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Numerical Simulation of Natural Convection of Nanofluid in a Square Cavity Including a Square Heater

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Abstract: ***This paper reports a numerical study of natural convection of nanofluid (Cu–water) in a square cavity with a centrally-placed heated block. The transport equations were solved using the finite volume discretization method with SIMPLE algorithm. The method used is validated against previous works. Results were presented in terms of streamlines, isotherms, and Nusselt number for different sizes of the heater. The other parameters governing the problem are the Rayleigh number $(10^3 \leq Ra \leq 10^6)$ the Prandtl number of the pure water $(Pr = 6.2)$ and the volume fraction of nanoparticles $(0 \leq \varphi \leq 0.05)$. The investigation revealed that increased the surface area of heater has enhanced the heat transfer. Moreover, it is found that at higher Rayleigh number, there is an optimum value of volume fraction of nanoparticles which result in the highest rate of heat transfer.***

Keywords: Cavity, Natural Convection, Isothermal Heater, Nanofluid.

1. Introduction

When Natural convection fluid flow and heat transfer are encountered in a wide range of engineering applications, such as heating and cooling nuclear systems of reactors, lubrication technologies, cooling of electronic devices, ventilation of rooms with radiators, cooling of containers and heat exchangers. Therefore, enhancement of heat transfers in the cavities in several application need to add a block inside. So far natural convection in a cavity has been intensively studied in literature. One of the first works on the numerical simulation of natural convection inside cavities is the pioneering work of De Vahl Davis [1]. He performed a numerical simulation on a square cavity with two vertical isothermal walls, one cold and one hot, and two horizontal adiabatic walls. This cavity with those boundary conditions is known as differentially heated cavity (DHC). Bouafia and Daube [2] studied natural convection in cavity filled with air having a heated solid body. They observed that the induced disturbances determined for weakly supercritical regimes indicate the existence of two instability types driven by different physical mechanisms: shear and buoyancy-driven instabilities, according to whether the flow develops in a square or in a tall cavity. Ha and Jung [3] considered a numerical study on three dimensional conjugate heat transfer of natural convection and conduction in a differentially heated cubic enclosure having an internal heated square partition. They investigated the effects of three-dimensionalities on the fluid flow and thermal characteristics in the enclosure.

In the present work, the main aim is to examine the free convection heat transfer of nanofluid in a square enclosure including a central square heater for different sizes. The computational procedure elaborated in this study is validated against the numerical results of other investigations. We studied the effects of sizes which are simulated in the range of volume fraction between 0% and 5%. The new models of the thermal conductivity and effective viscosity investigated by Corcione et al. [4] are used to estimate themophysical proprieties of the nanofluid. Our numerical results are presented in the form of plots of isotherms, streamlines and average Nusselt numbers to show the influence of nanofluid and the surface area of heater.

2. Problem Statement

The studied configurations and coordinate system of the considered enclosure in the present study are shown in **Fig. 1** Two vertical walls are cooled at constant temperatures $T_c = 290\ K$ and the other walls are thermally insulated. The cavity is a square with height H which contains a central heater with temperature $T_h = 310\ K$. Shape of the heater is square with dimensionless width w. The fluid in the enclosure is a water-based nanofluid containing Cu nanoparticles. It is assumed that the nanofluid is newtonian, incompressible and laminar and the base fluid and the nanoparticles are in a thermal equilibrium state. The thermo-physical properties of the nanofluid used in this study are evaluated at the average fluid temperature $(T_c + T_h)/2$ as listed in **Table 1.**

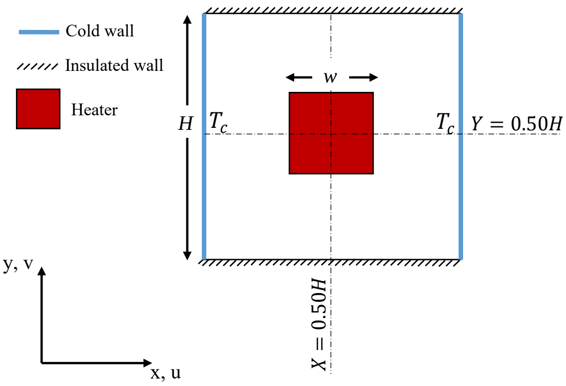

Figure 1: Schematic of the cavity with central heater and boundary conditions.

