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Three Dimensional Inverse Transient Thermoelastic Problem of A Square Plate

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Abstract: ***The present paper deals with the determination of the temperature distribution, unknown temperature at z=h of the square plate, thermoelastic displacement function , displacement components and thermal stresses of square plate occupying the space*** $D: 0 \le x \le a,\ 0 \le y \le a,\ 0 \le z \le h$, ***with known boundary conditions by applying finite Fourier sine transform , Fourier cosine transform and Laplace transform techniques. Numerical calculations are carried out for a particular case of square plate made of copper (pure) metal by assigning suitable values to the parameters and functions in the equations and results are depicted graphically.***

Keywords: Thermoelastic Problem, Thermal Stresses, Fourier Sine Transform, Fourier Cosine Transform, Laplace Transform.

1. Introduction

Komatsubara and Tanigawa and (1997) and **Adams and Bert (1999)** have studied the direct problem of thermoelasticity in a rectangular plate under thermal shock. **Khobragade and Wankhede** (2003) have studied the inverse steady state thermoelastic problem to determine the temperature displacement function and thermal stresses at the boundary of a thin rectangular plate. They have used the finite Fourier sine transform technique. **Dange and Khobragade** (2009) have studied three dimensional inverse steady-state thermoelastic problem of a thin rectangular plate. **Ghadle and Gaikwad** (2011) have studied three dimensional non-homogeneous thermoelastic problem in a thick rectangular plate due to internal heat generation . **Dange** (2014) has studied three dimensional thermoelastic problem of a square plate . In the present paper an attempt is made to determine temperature distribution ,unknown temperature at z=h of a square plate, thermoelastic displacement function , displacement components and thermal stresses of square plate occupying the space $D: 0 \le x \le a,\ 0 \le y \le a,\ 0 \le z \le h$, with known boundary conditions by applying finite Fourier sine transform and Fourier cosine transform and Laplace transform techniques. Numerical calculations are carried out for a particular case of square plate made of copper (pure) metal by assigning suitable values to the parameters and functions in the equations and results are depicted graphically.

Statement of the Problem

Consider a square plate occupying the space $D: 0 \le x \le a, 0 \le y \le a, 0 \le z \le h$. The displacement components u_x, u_y and u_z in the x, y, z direction respectively are in the integral form as

$$u_x = \int_0^a \frac{1}{E}\left(\frac{\partial^2 \cup}{\partial y^2} + \frac{\partial^2 \cup}{\partial z^2} - \nu \frac{\partial^2 \cup}{\partial x^2} + \alpha T\right) dx \tag{1}$$

$$u_y = \int_0^a \frac{1}{E}\left(\frac{\partial^2 \cup}{\partial z^2} + \frac{\partial^2 \cup}{\partial x^2} - \nu \frac{\partial^2 \cup}{\partial y^2} + \alpha T\right) dy \tag{2}$$

$$u_z = \int_0^h \frac{1}{E}\left(\frac{\partial^2 \cup}{\partial x^2} + \frac{\partial^2 \cup}{\partial y^2} - \nu \frac{\partial^2 \cup}{\partial z^2} + \alpha T\right) dz \tag{3}$$

Where E, ν and α are the Young's modulus, Poison's ratio and the linear coefficient of thermal expansion of the material of the plate respectively, $\cup(x, y, z, t)$ is the Airy's stress function which satisfies the differential equation ,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)^2 \cup(x, y, z, t) = -\alpha E\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) T(x, y, z, t) \tag{4}$$

Where $T(x, y, z, t)$ denotes the temperature of square plate satisfying the following differential equation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t} \tag{5}$$

