Cosmological Model with Decaying Λ in *f(R,T)* Theory of Gravity

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Abstract: ***This paper is devoted to study of exact solutions of Bianchi type-I Cosmological model with decaying Λ in $f(R,T)$ theory of gravity. Using assumption of constant deceleration parameter and power law variation of Hubble parameter, we get a solution whose physical as well as geometrical nature have been investigated using some physical quantities.***

Keywords: $f(R,T)$ theory of gravity, Bianchi type-I Cosmological model, deceleration parameter

1. Introduction

Einstein general relativity is the successful theory which explain gravitational phenomena but it fails to resolve some of the problems in cosmology such as the accelerating expansion of the universe. In order to explain the accelerated expansion of the universe, numbers of cosmological models have been proposed by different authors. $f(T)$ theory of gravity where T is the scalar torsion has been proposed to explain current accelerated expansion without involving dark energy. Ratbay M. [1] has shown that the acceleration of the universe can be understood by $f(T)$ gravity models. Wei H. et al [2] tried to constrain $f(T)$ theories with the fine structure constant. Bamba K. et al [3] studied the cosmological evolution of the equation of state for dark energy with the combination of exponential, logarithmic and $f(T)$ theories. Another example of modified theory of gravity $f(R)$ theory of gravity. Many researchers have investigated $f(R)$ gravity in different contest. Carrol et. al. [4] explained the presence of late time comic acceleration of the universe in $f(R)$ gravity. Nojiri and Odintsov[5,6] proved that the $f(R)$ theory of gravity provides very natural unification of the early time inflation and late time acceleration. Multamaki and Vilja [7,8] investigated static spherically symmetric vacuum solutions of the field equations and non- vacuum solutions by taking fluid respectively. Capozziello et al [9] used Noether symmetries to study spherically symmetric solutions in $f(R)$ theory of gravity. Sharif and Shamir [10] studied exact vacuum solutions in Bianchi type-I and V space-times in $f(R)$) theory of gravity. Shamir [11] discussed the plane symmetric vacuum Bianchi type-III cosmology in f (R) gravity. Aktas et al [12] have studied anisotropic models in $f(R)$ gravity. Reddy et. al. [13] studied vacuum solutions of Bianchi type-I &V models in $f(R)$ gravity with a special form of deceleration parameter.

New version of generalized theory of gravity Proposed by Harko et. al.[14] known as $f(R,T)$ theory of gravity. Gravitational Lagrangian involves the arbitrary function of the scalar Curvature *R* and the trace of the energy momentum tensor *T*. Adhav K.S. [15] studied the exact solution of $f(R,T)$ field equations for locally rotationally symmetric Bianchi type-I space time. Houndjo [16] reconstructed $f(R,T)$ gravity by taking $f(R,T) = f_1(R,T) + f_2(R,T)$. M. Farasat Shamir et al [17] obtained the exact solutions of Bianchi types-I & V models in $f(R,T)$ by using the assumption of constant deceleration parameter and variation of law of Hubble parameter. Shamir et. al. [18] and Chaubey & Sukla [19] have discussed Bianchi type–I & V and general class of Bianchi models respectively in f(R,T) gravity by considering *R + 2 f(T)*. Nasir Ahmed and Anirudha Pradhan [20] considered the cosmological consequences of the class for which $f(R,T) = f_1(R,T) + f_2(R,T)$.

Motivating with the above research work we study exact solutions of Bianchi type-I cosmological model with decaying Λ in f(R, T) theory of gravity for the class f(R,T) = $f_1(R) + f_2(T)$ with the conditions $f_1(R) = \mu R$ and $f_2(T) = \mu T$ where μ is constant using assumption of constant deceleration parameter and power law variation of Hubble parameter proposed by Berman M. S.[21]. We get a solution whose physical as well as geometrical nature have been investigated using some physical quantities.

2. Field Equations in $f(R,T)$ Theory of Gravity

The action for $f(R,T)$ theory of gravity is given by

$$S = \int \left(\frac{1}{16\pi G} f(R,T) + L_m \right) \sqrt{-g}\, d^4x, \qquad (1)$$

Where $f(R,T)$ is an arbitrary function of Ricci scalar *R* and $T = g^{ij} T_{ij}$ is trace of stress energy momentum tensor of matter T_{ij}, L_m is matter Lagrangian density. The field equations in $f(R,T)$ theory of gravity is given by

$$f_R(R,T)R_{ij} - \frac{1}{2} f(R,T)g_{ij} - (\nabla_i \nabla_j - g_{ij})$$
$$f\ (R,T) = 8\pi T_{ij} - f_T(R,T)(T_{ij} + \theta_{ij}) \quad (2)$$

Where $\theta_{ij} = -2T_{ij} + g_{ij} L_m - 2g^{\alpha\beta} \frac{\delta(\sqrt{-g}\ L_m)}{\delta g^{ij}}$ (3)

Here $f_R(R,T) \equiv \frac{\partial f_R(R,T)}{\partial R}$,

$f_T(R,T) \equiv \frac{\partial f_T(R,T)}{\partial T}$, $T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\partial\left(\sqrt{-g}\ L_m\right)}{\partial g^{ij}}$,

$\square \equiv \nabla^i \nabla_i$ called as De Alembert‟s operator and ∇_i is the covariant derivative.