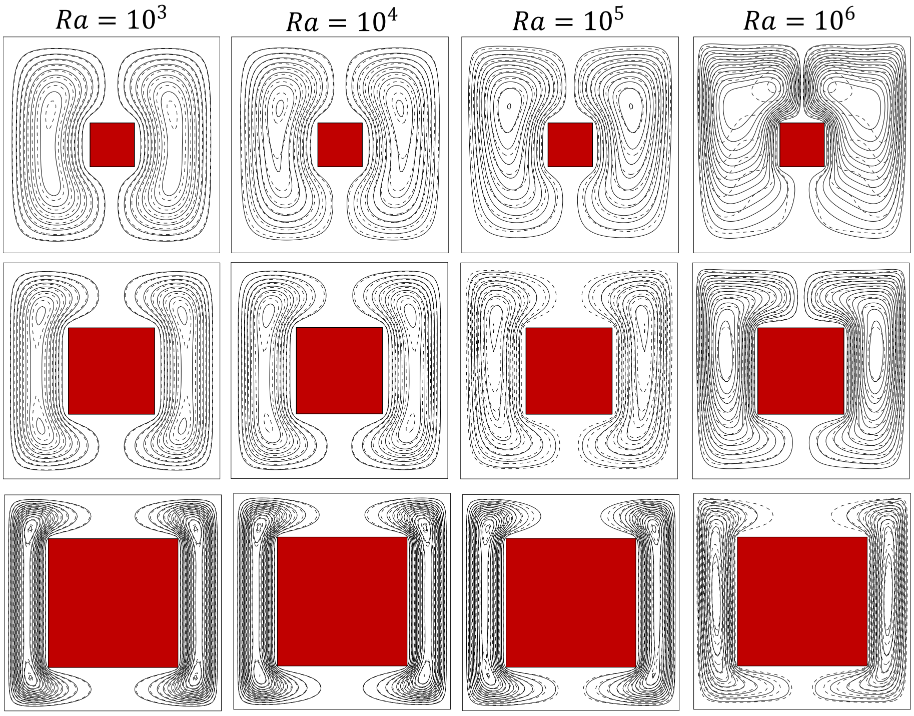

Figure 2: The streamlines inside the cavity corresponding to different sizes of the heated block, i.e. $W = w/H = 0.2, 0.4$ and 0.6 with case of pure fluid (dashed line) and Cu-water (solid line) nanofluid with $\varphi = 0.05$

Table 1: Thermo-physical properties of water and nanoparticles at T = 300 K [9]

	Copper (Cu)	Water (H_2O)
C_p(J/Kg K)	385	4179
ρ(Kg/m^3)	8933	997.1
K(W/mK)	401	0.613
$\beta \times 10^5 (K^{-1})$	1.67	27.6
$\mu \times 10^6 (\mathrm{kg\,m^{-1}\,s^{-1}})$	–	855
d_p(nm)	25	0.385

3. Mathematical Formulation

The governing equations including the two-dimensional transient equations of the continuity, momentum and energy for an incompressible flow are expressed in the following format:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \frac{(\rho\beta)_{nf}}{\rho_{nf}}(T - T_c) \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

Where the nanofluid effective density, heat capacity, thermal expansion coefficient and thermal diffusivity are calculated from the following equations [5-4]:

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s \tag{5}$$

$$\left(\rho C_p\right)_{nf} = (1-\varphi)\left(\rho C_p\right)_f + \varphi\left(\rho C_p\right)_s \tag{6}$$

$$(\rho\beta)_{nf} = (1-\varphi)(\rho\beta)_f + \varphi(\rho\beta)_s \tag{7}$$

$$\alpha_{nf} = k_{nf}/\left(\rho C_p\right)_{nf} \tag{8}$$

Corcione model [6-4] for dynamic viscosity and thermal conductivity of the nanofluid are given by:

$$\mu_{nf} = \mu_f/\left(1 - 34.87\left(d_p/d_f\right)^{-0.3}\varphi^{1.03}\right) \tag{9}$$

$$\frac{k_{nf}}{k_f} = 1 + 4.4Re_B^{0.4}Pr^{0.66}\left(\frac{T}{T_{fr}}\right)^{10}\left(\frac{k_p}{k_f}\right)^{0.03}\varphi^{0.66} \tag{10}$$

$$Re_B = \frac{\rho_f u_B d_p}{\mu_f} \tag{11}$$

$$u_B = \frac{2k_b T}{\pi\mu_f d_p^2} \tag{12}$$

The boundary conditions for natural convection written as:

$$\begin{aligned} &u = 0,\ v = 0,\ \partial T/\partial y = 0 \text{ on bottom wall of the cavity}\\ &u = 0,\ v = 0,\ \partial T/\partial y = 0 \text{ on upper wall of the cavity}\\ &v = 0,\ v = 0,\ T = T_c \text{ on right wall of the cavity}\\ &u = 0,\ v = 0,\ T = T_c \text{ on left wall of the cavity}\end{aligned} \tag{13}$$

