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Ion Slip and Dufour Effect on Unsteady Free Convection Flow past an Infinite Vertical Plate with Oscillatory Suction Velocity and Variable Permeability

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Abstract: ***Unsteady two dimensional free convection and mass transfer flow of a viscous, incompressible, electrically conducting fluid past an infinite vertical porous plate of time dependent permeability with oscillatory suction velocity and heat source in the presence of Dufour effect, Hall Effect and Ion slip effect is investigated. A uniform magnetic field is applied normal to the direction of the flow. The governing equations are solved using perturbation technique in Eckert number. Approximate solutions for the velocity field, temperature distribution, concentration distribution, skin friction, Nusselt number and Sherwood number are obtained. The results obtained are discussed for various emerging parameters such as Prandtl number (Pr), Grashof number (Gr), Modified Grashof number(Gm), Dufour number (Du), Schmidt number(Sc), Source strength parameter(S), Eckert number(Ec), Hall parameter(β_e) and Ion slip parameter(β_i) encountered in the problem under investigation.***

Keywords: Mass transfer, Free convection, Porous medium, Suction, Heat source, Ion slip effect.

1. Introduction

The MHD flow between two parallel plates, known as Hartmann flow is a classical problem whose solution has many applications in MHD power generators, MHD pumps, accelerators, aerodynamic heating, electrostatic precipitation, petroleum industry, purification of oil, fluid droplets and sprays etc. Lot of research work concerning the Hartmann flow was conducted under different physical effects. Hartmann and Lazarus (1937) studied the influence of a transverse uniform magnetic field on the flow of a conducting fluid between two infinite parallel, stationary and insulated plates. The MHD in the present form is due to the contributions of several notable authors like Shercliff (1965), Ferraro and Plumpton (1966) and Cramer and Pai (1973). In most of the cases, the Hall and ion-slip terms were ignored in applying Ohm's law, as they have no marked effect for small and moderate values of magnetic field. But, the current trend for the application of magneto hydrodynamics is towards a strong magnetic field. Under these conditions, the Hall current and ion-slip are important and have marked effect on the magnitude and direction of the current density and consequently on magnetic force term.

Hall and ion-slip currents are likely to be important in flows of laboratory plasma when a strong magnetic field of a uniform strength is applied and drawn the attention of the researchers due to their varied significance in liquid metals, electrolytes and ionized gases etc.

Magneto hydrodynamic unsteady free convection flow past an infinite vertical plate with constant suction and heat sink was studied by Sahoo.P.K, Datta. N and Biswal.S (2003). Ram. P. C (1990) made an analysis of Hall and Ion -Slip current effects on MHD free-convection flow of a partially ionized gas past an infinite vertical porous plate in a rotating frame of reference. Theoretical analysis was carried out to study the heat and mass transfer effects on elastico-viscous fluid flow in a vertical porous channel with rotation and Hall current by Saswati Purkayastha and Rita Choudhury (2014). Soundalgekar. V.M and Bhat. J. P (1984) made an approximate analysis of an Oscillatory MHD channel flow and heat transfer under transverse magnetic field. Soundalgekar (1978) has studied the Hall and ion-slip effects in MHD couette flow with heat transfer.

An analysis was made by Nimal Ghara et al (2012) on the unsteady MHD couette flow of an incompressible viscous electrically conducting fluid between two infinite non-conducting horizontal porous plates under the boundary layer approximations with both Hall current and ion-slip. The influence of Hall and ion slip effects on the magneto-hydrodynamic flow of a micropolar fluid past a non-conducting wedge was studied by Uddin.Z and Kumar.M (2013). Viscous dissipation and Joule heating effects on unsteady MHD combined heat and mass transfer flow through Hall and ion-slip currents along a semi-infinite vertical plate in a rotating system has been studied numerically by Wahiduzzaman.M (2015).