The matter source of universe we consider as perfect fluid. The energy momentum tensor for perfect fluid yields

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \quad (4)$$

where ρ, p are energy density and pressure of the fluid respectively and $u_i = \sqrt{g_{00}}(1,0,0,0)$ is four velocity in co-moving coordinates which satisfy the condition $u_i u^i = 1,\ u^i \nabla_j u_i = 0.$ For the matter lagrangian $L_m = -p$ with this value, equation (3) reduced as

$$\theta_{ij} = -pg_{ij} - 2T_{ij} \quad (5)$$

Harkoet. al. [14] gives three class of models to solve the field equation of $f(R,T)$ gravity out of which we used $f(R,T) = f_1(R) + f_2(T)$ with the conditions $f_1(R) = \mu R$, $f_2(T) = \mu T$ where μ is constant. By using $f_1(R) + f_2(T) = \mu(R+T)$ equation (2) can be written as.

$$R_{ij} - \frac{1}{2} Rg_{ij} = (\frac{8\pi + \mu}{\mu})T_{ij} + (p + \frac{1}{2}T)g_{ij}, \quad (6)$$

The Einstein field equation with cosmological constant can be written as

$$R_{ij} - \frac{1}{2} Rg_{ij} = -8\pi T_{ij} + \Lambda g_{ij}, \quad (7)$$

From equation (6) and (7), we have

$$\Lambda = \Lambda(T) = (p + \frac{1}{2}T) \quad (8)$$

3. Exact solutions of Bianchi type-I Space-Time

In this section we find exact solutions of Bianchi type-I space time in $f(R,T)$ theory of gravity. The line element of Bianchi type-I space-time is given by

$$ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2, \quad (9)$$

where A, B and C are functions of t only.

From equation (6), we have obtain field equations as

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = (\frac{8\pi + \mu}{\mu})\rho + \Lambda, \quad (10)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \Lambda - (\frac{8\pi + \mu}{\mu})p, \quad (11)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = \Lambda - (\frac{8\pi + \mu}{\mu})p, \quad (12)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \Lambda - (\frac{8\pi + \mu}{\mu})p, \quad (13)$$

where overhead prime denotes derivative w.r.to. t. The conservation equation $T^{ij}_{;j} = 0$, which gives us

$$\dot{\rho} + (p + \rho)[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}] = 0 \quad (14)$$

The system of these five non-linear differential equations consists of six undefined function i.e. A, B, C, p, ρ and Λ. Hence to find derministic solution one more condition is necessary , so we consider well known relation between Hubble parameter H and average scale factor a given as

$$H = la^{-n}, \text{ where } l > 0 \text{ and } n \geq 0 \quad (15)$$

Subtracting equation (11) from equation (12), equation (12) from equation (13), equation (11) from equation (13), we have

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{C}}{C}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0, \quad (16)$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0, \quad (17)$$

$$\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right) = 0. \quad (18)$$

On solving above equations, we get

$$\frac{B}{A} = d_1 \exp\left[c_1 \int \frac{dt}{a^3}\right], \quad (19)$$

$$\frac{C}{B} = d_2 \exp\left[c_2 \int \frac{dt}{a^3}\right], \quad (20)$$

$$\frac{A}{C} = d_3 \exp\left[c_3 \int \frac{dt}{a^3}\right], \quad (21)$$

where c_1, c_2, c_3 and d_1, d_2, d_3 are constants of integration which satisfy the relation

$$c_1 + c_2 + c_3 = 0,\ d_1 d_2 d_3 = 1. \quad (22)$$

Using equations (19), (20) and (21), the metric functions are

$$A = ap_1 \exp\left[q_1 \int \frac{dt}{a^3}\right], \quad (23)$$

$$B = ap_2 \exp\left[q_2 \int \frac{dt}{a^3}\right], \quad (24)$$

$$C = ap_3 \exp\left[q_3 \int \frac{dt}{a^3}\right], \quad (25)$$

Where

$$p_1 = (d_1^{-2} d_2^{-1})^{1/3},\ p_2 = (d_1 d_2^{-1})^{1/3},\ p_3 = (d_1 d_2^2)^{1/3}, \quad (26)$$

And $q_1 = -\frac{2c_1 + c_2}{3}$, $q_2 = \frac{c_1 - c_2}{3}$, $q_3 = \frac{c_1 + 2c_2}{3}$, (27)

satisfying the relations

$$p_1 p_2 p_3 = 1 \;,\; q_1 + q_2 + q_3 = 0 \quad (28)$$

4. Some Important Physical Quantities

In this section we define some important physical quantities.

