3353	1

Therefore, the best selection is D followed by A, A is followed by B and B is followed by C. Finally we observe that although the original AHP, the ideal AHP and the moderate AHP have the same ranking for the said 4 alternatives, even though they assigned different final priority vectors for these alternatives.

5. Conclusion

The AHP is used for ranking with priority vectors of pairwise comparison matrices. It provides an effective solution for solving MCDM problem. It should be noted that, there is a AHP calculation software (web system) by CGI system that calculates the weights and CI values of the AHP models from pairwise comparison matrices we can involve any relative importance of criteria and that of alternatives in the moderate AHP. Also moderate AHP allows for a sensitivity analysis in term of the relative priorities, by adjusting the ranking values.

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