In the above mentioned works, the Diffusion thermo (Dufour) effect was not taken into account in the species continuity equation. The energy flux caused due to composition gradient is known as Diffusion thermo effect or Dufour effect. The experimental study on diffusion thermo effect on heat transfer related problems was first performed by L.Dufour in 1872. Temperature gradients can also create mass fluxes and is known as Soret or thermo-diffusion effect. Generally, the thermo-diffusion and the diffusion-thermo

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effects are of smaller order magnitude than the effects prescribed by Fourier's or Fick's laws and are often neglected in heat and mass transfer processes, there are, however, exceptions. The Soret effect, for instance, has been utilized for isotope separation and in mixtures between gases with very light molecular weight (H2, He). For medium molecular weight (N2, air), the Dufour effect was found to be of considerable magnitude such that it cannot be neglected (Eckert and Drake,1972).

An attempt has been made to study the unsteady MHD free convective flow past a vertical porous plate immersed in a porous medium with Hall current, Diffusion thermo and heat source by Ahmed. N, Kalita. H and Barua. D. P (2010). Moorthy .M.B.K, Kannan. T and Senthilvadivu. K (2013) analyzed the Soret and Dufour effects on Natural convection heat and mass transfer flow past a horizontal surface in a porous medium with variable viscosity. The boundary layer fluid flow in a channel with heat source, Soret effects and slip condition was studied by Uwanta. I. J and Isah. B.Y (2012). Therefore, the objective of the present paper is to study the ion slip and Dufour effect on unsteady free convective flow past an infinite vertical plate with oscillatory suction velocity and variable permeability.

2. Mathematical Formulation

Consider an unsteady, free convective, viscous, incompressible, electrically conducting fluid through porous medium past an infinite vertical porous plate with oscillatory suction velocity, variable permeability and heat source in the presence of Dufour effect, Hall effect and Ion slip effect. Let the x-axis be taken along the plate in the direction of the flow and y-axis normal to the plate.

A uniform magnetic field is introduced normal to the direction of flow. All the fluid properties are assumed constant except the influence of density variation with temperature. The basic flow medium is entirely due to the buoyancy force caused by temperature difference between the wall and the medium. The surface temperature and concentration level near the plate are raised uniformly.

Let $\overline{B} = (0, B_0, 0)$ and $\overline{V} = (u', v', 0)$ The fundamental equations are

Continuity Equation:

$$\nabla.\overline{V} = 0 \tag{1}$$

Momentum Equation:

$$\rho\left[\frac{\partial\overline{V}}{\partial t} + (\overline{V}.\nabla)\overline{V}\right] = -\nabla p + \mu\nabla^2\overline{V} + \rho g\beta\left(T' - T'_\infty\right) + \overline{J}\times\overline{B} - \frac{\mu}{K}\overline{V} \tag{2}$$

Energy Equation:

$$\rho C_p\left[\frac{\partial T}{\partial t} + (\overline{V}.\nabla)T\right] = -K\nabla^2 T - p(\nabla.\overline{V}) + S^*\left(T' - T'_\infty\right) + \mu(\nabla.\overline{V})^2 \tag{3}$$

Species concentration equation:

$$\rho\left[\frac{\partial C}{\partial t} + (\overline{V}.\nabla)C\right] = \rho D\nabla^2 C \tag{4}$$

Generalized ohm's law with Hall and Ion slip effect:

$$\overline{J} = \sigma\left(\overline{E} + \overline{V}\times\overline{B}\right) - \frac{\omega_e\tau_e}{|B|}\left(\overline{J}\times\overline{B}\right) + \frac{\omega_e\tau_e\omega_i\tau_i}{|B|^2}\left(\left(\overline{J}\times\overline{B}\right)\times\overline{B}\right) \tag{5}$$

where $\overline{J}\times\overline{B}$ is the Lorentz force per unit volume.