Where, k is thermal diffusivity of the material.
The initial condition is

$$T(x, y, z, t)\big|_{t=0} = 0 \tag{6}$$

The boundary conditions are

$$\left[T(x, y, z, t)\right]_{x=0} = 0 \tag{7}$$

$$\left[T(x, y, z, t)\right]_{x=a} = 0 \tag{8}$$

$$\left[\frac{\partial T}{\partial y}\right]_{y=0} = 0 \tag{9}$$

$$\left[\frac{\partial T}{\partial y}\right]_{y=a} = 0 \tag{10}$$

$$\left[T(x, y, z, t)\right]_{z=0} = u(x, y, t) \tag{11}$$

$$\left[T(x, y, z, t)\right]_{z=\xi} = f(x, y, t) \tag{12}$$

$$\left[T(x, y, z, t)\right]_{z=h} = g(x, y, t) \tag{13}$$

The stresses components are given by

$$\sigma_{xx} = \left(\frac{\partial^2 \cup}{\partial y^2} + \frac{\partial^2 \cup}{\partial z^2}\right) \tag{14}$$

$$\sigma_{yy} = \left(\frac{\partial^2 \cup}{\partial z^2} + \frac{\partial^2 \cup}{\partial x^2}\right) \tag{15}$$

$$\sigma_{zz} = \left(\frac{\partial^2 \cup}{\partial x^2} + \frac{\partial^2 \cup}{\partial y^2}\right) \tag{16}$$

The equations (1) to (16) constitute the mathematical formulation of the problem under consideration.

Solution of the Problem

Applying Fourier sine transform over x to the equation (5) (7) and (8). Applying Fourier Cosine transform over y to the equations (9) (10), taking Laplace transform and then their inverses one obtains the expression for temperature and unknown temperature $g(x, y, t)$ as

$$T(x,y,z,t)=\frac{8k\pi}{\xi^2a^2}\sum_{m-1}^{\infty}\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}\left[\frac{l}{\cos l\pi}\right]$$
$$\left\{\left[\sin\frac{l\pi}{\xi}(z-\xi)\int_0^t \bar{\bar{u}}(m,n,t')e^{-k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right](t-t')}dt'\right.\right.$$
$$\left.\left.-\left[\sin\left(\frac{l\pi}{\xi}\right)z\int_0^t \bar{\bar{f}}(m,n,t')e^{-k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right](t-t')}dt'\right]\right\}$$
$$\times\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{a} \tag{17}$$

$$g(x,y,t)=\frac{8k\pi}{\xi^2a^2}\sum_{m-1}^{\infty}\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}\left[\frac{l}{\cos l\pi}\right]$$
$$\left\{\left[\sin\frac{l\pi}{\xi}(h-\xi)\int_0^t \bar{\bar{u}}(m,n,t')e^{-k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right](t-t')}dt'\right.\right.$$
$$\left.\left.-\left[\sin\left(\frac{l\pi}{\xi}\right)h\int_0^t \bar{\bar{f}}(m,n,t')e^{-k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right](t-t')}dt'\right]\right\}$$
$$\times\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{a} \tag{18}$$

Determination Of Thermoelastic Displacement Function :

Substituting the values of $T(x, y, z, t)$ from equation (17) in equation (4) one obtains,

$$U(x,y,z,t)=8\alpha kE\sum_{m-1}^{\infty}\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}\left[\frac{l}{\pi\cos(l\pi)}\right]\left[\frac{1}{[(m^2+n^2)\xi^2+l^2a^2]}\right]$$
$$\left\{\left[\sin\frac{l\pi}{\xi}(z-\xi)\int_0^t \bar{\bar{u}}(m,n,t')e^{-k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right](t-t')}dt'\right.\right.$$
$$\left.\left.-\left[\sin\left(\frac{l\pi}{\xi}\right)z\int_0^t \bar{\bar{f}}(m,n,t')e^{-k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right](t-t')}dt'\right]\right\}$$
$$\times\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{a} \tag{19}$$