The following dimensionless variables for natural convection are defined based on properties of pure fluid:

$$\tau=\frac{t}{H/U_{ref}},\quad X=\frac{x}{H},\quad Y=\frac{y}{H},\quad U=\frac{u}{U_{ref}},\quad V=\frac{v}{U_{ref}},\quad P=\frac{p}{\rho_{nf}U_{ref}^2},\quad \theta=\frac{T-T_c}{T_h-T_c} \tag{14}$$

Where U_{ref} is considered to be α_f/H for natural convection. Dimensionless numbers for the system are defined as:

$$Gr=\frac{g\beta_f(T_h-T_c)H^3}{\nu_f^2},\quad Ra=Gr.Pr \tag{15}$$

$$Ra=\frac{g\beta_f(T_h-T_c)H^3}{\alpha_f\nu_f},\quad Pr=\frac{\nu_f}{\alpha_f}, \tag{16}$$

The governing equations (1)–(4) are written in the following dimensionless form:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{17}$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+Pr\frac{\rho_f}{\rho_{nf}}\frac{\mu_{nf}}{\mu_f}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{18}$$

$$\frac{\partial V}{\partial \tau}+U\frac{\partial U}{\partial Y}+V\frac{\partial V}{\partial Y}=-\frac{\partial p}{\partial Y}+Pr\frac{\rho_f}{\rho_{nf}}\frac{\mu_{nf}}{\mu_f}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra.Pr\frac{(\rho\beta)_{nf}}{\rho_{nf}}\theta \tag{19}$$

$$\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial x}+V\frac{\partial \theta}{\partial y}=\frac{\alpha_{nf}}{\alpha_f}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{20}$$

Dimensionless form of the boundary conditions can be written as:

$$\begin{array}{ll} U=0,\ V=0, \partial\theta/\partial y=0 & \text{on bottom wall of the cavity} \\ U=0,\ V=0,\ \partial\theta/\partial y=0 & \text{on upper wall of the cavity} \\ V=0,\ V=0,\ \theta=0 & \text{on right wall of the cavity} \\ U=0,\ V=0,\ \theta=0 & \text{on left wall of the cavity} \end{array} \tag{21}$$

The total mean Nusselt number of all cavity's wall is defined as:

$$\overline{Nu}_{tot}=\frac{1}{H}\int_0^H\frac{k_{nf}(\varphi)}{k}\left\{\left|\frac{\partial\theta}{\partial X}\right|_{left}+\left|\frac{\partial\theta}{\partial X}\right|_{right}\right\}dY \tag{22}$$

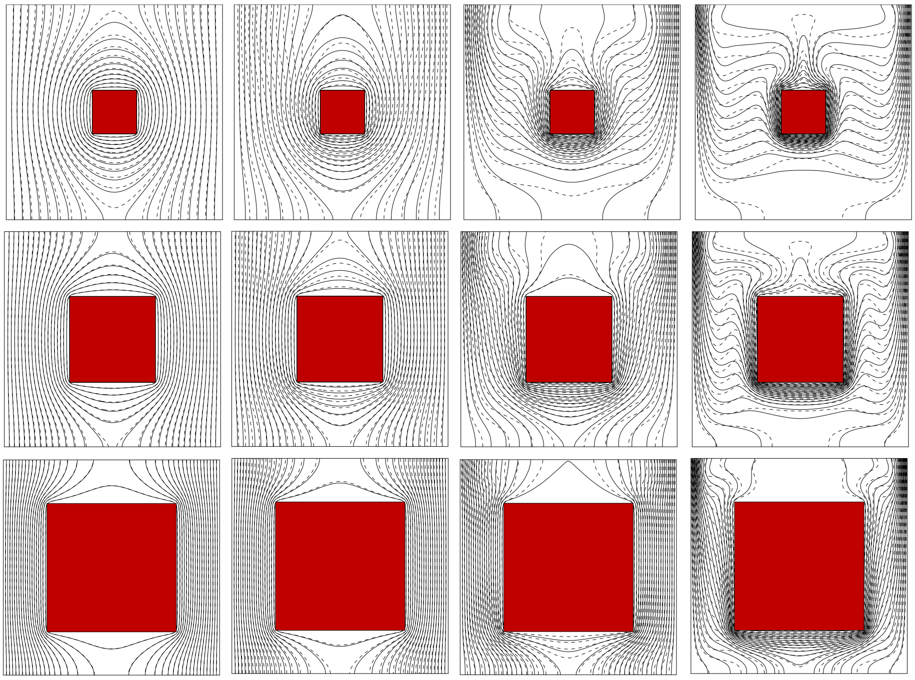

Figure 3: The isotherms inside the cavity corresponding to different sizes of the heated block, i.e. $W=w/H=0.2, 0.4$ and 0.6 with case of pure fluid (dashed line) and Cu-water (solid line) nanofluid with $\varphi=0.05$