Under the above mentioned assumptions with Boussinesq's approximation taken into account the governing equation for momentum, energy and concentration are

Equation of Continuity:

$$\frac{\partial V'}{\partial y'} = 0 \tag{6}$$

That is $V' = -V_0'\left(1 + \varepsilon e^{i\omega t}\right)$ (7)

Equation of Momentum:

$$\frac{\partial u'}{\partial t'} + V'\frac{\partial u'}{\partial y'} = g\beta(T' - T'_\infty) + g\beta'(C' - C'_\infty) + \upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2(1 + \beta_e\beta_i)}{\rho\left((1 + \beta_e\beta_i)^2 + \beta_e^2\right)}u' - \frac{\upsilon}{K'}u' \tag{8}$$

Equation of Energy:

$$\frac{\partial T'}{\partial t'} + V'\frac{\partial T'}{\partial y'} = \frac{K}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + S'(T' - T'_\infty) + \frac{\upsilon}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{D_m K_T}{C_s C_p}\frac{\partial^2 C'}{\partial y'^2} \tag{9}$$

On disregarding the Joule's heating.

Equation of Concentration:

$$\frac{\partial C'}{\partial t'} + V'\frac{\partial C'}{\partial y'} = D_m\frac{\partial^2 C'}{\partial y'^2} \tag{10}$$

the boundary conditions are given by

$$u' = 0, V' = -V_0'(1 + \varepsilon e^{i\omega' t'}),$$
$$T' = T_w' + \varepsilon(T_w' - T_\infty')e^{e\omega' t'}$$
$$C' = C_w' + \varepsilon(C_w' - C_\infty')e^{e\omega' t'} \qquad \text{at } y' = 0$$
$$u' \to 0, T' \to T_w', C' \to C_w' \qquad \text{as } y' \to \infty \tag{11}$$

Consider the following non dimensional parameters

$$y = \frac{y'V_0'}{\upsilon},\ t = \frac{t'V_0'^2}{4\upsilon},\ \omega = \frac{4\upsilon\omega'}{V_0'^2},\ u = \frac{u'}{V_0'},\ \upsilon = \frac{\mu}{\rho}$$
$$V = \frac{V'}{V_0'}, T = \left(\frac{T' - T'_\infty}{T'_w - T'_\infty}\right), C = \left(\frac{C' - C'_\infty}{C'_w - C'_\infty}\right) \tag{12}$$

Introducing the non-dimensional parameters into equation (8)-(10) it reduces to

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y}\left(1 + \varepsilon e^{i\omega t}\right) = GrT + GmC + \frac{\partial^2 u}{\partial y^2} - M_1 u - \frac{u}{K_0\left(1 + \varepsilon e^{i\omega t}\right)} \tag{13}$$

$$\frac{\Pr}{4}\frac{\partial T}{\partial t} - \Pr\left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + \frac{\Pr}{4}ST + \Pr Ec\left(\frac{\partial u}{\partial y}\right)^2 + \Pr Du\left(\frac{\partial^2 C}{\partial y^2}\right) \tag{14}$$

where $M_1 = \dfrac{M(1 + \beta_e\beta_i)}{((1 + \beta_e\beta_i)^2 + \beta_e^2)}$

$$\frac{1}{4}\frac{\partial C}{\partial t} - \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{15}$$

Where $\Pr = \dfrac{\mu C_p}{K}$: Prandtl number

$Gr = \dfrac{\upsilon g\beta(T'_w - T')}{V_0'^3}$: Grashof number

$Gm = \dfrac{\upsilon g\beta(C'_w - C')}{V_0'^3}$: Mass Grashof number

$Du = \dfrac{D_m K_T(C'_w - C'_\infty)}{C_s C_p \upsilon(T'_w - T'_\infty)}$: Dufour number

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$Sc = \frac{\upsilon}{D_m}$: Schmidt number

$S = \frac{4S'\upsilon}{V_0'^2}$: Source strength parameter

$Ec = \frac{V_0'^2}{C_p(T_w' - T_\infty')}$: Eckert number

$M = \left(\frac{\sigma B_0^2}{\rho}\right)\frac{\upsilon}{V_0'^2}$: Magnetic Parameter