Determinations Of Displacement Components u_x, u_y, u_z **:**

Substituting the values (19) in the equation (1) to (3) one obtains

$$u_x=\frac{8\alpha k\pi}{a^2}\sum_{m-1}^{\infty}\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}\left[\frac{l}{\cos(l\pi)}\right]\left[\frac{(1+\nu)m^2[(-1)^{m+1}+1]}{(m^2+n^2)\xi^2+l^2a^2}\right]$$
$$\left\{\left[\sin\frac{l\pi}{\xi}(z-\xi)\int_0^t \bar{\bar{u}}(m,n,t')e^{-k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right](t-t')}dt'\right.\right.$$
$$\left.\left.-\left[\sin\left(\frac{l\pi}{\xi}\right)z\int_0^t \bar{\bar{f}}(m,n,t')e^{-k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right](t-t')}dt'\right]\right\}$$
$$\times\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{a} \tag{20}$$

$$u_y=0 \tag{21}$$

$$u_z=\frac{8\alpha k\pi}{\xi^2}\sum_{m-1}^{\infty}\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}\left[\frac{l}{\cos l\pi}\right]$$
$$\left[\frac{(1+\nu)l^2}{(m^2+n^2)\xi^2+l^2a^2}\right][(-1)^l-1]$$
$$\left\{\int_0^t\left(\bar{\bar{u}}(m,n,t')+\bar{\bar{f}}(m,n,t')\right)e^{-k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right](t-t')}dt'\right\}$$
$$\times\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{a} \tag{22}$$

Determination Of Stress Function $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ **:**

Substituting values of (19) in equations (14) to (16) one obtains

$$\sigma_{xx}=\frac{-8\alpha k\pi E}{\xi^2a^2}\sum_{m-1}^{\infty}\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}\left[\frac{l}{\cos l\pi}\right]$$
$$\left[\frac{\xi^2n^2+l^2a^2}{[(m^2+n^2)\xi^2+a^2l^2]}\right]$$
$$\left\{\left[\sin\frac{l\pi}{\xi}(z-\xi)\int_0^t \bar{\bar{u}}(m,n,t')e^{-k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right](t-t')}dt'\right.\right.$$
$$\left.\left.-\left[\sin\left(\frac{l\pi}{\xi}\right)z\int_0^t \bar{\bar{f}}(m,n,t')e^{-k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right](t-t')}dt'\right]\right\}$$
$$\times\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{a} \tag{23}$$

$$\sigma_{yy}=\frac{-8\alpha k\pi E}{\xi^2a^2}\sum_{m-1}^{\infty}\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}\left[\frac{l}{\cos l\pi}\right]$$
$$\left[\frac{\xi^2m^2+l^2a^2}{[(m^2+n^2)\xi^2+a^2l^2]}\right]$$

$$\left\{\left[\sin\frac{l\pi}{\xi}(z-\xi)\int_0^t \bar{\bar{u}}(m,n,t')e^{-k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right](t-t')}dt'\right]\right.$$

$$\left.-\left[\sin\left(\frac{l\pi}{\xi}\right)z\int_0^t \bar{\bar{f}}(m,n,t')e^{-k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right](t-t')}dt'\right]\right\}$$

$$\times\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{a} \qquad (24)$$

$$\sigma_{zz}=\frac{-8\alpha k\pi E}{a^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}\left[\frac{l}{\cos l\pi}\right]\left[\frac{(m^2+n^2)}{\left[(m^2+n^2)\xi^2+a^2l^2\right]}\right]$$

$$\left\{\left[\sin\frac{l\pi}{\xi}(z-\xi)\int_0^t \bar{\bar{u}}(m,n,t')e^{-k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right](t-t')}dt'\right]\right.$$

$$\left.-\left[\sin\left(\frac{l\pi}{\xi}\right)z\int_0^t \bar{\bar{f}}(m,n,t')e^{-k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right](t-t')}dt'\right]\right\}$$

$$\times\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{a} \qquad (25)$$

Special Case and Numerical Results and Discussion:

Setting ,

$$u(x,\,y,t)=(1-e^t)x(a-x)y^2(3a-2y)$$

$$f(x,\,y,t)=(1-e^t)x(a-x)y^2(3a-2y)e^{\xi}$$

In the equation (17) , one obtains

$$T(x,\,y,\,z,\,t)=\frac{32ka^4}{\xi^2\pi^5}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}\left[\frac{l}{\cos l\pi}\right]\left[\frac{(3a-2)}{m^3n^2}\right]$$

$$\left[\frac{(3a-2)}{m^3n^2}\right]\left[(-1)^n+(-1)^{m+n}\right]$$

$$\left[\frac{k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right](1+e^t)-e^{-k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right]t}+1}{k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right]\left[k\left[\frac{(m^2+n^2)\pi^2}{a^2}+\left(\frac{l\pi}{\xi}\right)^2\right]+1\right]}\right]$$

$$\left[\sin\frac{l\pi}{\xi}(z-\xi)-e^{\xi}\sin\left(\frac{l\pi}{\xi}\right)z\right]\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{a} \qquad (26)$$

Dimensions of the Square Plate*:*

Length, breadth of square plate a = 2m

Thickness of the square plate h=1m

$\xi=0.5m$

To interpret the numerical computations, we consider material properties of copper (pure) square plate with the material properties.

Poisson ratio, υ = 0.35

Thermal expansion coefficient, α(cm/cm-^{0}C) =16.5× 10^{-6}

Thermal diffusivity, κ (cm^2/sec) = 112.34× 10^{-6}

Young's Modulus E=120GPa

2. Discussions

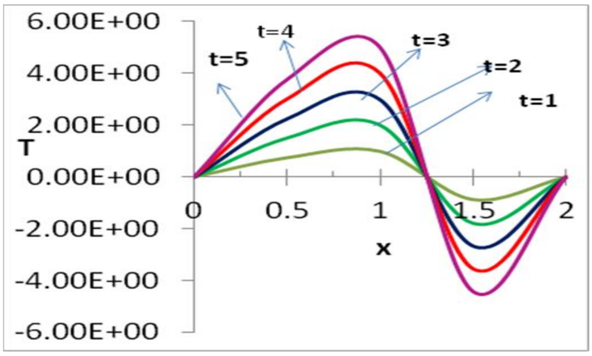

Figure 1: Graph of T versus x for different values oft

Figure 1: Represents the graphs of T versus x for different values of t.It is obeserved that $T(x,y,z,t)$ develops tensile stress from $x=0$ to $x=1.25$ and compressive stresses in the region $x=1.25$ to $x=2$ in the square region for different values of t.

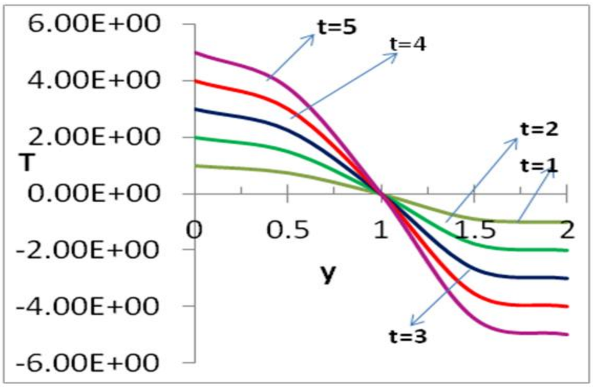

Figure 2: Graph of T versus y for different valuesof t

Figure 2: Represents the graphs of $T(x,y,z,t)$ versus y for different values of t.It is obeserved that $T(x,y,z,t)$ goes on decreasing from y=0 to y=2 and $T(x,y,z,t)$ is zero at point y=1 in the square region for different values of t.