The average scale factor and the volume scale factor are defined respectively as under

$$a = (ABC)^{\frac{1}{3}} \;,\; V = a^3 = ABC \,. \quad (29)$$

The generalized mean Hubble parameter H is defined by

$$H = (\ln a)_t = \frac{\dot{a}}{a} = \frac{1}{3}[H_1 + H_2 + H_3], \quad (30)$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters in the directions of x, y, z axes respectively.

The mean anisotropy parameter $\bar{A}$ is given by

$$\bar{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \quad (31)$$

Where $\Delta H_i = H_i - H$

The expansion scalar θ and shear scalar σ^2 are defined as under

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \quad (32)$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}, \quad (33)$$

Where

$$\sigma_{ij} = \frac{1}{2}\left[\nabla_j u_i + \nabla_i u_j\right] - \frac{1}{3}\theta g_{ij}, \quad (34)$$

From equation (15) and (30), we have

$$\dot{a} = la^{1-n}, \quad (35)$$

Hubble‟s law of variation parameter gives the constant value of deceleration parameter. This law yields two type of universe (i) power law universe (ii) exponential law universe. The exponential law presents the dynamics of future universe and such type of model does not have reliability with present day observations. Since we are looking for a model, describing the late time acceleration of universe, hence we used the power law expansion for describing nature of the present universe.

Therefore on solving equation (35), the value of average scale factor as

$$a = (nlt)^{1/n}, \quad (36)$$

The constant of integration are omitted by assuming $a = 0$ at $t = 0$.

The metric coefficients A, B and C turn out to be

$$A = p_1 (nlt)^{1/n} \exp\left[\frac{q_1 (nlt)^{\frac{n-3}{n}}}{l(n-3)}\right], n \neq 3 \quad (37)$$

$$B = p_2 (nlt)^{1/n} \exp\left[\frac{q_2 (nlt)^{\frac{n-3}{n}}}{l(n-3)}\right], n \neq 3 \quad (38)$$

$$C = p_3 (nlt)^{1/n} \exp\left[\frac{q_3 (nlt)^{\frac{n-3}{n}}}{l(n-3)}\right], n \neq 3 \quad (39)$$

The mean generalized Hubble parameter and the volume scale factor become

$$H = \frac{l}{nlt}, \; V = (nlt)^{3/n}. \quad (40)$$

The mean anisotropy parameter $\bar{A}$ turns out to be

$$\bar{A} = \frac{q_1^2 + q_2^2 + q_3^2}{3l^2 (nlt)^{(6-2n)/n}}. \quad (41)$$

The deceleration parameter q in cosmology is the measure of the cosmic accelerated expansion of the universe and is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = n - 1, \quad (42)$$

which is a constant.

A positive sign of q, i.e. $n > 1$ corresponds to the standard decelerating model whereas the negative sign of q, i.e. $0 < n < 1$ indicates inflation. The expansion of the universe at a constant rate corresponds to $q = 0$, i.e. $n = 1$. Also, recent observations of SN Ia, reveal that the present universe is accelerating and value of DP lies somewhere in the range $-1 < q < 0$.

The expansion θ and shear scalar σ^2 are given by

$$\theta = \frac{3l}{nlt} \text{ and } \sigma^2 = \frac{q_1^2 + q_2^2 + q_3^2}{2(nlt)^{6/n}}, \quad (43)$$

Thus the energy density of the universe becomes

$$\rho = \frac{\mu}{8\pi + \mu}\left[\frac{\mu(n+3) + 24\mu}{2(\mu + 4\mu)}\frac{l^2}{(nlt)^2} - \frac{q_1 q_2 + q_2 q_3 + q_3 q_1}{(nlt)^{6/n}}\right], \quad (44)$$

The pressure of the universe becomes

$$p = \frac{\mu}{8\pi + \mu}\left[\frac{q_1 q_2 + q_2 q_3 + q_3 q_1}{(nlt)^{6/n}} + \frac{\mu(n-1) + 8\pi(n-2)}{2(\mu + 4\pi)}\frac{l^2}{(nlt)^2}\right] \quad (45)$$

The cosmological density of the universe calculated as

$$\Lambda = \frac{(3-n)\mu l^2}{2(\mu + 4\pi)(nlt)^2}, \quad (46)$$

5. Physical Behavior of the Model

From the **Fig(1),** we observe that pressure is increasing function of time. It starts from a large negative value and approaches to a small negative value near zero. From the discovery of the accelerated expansion of the universe, it is generally assumed that this cosmic acceleration is due to some kind of energy matter with negative pressure known as „dark energy‟.