$\beta_e = \omega_e \tau_e$: Hall parameter

$\beta_i = \omega_i \tau_i$: Ion slip parameter

the modified boundary conditions are

$$u = 0, T = 1 + \varepsilon e^{i\omega t}, C = 1 + \varepsilon e^{i\omega t} \text{ at } y = 0 \text{ and},$$
$$u \to 0, T \to 0, C \to 0 \quad \text{as} \quad y \to \infty \tag{16}$$

To solve equations (13), (14) and (15) we assume from V.M.Soundalgekar, J.P.Bhat (1984) that

$$u(y,t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y) \tag{17}$$
$$T(y,t) = T_0(y) + \varepsilon e^{i\omega t} T_1(y) \tag{18}$$
$$C(y,t) = C_0(y) + \varepsilon e^{i\omega t} C_1(y) \tag{19}$$

Substituting (17) - (19) in equation (13) - (15), equating like terms and neglecting the co-efficient of higher powers of ε we get

$$\frac{\partial^2 u_0}{\partial y^2} + \frac{\partial u_0}{\partial y} - A_1 u_0 = -GrT_0 - GmC_0 \tag{20}$$
$$\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial u_1}{\partial y} - A_2 u_1 = -GrT_1 - GmC_1 - \frac{\partial u_0}{\partial y} - \frac{1}{K_0} u_0 \tag{21}$$
$$\frac{\partial^2 T_0}{\partial y^2} + \Pr\frac{\partial T_0}{\partial y} + \frac{\Pr S}{4} T_0 = -\Pr Ec\left(\frac{\partial u_0}{\partial y}\right)^2 - \Pr Du \frac{\partial^2 C_0}{\partial y^2} \tag{22}$$
$$\frac{\partial^2 T_1}{\partial y^2} + \Pr\frac{\partial T_1}{\partial y} - \frac{\Pr}{4}(iw - S)T_1 = -2\Pr Ec \frac{\partial u_0}{\partial y}\frac{\partial u_1}{\partial y} - \Pr\frac{\partial T_0}{\partial y}$$
$$-\Pr Ec\left(\frac{\partial u_1}{\partial y}\right)^2 - \Pr Du \frac{\partial^2 u_1}{\partial y^2} \tag{23}$$
$$\frac{\partial^2 C_0}{\partial y^2} + Sc\frac{\partial C_0}{\partial y} = 0 \tag{24}$$
$$\frac{\partial^2 C_1}{\partial y^2} + Sc\frac{\partial C_1}{\partial y} - \frac{i\omega}{4} ScC_1 = -Sc\frac{\partial C_0}{\partial y} \tag{25}$$

Where $A_1 = M_1 + \frac{1}{K_0}; A_2 = A_1 + \frac{i\omega}{4}$

Using multi parameter perturbation technique and assuming Ec << 1 we write,

$$u_0 = u_{00} + Ecu_{01}, T_0 = T_{00} + EcT_{01}, C_0 = C_{00} + EcC_{01} \tag{26}$$
$$u_1 = u_{10} + Ecu_{11}, T_1 = T_{10} + EcT_{11}, C_1 = C_{10} + EcC_{11} \tag{27}$$

Using equation (26) and (27) in equations (20) - (25) and equating the coefficient of Ec^0 and Ec^1 only, we get the following sets of differential equations for $u_{00}, u_{01}, u_{10}, u_{11}$, $T_{00}, T_{01}, T_{10}, T_{11}$ and $C_{00}, C_{01}, C_{10}, C_{11}$