4. Numerical Details

The discretization procedure of the governing equations (Eqs. (17)–(20)) and boundary conditions described by Eq. (21) is based on a finite volume formulation, given by Patankar [10] on a staggered grid. SIMPLE (Semi-Implicit Method for Pressure Linked Equations) is used to solve the coupled pressure–velocity equation while Hybrid Differencing Scheme (HDS) of Spalding [11] is used for the convective terms. Line by line application of TDMA (Tri-Diagonal Matrix Algorithm) method [11] is applied on equation systems until sum of the residuals became less than 10^{-6}. The developed algorithm was implemented in FORTRAN program.

4.1 Grid independence study

In order to determine a proper grid for the numerical simulation, a square cavity filled with Cu–water nanofluid ($\varphi=0.05$) having central heater with size $w=0.2$ is analyzed in two extreme Rayleigh numbers ($Ra=10^4$ and 10^6). The mean Nusselt number obtained using different grid numbers for particular cases is presented in **Table 2**. As can be observed from the table, a uniform 103×103 grid is sufficiently fine for the numerical calculation

Table 2: Effect of the grid size on $\overline{Nu}_{tot}$ for the cavity filled with the Cu–water nanofluid ($\varphi=0.05$) having central heater with size $w=0.2$.

Ra	63×63	83×83	103×103	123×123
10^4	3.034	3.214	3.248	3.248
10^6	9.481	9.655	9.694	9.697

4.2 Validations

The present numerical scheme was validated against various numerical results available in the literature. the benchmark problem of natural convection in a square cavity which considered by De Vahl Davis [2] filled with Air ($Pr = 0.71$). Table 3 demonstrates an excellent comparison of the average Nusselt number between the present results and the numerical results found in the literature [8-7].

Table 3: Comparison of $\overline{Nu}$ between the present results and those reported in the literature for a DHC at different Rayleigh numbers.

Ra	10^3	10^4	10^5	10^6
de Vahl Davis [2]	1.118	2.243	4.519	8.799
Relative Error (%)	0.17	0.13	0.75	0.54
Barakos and Mitsoulis [7]	1.114	2.245	4.510	8.806
Relative Error (%)	0.17	0.22	0.55	0.62
Dixit and Babu [8]	1.118	2.256	4.519	8.817
Relative Error (%)	0.17	0.71	0.75	0.75
Present study	**1.116**	**2.240**	**4.485**	**8.751**
Grid size	83^2	83^2	83^2	103^2

5. Results and discussion

In the present study, numerical results of natural convection fluid flow and heat transfer of Cu-water nanofluid inside a square cavity with central heater are investigated. The results have been obtained for three different sizes of the heated block as: $w = 0.2H$, $w = 0.4H$ and $w = 0.6H$. For all considered sizes of the heated block, the width w is fixed, while the Rayleigh number is ranging from 10^3 to 10^6 and the volume fraction of the nanoparticles is varying from 0 to 0.05.

Fig. 2 and **3** displays effects of size on the streamlines and isotherms for different values of Rayleigh number. **Fig. 2** shows at $Ra = 10^3$ and 10^4 (case of dominating conduction) two overall rotating symmetric eddies, by increasing size of the heated block from $w = 0.2H$ to $0.6H$, we can see clearly that the rotating symmetric eddies became with two inner vortices respectively. The isotherms shown in **Fig. 3** are uniformly distributed which indicate that the heat transfer in the cavity was governed mainly by the conduction mode. At $Ra = 10^4$ for size $w = 0.6H$, the patterns of the isotherms (**Fig. 3**) and streamlines are about the same as those for $Ra = 10^3$, it is worth mentioning that at $Ra = 10^3$, 10^4 and 10^5 increasing size of heated block to $w = 0.6H$, weakens strength of the flow circulation hence the conduction becomes stronger and significantly dominant. By increasing Rayleigh numbers up to 10^5 (and therefore the buoyancy force), the role of convection in heat transfer becomes more significant, in **Fig. 3** for size $w = 0.1H$ and $0.2H$, a plume starts to appear on the top of the heated block, the presence of high-temperature gradients across this plum, makes the upper part much more intense concerning the thermal gradient than the lower part. The flow at the bottom of the cavity is weak compared with that at the top regions, which suggests stratification effects in the lower part of cavity.

