International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2013): 6.14 | Impact Factor (2013): 4.438

About the Erdos-Strauss Conjecture

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Abstract: ***In this document, we deal with the Erdos conjecture.***

Keywords: Erdos ; Conjecture.

1. The Proof

The erdos conjecture states that

$$\forall n \in N, \exists x, y, z \in N \mid \frac{4}{n} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

There will be no calculus, only formulas

$\forall n \in N, \exists k \mid n = 6k$ or n=6k+1 or n=6k+2 or n=6k+3 or n=6k+4 or n=6k+5

$$\frac{4}{6k} = \frac{1}{3k} + \frac{1}{3k+1} + \frac{1}{3k(3k+1)}$$

$$\frac{4}{6k+2} = \frac{1}{6k+2} + \frac{1}{2k+1} + \frac{1}{(2k+1)(6k+2)}$$

$$\frac{4}{6k+3} = \frac{1}{6k+3} + \frac{1}{2k+2} + \frac{1}{(2k+2)(2k+1)}$$

$$\frac{4}{6k+4} = \frac{1}{3k+2} + \frac{1}{3k+3} + \frac{1}{(3k+3)(3k+2)}$$

$$\frac{4}{6k+5} = \frac{1}{6k+5} + \frac{1}{2(k+1)} + \frac{1}{2(k+1)(6k+5)}$$

And

$$\frac{1}{k+c} + \frac{1}{\frac{(k+c)b(6k+1)}{(4b-1)(k+c)-(6k+1)b}}$$

$$= \frac{1}{k+c}\left(1 + \frac{1}{\frac{b(6k+1)}{(4b-1)(k+c)-(6k+1)b}}\right)$$

$$= \frac{1}{k+c}\left(1 + \frac{(4b-1)(k+c)-(6k+1)b}{(6k+1)b}\right)$$

$$= \frac{1}{k+c}\left(\frac{(4b-1)(k+c)}{(6k+1)b}\right) = \frac{4b-1}{b(6k+1)} = \frac{4}{6k+1} - \frac{1}{b(6k+1)}$$

$$\Rightarrow \frac{4}{6k+1} = \frac{1}{b(6k+1)} + \frac{1}{k+c} + \frac{1}{\frac{(k+c)b(6k+1)}{(4b-1)(k+c)-(6k+1)b}}$$

We choose for each k, b,c for which

$\frac{(k+c)b(6k+1)}{(4b-1)(k+c)-(6k+1)b}$ is a positive integer and then Erdos-Straus conjecture is proved ! It is always possible:

$$k = 1 \Rightarrow \frac{(k+c)b(6k+1)}{(4b-1)(k+c)-(6k+1)b} = \frac{(1+c)b7}{(4b-1)(1+c)-7b}$$

$$= \frac{(1+c)b7}{4bc-c-3b-1}$$

$$c = 1; b = 3 \Rightarrow 4bc-c-3b-1 = 1 \Rightarrow \frac{4}{7} = \frac{1}{7} + \frac{1}{2} + \frac{1}{14}$$

$$k = 2 \Rightarrow \frac{(k+c)b(6k+1)}{(4b-1)(k+c)-(6k+1)b} = \frac{(2+c)b13}{(4b-1)(2+c)-13b}$$

$$= \frac{(2+c)b13}{4bc-5b-c-2}$$

$$c = 2; b = 2 \Rightarrow 4bc-5b-c-2 = 2 \Rightarrow \frac{4}{13} = \frac{1}{26} + \frac{1}{4} + \frac{1}{52}$$

$$k = 3 \Rightarrow \frac{(k+c)b(6k+1)}{(4b-1)(k+c)-(6k+1)b} = \frac{(3+c)b19}{(4b-1)(3+c)-19b}$$

$$= \frac{(3+c)b19}{4bc-7b-c-3}$$

$$c = 2; b = 6 \Rightarrow 4bc-7b-c-3 = 1 \Rightarrow \frac{4}{19} = \frac{1}{114} + \frac{1}{6} + \frac{1}{684}$$

Etc… until infinity !

In all cases, we have three egytian fractions.

2. Conclusion

By series of formulas, we have proved the Erdos conjecture.

References

[1] Yamamoto, Koichi (1965), *On the Diophantine equation 4/n = 1/x + 1/y + 1/z, Memoirs of the Faculty of Science.*