$$u_{00}'' + u_{00}' - A_1 u_{00} = -GrT_{00} - GmC_{00} \tag{28}$$
$$u_{01}'' + u_{01}' - A_1 u_{01} = -GrT_{01} - GmC_{01} \tag{29}$$
$$u_{10}'' + u_{10}' - A_2 u_{10} = -GrT_{10} - GmC_{10} - u_{00}' - \frac{1}{K_0} u_{00} \tag{30}$$
$$u_{11}'' + u_{11}' - A_2 u_{11} = -GrT_{11} - GmC_{11} - u_{01}' - \frac{1}{K_0} u_{01} \tag{31}$$
$$T_{00}'' + \Pr T_{00}' + \frac{\Pr S}{4} T_{00} + \Pr DuC_{00}'' = 0 \tag{32}$$
$$T_{01}'' + \Pr T_{01}' + \frac{\Pr S}{4} T_{01} = -\Pr(u_{00}')^2 - \Pr DuC_{01}'' \tag{33}$$
$$T_{11}'' + \Pr T_{11}'' - \frac{\Pr}{4}(i\omega - S)T_{11} = -2\Pr u_{00}' u_{10}' - \Pr T_{01}' - \Pr DuC_{11}'' - \Pr u_{10}^2 \tag{34}$$
$$T_{10}'' + \Pr T_{10}'' - \frac{\Pr}{4}(i\omega - S)T_{10} = -\Pr T_{00}' - \Pr DuC_{10}'' \tag{35}$$
$$C_{00}'' + ScC_{00}' = 0 \tag{36}$$
$$C_{01}'' + ScC_{01}' = 0 \tag{37}$$
$$C_{10}'' + ScC_{10}' - \frac{i\omega}{4} ScC_{10} = -ScC_{00}' \tag{38}$$
$$C_{11}'' + ScC_{11}' - \frac{i\omega}{4} ScC_{11} = -ScC_{01}' \tag{39}$$

Solving these differential equations (28) - (39) with the help of the corresponding boundary conditions and then substitute the values in the relations (26) and (27), we obtain the velocity, temperature and concentration as,

$$u_0 = ((D_1 - D_2)e^{-a_8 y} - D_1 e^{-a_4 y} + D_2 e^{-Scy}) + Ec(D_{00} e^{-a_8 y} - D_{13} e^{-a_4 y}$$
$$- D_{14} e^{-2a_8 y} - D_{15} e^{-2a_4 y} - D_{16} e^{-2Scy} + D_{17} e^{-(a_4 + a_8)y} + D_{18} e^{-(a_4 + Sc)y}$$
$$- D_{19} e^{-(Sc + a_8)y} - D_{20} e^{-a_8 y} - D_{21} e^{-Scy}) \tag{40}$$

$$u_1 = (D_{31} e^{-a_{10} y} - D_{26} e^{-a_6 y} + D_{27} e^{-a_4 y} + D_{28} e^{-Scy} + D_{29} e^{-a_2 y} + D_{30} e^{-a_8 y})$$
$$+ Ec(D_{80} e^{-a_{10} y} - D_{56} e^{-a_6 y} + D_{57} e^{-(a_8 + a_{10})y} - D_{58} e^{-(a_6 + a_8)y}$$
$$+ D_{59} e^{-(a_4 + a_8)y} + D_{60} e^{-(a_8 + Sc)y} + D_{61} e^{-(a_2 + a_8)y} + D_{62} e^{-2a_8 y}$$
$$+ D_{63} e^{-(a_4 + a_{10})y} + D_{64} e^{-(a_4 + a_6)y} + D_{65} e^{-2a_4 y} + D_{66} e^{-(a_4 + Sc)y}$$
$$- D_{67} e^{-(a_2 + a_4)y} + D_{68} e^{-(a_{10} + Sc)y} - D_{69} e^{-(a_6 + Sc)y} + D_{70} e^{-2Scy}$$
$$+ D_{71} e^{-(a_2 + Sc)y} + D_{72} e^{-a_4 y} + D_{73} e^{-a_8 y} + D_{74} e^{-Scy} + D_{75} e^{-a_2 y}$$
$$+ D_{76} e^{-2a_{10} y} + D_{77} e^{-2a_6 y} + D_{78} e^{-2a_2 y} - D_{79} e^{-(a_6 + a_{10})y}) \tag{41}$$