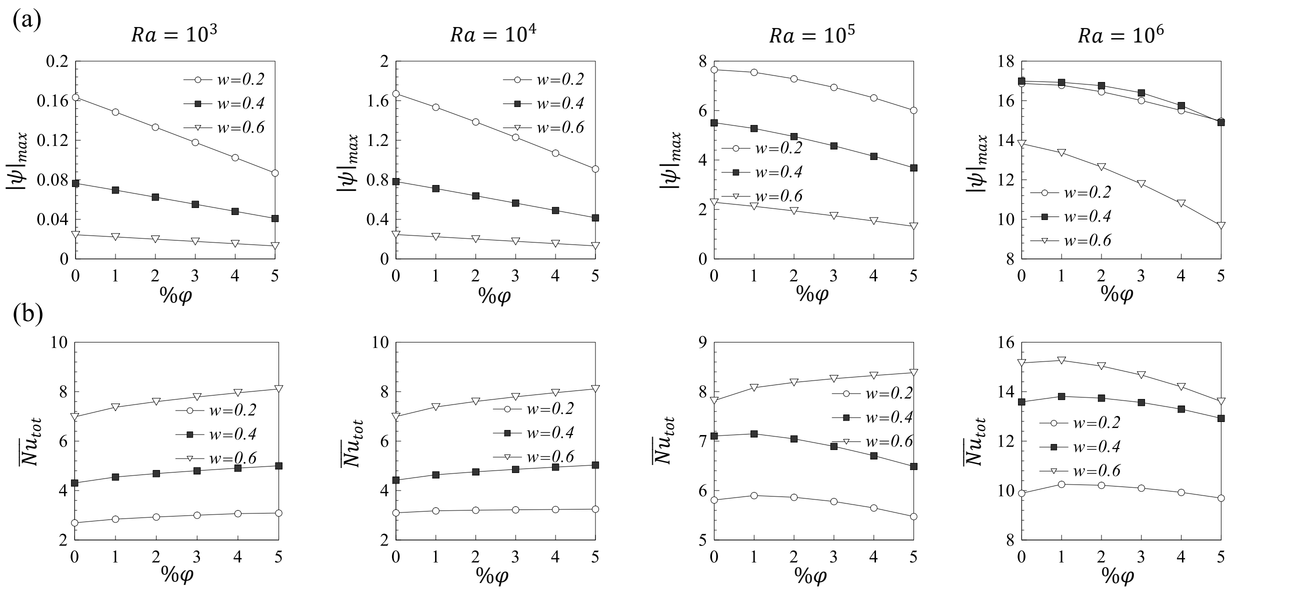

Figure 4: Variation of (a) ψ_{max} and (b) $\overline{Nu}_{tot}$ corresponding to different sizes of the heated block, i.e. $W = w/H = 0.2, 0.4$ and 0.6 for different Rayleigh numbers and volume fraction of the nanoparticles.

Fig. 4 shows values of ψ_{max} and $\overline{Nu}_{tot}$ at different Rayleigh number and volume fraction. The maximum stream function ψ_{max} shows in **Fig. 4**(a) decreases at all Rayleigh number by increasing volume fraction of nanoparticles, we can explain the decrease of ψ_{max} by Eq. (9) which indicates that the increasing volume fraction of the nanoparticles, leads to increase in the viscosity of the fluid. We can see in **Fig. 4**(b) that by increasing size of the heated block and Rayleigh number, the heat transfer rate enhances. We have seen at small values of Rayleigh number ($Ra = 10^3$ and 10^4) that the conduction is significantly dominant therefore enhancement of heat transfer rate is maximum for $\varphi = 5\%$. At high Rayleigh number ($Ra = 10^5$ and 10^6) the optimum value of φ is about 1% in most cases.

6. Conclusion

This study investigates the role of natural convection heat transfer of nanofluid (Cu– water) in a square cavity with heated block inside. According to the presented results, the following conclusions are drawn:

- By increasing Ra, the heat transfer rate increases, Moreover, it is found that the cavity heat transfer increases, with increase in the width of the square heater.
- At small values of Rayleigh number ($Ra = 10^3$ and 10^4) which the conduction is significantly dominant therefore enhancement of heat transfer rate is maximum for $\varphi = 5\%$.
- At high Rayleigh number ($Ra = 10^5$ and 10^6) which the convection is significantly dominant there is an optimum value of φ where the heat transfer is getting its maximum. φ_{opt} is about 1% in most cases.

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