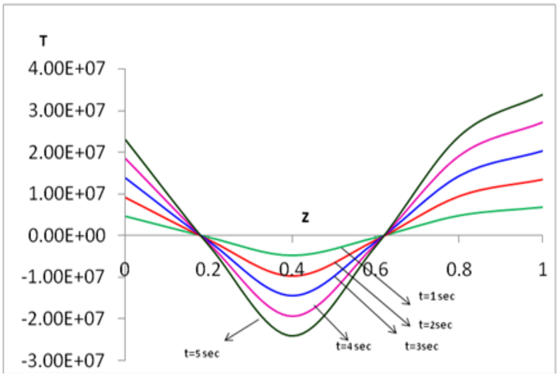

Figure3: Graph of T versus z for different values of t

Figure 3: Represents the graphs of $T(x,y,z,t)$ versus z for different values of t.It is obeserved that $T(x,y,z,t)$ goes on decreasing from z=0 to z=0.2 and $T(x,y,z,t)$ is goes on increasing from z=0.6.Also $T(x,y,z,t)$ develops compressive stresses from $z=0.2$ to $z=0.6$ in the square region for different values of t.

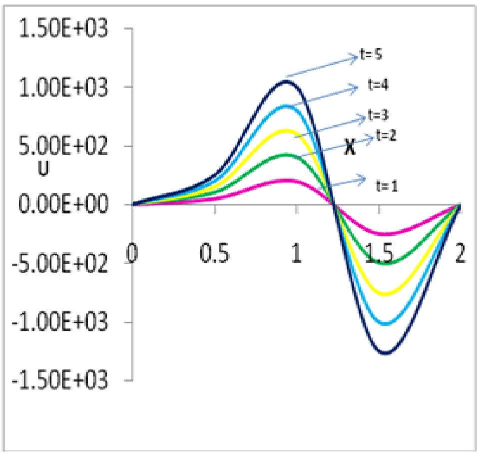

Figure 4: Graph of U versus x for different values of t

Figure 4: Represents the graphs of $U(x,y,z,t)$ versus x for different values of t.It is obeserved that $U(x,y,z,t)$ develops tensile stress from $x=0$ to $x=1.25$ and compressive stresses from $x=1.25$ to $x=2$ in the square region for different values of t.

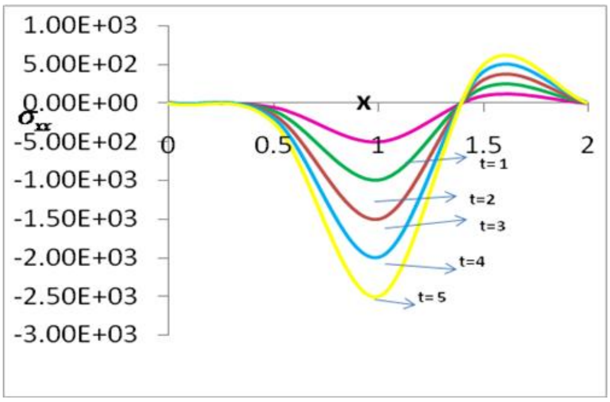

Figure 5: Graph of σ_{xx} versus x for different values of t

Figure 5: Represents the graphs of σ_{xx} versus x for different values of t.It is obeserved that σ_{xx} is approximatly zero from $x=0$ to $x=0.5$. Also σ_{xx} develops tensile stress from $x=1.4$ to $x=2$ and compressive stresses from $x=0.5$ to $x=1.4$ in the square region for different values of t.