$$T_0 = (1 + D_0)e^{-a_4 y} - D_0 e^{-Scy} + Ec(D_{12} e^{-a_4 y} + D_3 e^{-2a_8 y} + D_4 e^{-2a_4 y}$$
$$+ D_5 e^{-2Scy} - D_6 e^{-(a_4 + a_8)y} - D_7 e^{-(a_4 + Sc)y} + D_8 e^{-(Sc + a_8)y}$$
$$+ D_9 e^{-a_8 y} + (D_{10} - D_{11})e^{-Scy}) \tag{42}$$

$$T_1 = D_{25} e^{-a_6 y} + D_{22} e^{-a_4 y} - D_{23} e^{-Scy} - D_{24} e^{-a_2 y} + Ec(D_{55} e^{-a_6 y}$$
$$- D_{32} e^{-(a_8 + a_{10})y} + D_{33} e^{-(a_6 + a_8)y} - D_{34} e^{-(a_4 + a_8)y} - D_{35} e^{-(a_8 + Sc)y}$$
$$- D_{36} e^{-(a_8 + a_2)y} - D_{37} e^{-2a_8 y} + D_{38} e^{-(a_4 + a_{10})y} - D_{39} e^{-(a_4 + a_6)y}$$
$$+ D_{40} e^{-2a_4 y} + D_{41} e^{-(a_4 + Sc)y} + D_{42} e^{-(a_2 + a_4)y} - D_{43} e^{-(a_{10} + Sc)y}$$
$$+ D_{44} e^{-(a_6 + Sc)y} - D_{45} e^{-2Scy} - D_{46} e^{-(a_2 + Sc)y} + D_{47} e^{-a_4 y} + D_{48} e^{-a_8 y}$$
$$+ D_{49} e^{-Scy} + D_{50} e^{-a_2 y} - D_{51} e^{-2a_{10} y} - D_{52} e^{-2a_6 y} - D_{53} e^{-2a_2 y} + D_{54} e^{-(a_6 + a_{10})y}) \tag{43}$$

$$C_0 = e^{-Scy} + Ec(e^{-Scy}) \tag{44}$$

$$C_1 = \left(1 - \frac{4iSc}{\omega}\right)e^{-a_2 y} + \frac{4iSc}{\omega} e^{-Scy} + Ec\left(\left(1 - \frac{4iSc}{\omega}\right)e^{-a_2 y} + \frac{4iSc}{\omega} e^{-Scy}\right)$$
(45)

where the constants $a_2, a_4, a_6, a_8, a_{10}, D_1, D_2, D_3, D_4$..... used above are function of the physical parameters involved in the problem given in Appendix.

Skin friction, rate of heat and mass transfer:

Skin Friction coefficient (τ) at the plate is

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = u_0'(0) + \varepsilon e^{i\omega t} u_1'(0) \tag{46}$$

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Heat transfer coefficient (Nu) at the plate is

$$Nu = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = T_0^{'}(0) + \varepsilon e^{i\omega t} T_1^{'}(0) \tag{47}$$

Mass transfer coefficient (Sh) at the plate is

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = C_0^{'}(0) + \varepsilon e^{i\omega t} C_1^{'}(0) \tag{48}$$

3. Results and Discussion

The computational analysis is carried out to discuss the behavior of the velocity field, temperature field, concentration field, skin-friction coefficient, rate of heat transfer and rate of mass transfer for different variations in physical parameters $\Pr$, Gr, Gm, Du, Sc ,S, Ec, K_0, b_e, b_i.

Fig 1. Clearly shows the mean velocity profile u_0 for various Dufour number (Du). It is observed that, the profile decreases with the increase in Dufour number (Du).

Fig 2. Shows the transient velocity profile u_1 for various Dufour number (Du). It is evident that, the profile decreases with the increase in Dufour number (Du) and tends to zero as y increases.