The energy density has been graphed versus time in **Fig(2),** It is evident that the energy density remains always positive and decreasing function of time and it converges to zero as $t \rightarrow \infty$ as expected.

Also the **Fig(3),** is the plot of cosmological term versus time. We observe that Λ is decreasing function of time t and it approaches a small positive value at late time (i.e. at present epoch). Hence, with a positive value of the cosmological term Λ ,the expansion will take place and the universe accelerate as we expected.

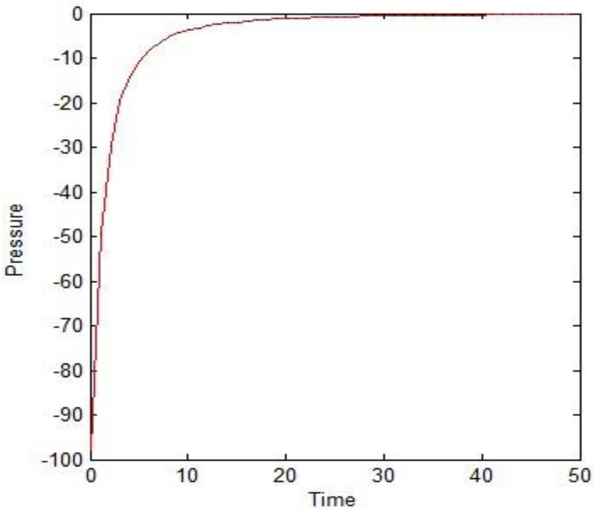

Figure 1: Pressure vrs Time

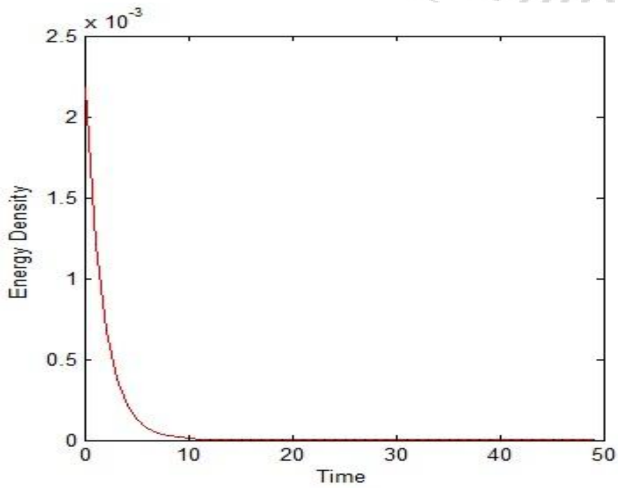

Figure 2: Energy density vrs Time

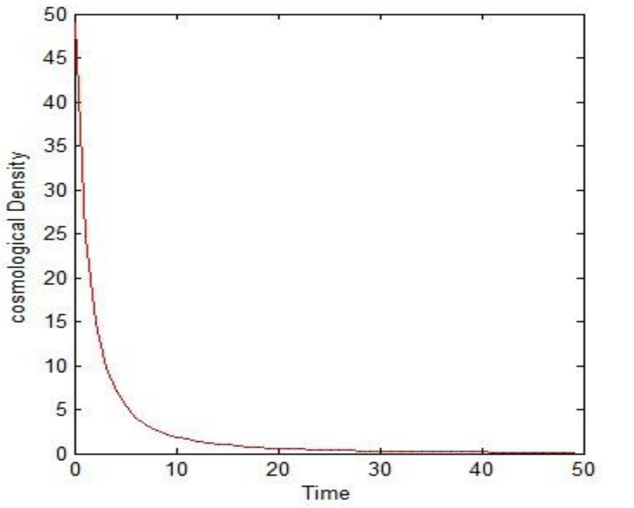

Figure 3: Cosmological density vrs Time

6. Concluding Remark

In this paper we have obtained exact solutions of Bianchi type-I space time in $f(R,T)$ theory of gravity using assumption of constant deceleration parameter and variation law of Hubble parameter. The solution is obtained by using power law expansion.

The cosmological importance of the model is discussed below.

1) Scale factor A,B,C are zero at $t=0$ hence the model has a point type singularity.
2) The mean generalized Hubble parameter H and mean anisotropy parameter $\overline{A}$ are all infinite at this point of singularity.
3) At $t=0$ volume scale factor V vanishes whereas expansion scalar θ is infinite, this shows that the universe starts evolving with zero volume at $t=0$.
4) The cosmological constant Λ is decreasing function of time and it approaches to a small positive value. This sort of behavior of the cosmological constant is in accordance to the present accelerated behavior of the universe.

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