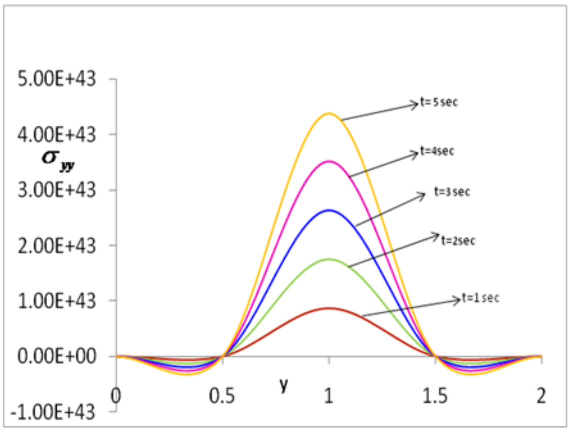

Figure 6: Graph of σ_{yy} versus y for different values of t

Figure 6: Represents the graphs of σ_{yy} versus y for different values of t.It is obeserved that σ_{yy} is approximatly zero from $y=0$ to $y=0.5$ & $y=1.5$ to $y=2$. Also σ_{yy} develops tensile stress from $y=0.5$ to $y=1.52$ in the square region for different values of t.

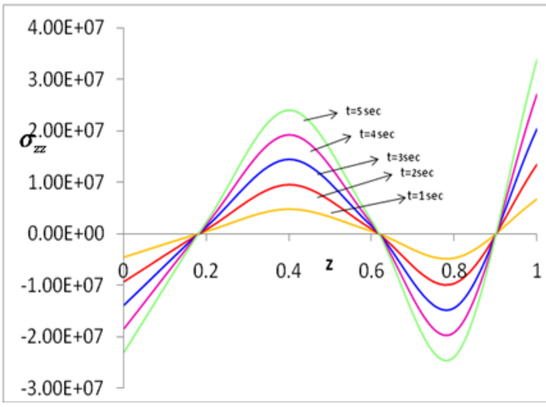

Figure 7: Graph of σ_{zz} versus z for different values of t

Figure 7 : Represents the graphs of σ_{zz} versus z for different values of t.It is obeserved that σ_{zz} goes on increasing from $z=0$ to $z=0.2$ and from $z=0.9$ to $z=1$ .Also σ_{zz} develops tensile stress from $z=0.2$ to $z=0.6$ and compressive stresses from $z=0.6$ to $z=0.9$ in the square region for different values of t.

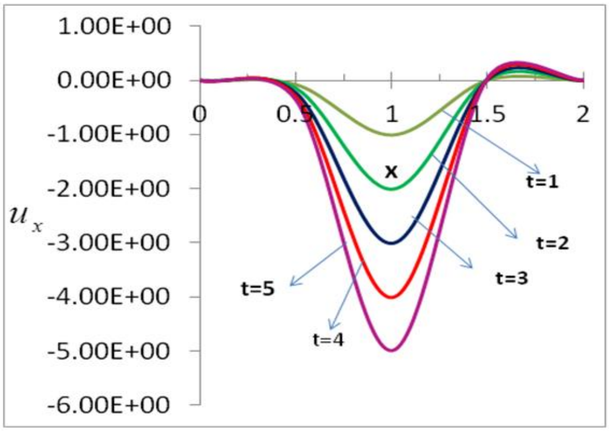

Figure 8: Graph of u_x versus x for different values of t

Figure 8: Represents the graphs of u_x versus x for different values of t.It is obeserved that u_x is approximatly zero from $x = 0$ to $x = 0.5$ and $x = 1.5$ to $x = 2$. Also u_x develops compressive stresses from $x = 0.5$ to $x = 1.5$ in the square region for different values of t.

3. Conclusion

In this study I treated the three dimensional inverse transient thermoelastic problem of square plate with stated boundary conditions. Under these conditions the temperature distribution $T(x, y, z, t)$,unknown temperature $g(x, y, t)$ at z=h ,The thermoelastic displacement $U(x, y, z, t)$, displacement components u_x ,u_y ,u_z in X, Y, Z axes respectively and thermal stresses σ_{xx}, σ_{yy}, σ_{zz} have been determined with the help of finite Fourier Sine transform ,Fourier Cosine transform and Laplace transform techniques. Any particular case can be derived by assigning suitable values to the parameters and functions in the expressions .I concluded that the system of equations proposed in this study can be adopted to design of useful structures or machines in engineering application in the determination of thermoelastic behavior and illustrated graphically.

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