Fig 3.Shows the mean velocity profile u_0 for different Hall parameter (b_e). It is clearly known that, the fluid velocity profile increases with the increase of Hall parameter (b_e).

Fig 4. Shows the transient velocity profile u_1 for different Hall parameter (b_e). It is known that, the fluid velocity profile decreases with the increase of Hall parameter (b_e).

Fig 5. Clearly shows the mean velocity profile u_0 for different Permeability parameter (K_0). It is clearly known that, the fluid velocity profile increases with the increase of permeability parameter (K_0).

Fig 6. Shows the transient velocity profile u_1 for different Permeability parameter (K_0). It is clearly known that, the fluid velocity profile decreases with the increase of permeability parameter (K_0).

Fig 7. Shows the mean velocity profile u_0 for different Ion slip parameter (b_i). It is clearly known that, the profile increases with the increase of Ion slip parameter (b_i).

Fig 8. Shows the transient velocity profile u_1 for different Ion slip parameter (b_i). It is clearly known that, the profile decreases to a particular extent and reverse their progress and at last converge with the increase of Ion slip parameter (b_i).

Fig 9. Shows the mean velocity profile u_0 for different Eckert number (Ec). It is clearly known that, the velocity profile increases with the increase in Eckert number (Ec).

Fig 10. Shows the transient velocity profile u_1 for different Eckert number (Ec). It is clearly known that, the velocity profile decreases with the increase in Eckert number (Ec).

Fig 11. Exhibits the mean temperature profile T_0 for various Dufour number (Du). It shows that the profile increase with the increase in Dufour number (Du).
Fig 12. Exhibits the mean temperature profile T_0 for various Hall parameter (b_e). It shows that the profile increase with the increase in Hall parameter (b_e).

Fig 13. Exhibits the transient temperature profile T_1 for various Hall parameter (b_e). It shows that the profile decreases with the increase in Hall parameter (b_e).

Fig 14. Exhibits the mean temperature profile T_0 for various Permeability parameter (K_0). It shows that the profile increase to certain extent and decreases with the increase in Permeability parameter (K_0) then converges to zero as $y \to \infty$.

Fig 15.Exhibits the transient temperature profile T_1 for various Permeability parameter (K_0). It shows that the profile decreases with the increase in Permeability parameter (K_0) and the profile tends to zero as $y \to \infty$.

Fig 16. Exhibits the mean temperature profile T_0 for various Ion slip parameter (b_i). It shows that the profile increases with the increase in Ion slip parameter (b_i) till T_0 =2 then decreases and comes to zero as $y \to \infty$.

Fig 17. Exhibits the transient temperature profile T_1 for various Ion slip parameter (b_i). It shows that the profile decreases with the increase in Ion slip parameter (b_i) and the profile tends to zero as $y \to \infty$.

Fig 18. Exhibits the mean temperature profile T_0 for various Eckert number (Ec). It shows that the profile increases with the increase in Eckert number (Ec) and tends to zero as $y \to \infty$.

Fig 19. Exhibits the transient temperature profile T_1 for various Eckert number (Ec). It shows that the profile decreases with the increase in Eckert number (Ec) and tends to zero as $y \to \infty$.

Fig 20. Shows the mean concentration profile C_0 for different Schmidt number (Sc). It is clear that, the profile increases with the decrease of Schmidt number (Sc).

Fig 21. Shows the Skin friction profile for various Dufour number (Du). It is observed that, the profile increase with the increase in Dufour number (Du).

Fig 22. Shows the Skin friction profile for various Hall parameter (b_e). It is observed that, the profile increase with the increase in Hall parameter (b_e).

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Fig 23. Shows the Skin friction profile for various Permeability parameter (K_0). It is observed that, the profile increase with the increase in Permeability parameter (K_0).

Fig 24. Shows the Skin friction profile for various Eckert number (Ec). It is observed that, the profile increase with the increase in Eckert number (Ec).

