Some new FEM packages for analyzing large deformation problems like LS-DYNA, DEFORM etc. were developed around this time. Further, the module for analyzing large deformation problems was incorporated in existing FEM packages like NASTRAN, ANSYS, ABAQUS etc.

4. Basic Steps in FEM

The finite element method involves the following steps.

First, the governing differential equation of the problem is converted into an integral form. These are two techniques to achieve this : (i) Variational Technique and (ii) Weighted Residual Technique. In variational technique, the calculus of variation is used to obtain the integral form corresponding to the given differential equation. This integral needs to be minimized to obtain the solution of the problem. For structural mechanics problems, the integral form turns out to be the expression for the total potential energy of the structure. In weighted residual technique, the integral form is constructed as a weighted integral of the governing differential equation where the weight functions are known and arbitrary except that they satisfy certain boundary conditions. To reduce the continuity requirement of the solution, this integral form is often modified using the divergence theorem. This integral form is set to zero to obtain the solution of the problem. For structural mechanics problems, if the weight function is considered as the virtual displacement, then the integral form becomes the expression of the virtual work of the structure.

In the second step, the domain of the problem is divided into a number of parts, called as elements. For one-dimensional (1-D) problems, the elements are nothing but line segments having only length and no shape. For problems of higher dimensions, the elements have both the shape and size. For two-dimensional (2D) or axi-symmetric problems, the elements used are triangles, rectangles and quadrilateral having straight or curved boundaries. Curved sided elements are good choice when the domain boundary is curved. For three-dimensional (3-D) problems, the shapes used are tetrahedron and parallelepiped having straight or curved surfaces. Division of the domain into elements is called a mesh.

In this step, over a typical element, a suitable approximation is chosen for the primary variable of the problem using interpolation functions (also called as shape functions) and the unknown values of the primary variable at some pre-selected points of the element, called as the nodes. Usually polynomials are chosen as the shape functions. For 1-D elements, there are at least 2 nodes placed at the end-points. Additional nodes are placed in the interior of the element. For 2-D and 3-D elements, the nodes are placed at the vertices (minimum 3 nodes for triangles, minimum 4 nodes for rectangles, quadrilaterals and tetrahedral and minimum 8 nodes for parallelepiped shaped elements). Additional nodes are placed either on the boundaries or in the interior. The values of the primary variable at the nodes are called as the degrees of freedom.

To get the exact solution, the expression for the primary variable must contain a complete set of polynomials (i.e., infinite terms) or if it contains only the finite number of terms, then the number of elements must be infinite. In either case, it results into an infinite set of algebraic equations. To make the problem tractable, only a finite number of elements and an expression with only finite number of terms are used. Then, we get only an approximate solution. (Therefore, the expression for the primary variable chosen to obtain an approximate solution is called an approximation). The accuracy of the approximate solution, however, can be improved either by increasing the number of terms in the approximation or the number of elements.

In the fourth step, the approximation for the primary variable is substituted into the integral form. If the integral form is of variational type, it is minimized to get the algebraic equations for the unknown nodal values of the primary variable. If the integral form is of the weighted residual type, it is set to zero to obtain the algebraic equations. In each case, the algebraic equations are obtained element wise first (called as the element equations) and then they are assembled over all the elements to obtain the algebraic equations for the whole domain (called as the global equations).

In this step, the algebraic equations are modified to take care of the boundary conditions on the primary variable. The modified algebraic equations are solved to find the nodal values of the primary variable.

In the last step, the post-processing of the solution is done. That is, first the secondary variables of the problem are calculated from the solution. Then, the nodal values of the primary and secondary variables are used to construct their graphical variation over the domain either in the form of graphs (for 1-D problems) or 2-D/3-D contours as the case may be.

5. Advantages of FEM Over Other Numerical Methods

Advantages of the finite element method over other numerical methods are as follows:

1) The method can be used for any irregular-shaped domain and all types of boundary conditions.
2) Domains consisting of more than one material can be easily analyzed.
3) Accuracy of the solution can be improved either by proper refinement of the mesh or by choosing approximation of higher degree polynomials.
4) The algebraic equations can be easily generated and solved on a computer. In fact, a general purpose code can be developed for the analysis of a large class of problems.

6. Modeling Boundary Value Problem Using FEM

To illustrate the development of integral formulations, the following model boundary value problem is considered. It represents the axial extension (or compression) of a bar shown in Fig. 2

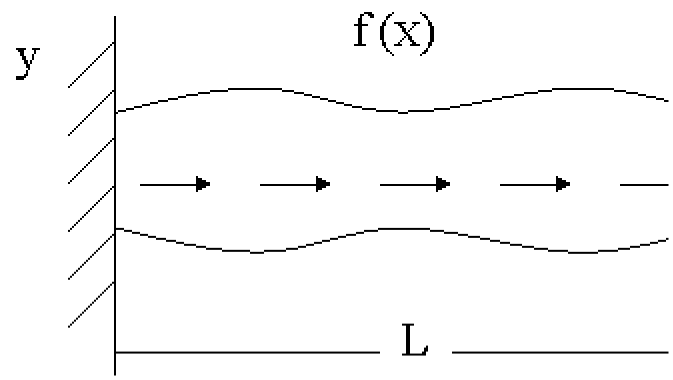

Figure 2: Axial Extension of a bar

The bar has a variable area of cross-section which is denoted by the function A(x). The length of the bar is L. The Young's modules of the bar material is E . The bar is fixed at the end x = 0. The forces acting on the bar are (i) a distributed force f(x), which varies with x and (ii) a point force P at the end x = L . The axial displacement of a cross-section at x, denoted by u(x), is governed by the following boundary value problem consisting of a differential equation (DE) and two boundary conditions (BC):

DE :$-\frac{d}{dx}\left(EA(x)\frac{du}{dx}\right) = f(x)$ $0<x<L$ (3.1 a)

BC: (i) u = 0 at x=0 (3.1 b)

(ii) $EA(x)\frac{du}{dx}$at x=L (3.1 c)

The differential equation represents the equilibrium of a small element of the bar expressed in terms of the displacement using the stress-strain and strain-displacement relations. The boundary condition (2.1b) is a geometric or kinematic boundary condition. Since, it is a condition on the primary variable u(x), it is called as Dirichlet boundary condition. The second boundary condition (condition 1c) is a force boundary condition, or a condition on the secondary variable (i.e., axial force). Since, it is a condition on a derivative of the primary variable; it is called as the Neumann boundary condition.

7. Conclusion

Finite element method is the most widely used method for optimization problems. As seen from the works of [1] that optimization could also lead to efficiency of the overall setup , an effort has been made by us in this research work to implement the concepts of FEM to a resistance and temperature problem of industrial furnace which is the heart of almost any industry . Also the above said method helped us learning the basics of Matlab and would increase our skill set.

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