Fig 25. Shows the Skin friction profile for various Ion slip parameter (b_i). It is observed that, the profile increase with the increase in Ion slip parameter (b_i).

Fig 26. Shows the rate of heat transfer profile for various Dufour number (Du). It is observed that, the profile decreases with the increase in Dufour number (Du).

Fig 27. Shows the rate of heat transfer profile for various Hall parameter (b_e). It is observed that, the profile decreases with the increase in Hall parameter (b_e).

Fig 28. Shows the rate of heat transfer profile for various Eckert number (Ec). It is observed that, the profile decreases with the increase in Eckert number (Ec).

Fig 29. Shows the rate of heat transfer profile for various Schmidt number (Sc). It is observed that, the profile increase with the increase in Schmidt number (Sc).

Fig 30. Shows the rate of heat transfer profile for various Ion slip parameter (b_i). It is observed that, the profile decreases with the increase in Ion slip parameter (b_i).

Fig 31. Shows the rate of heat transfer profile for various Permeability parameter (K_0). It is observed that, the profile decreases with the increase in Permeability parameter (K_0).

Fig 32. Shows the rate of concentration profile for different Schmidt number (Sc). It is clearly known that, the profile increase with the decreases in Schmidt number (Sc).

4. Conclusion

In this paper, unsteady free convective flow past an infinite vertical porous plate with Hall current effect, Ion slip effect and Dufour effect is studied. The flow is considered in the presence of transverse magnetic field. The following conclusions are made based on the result and discussion.

1) The mean velocity profile u_0 increases with increasing Hall parameter, Permeability parameter, Ion slip parameter and Eckert number. The trend is just reversed with respect to Dufour number.
2) The transient velocity profile u_1 decreases with the increase in Dufour number, Hall parameter, Permeability parameter, Ion slip parameter and Eckert number.
3) The mean temperature profile T_0 increases with the increases in Dufour number, Hall parameter, Ion slip parameter and Eckert number.
4) The mean temperature profile T_0 decreases with the increases in Permeability parameter.
5) The transient temperature profile T_1 decreases with the increases in Hall parameter, Permeability parameter, Ion slip parameter and Eckert number.
6) The mean concentration profile C_0 increases with the decreases in Schmidt number.
7) The skin-friction increases with increase of Dufour number, Hall parameter, Permeability parameter, Eckert number and Ion slip parameter.
8) There is a fall in the rate of heat transfer profile due to increase of Dufour number, Hall parameter, Eckert number, Ion slip parameter and Permeability parameter. The rate of heat transfer increases with increase in Schmidt number.
9) The rate of concentration increases with the decrease in Schmidt number.

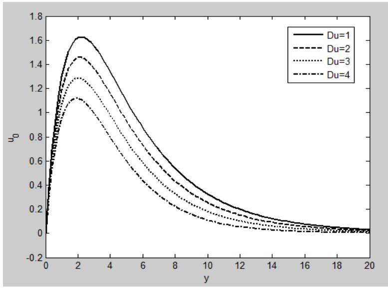

Figure 1: Mean velocity profile u_0 for different Du
Pr = 0.71, Gm=3,ω =10, Sc=0.22, ε=0.05, S =3, b_e = 0.2, Ec= 0.001, Gr= 10, M = 1, K_0 = 10, b_i =1

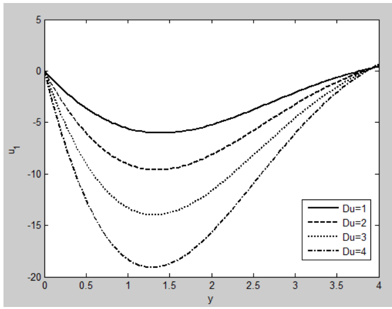

Figure 2: Transient velocity profile u_1 for different Du
Pr = 0.71, Gm=3, ω = 10, Sc=0.22, ε = 0.005, S =1, b_e = 2, Ec= 0.001, Gr =10, M = 1, K_0 = 10, b_i =0.1

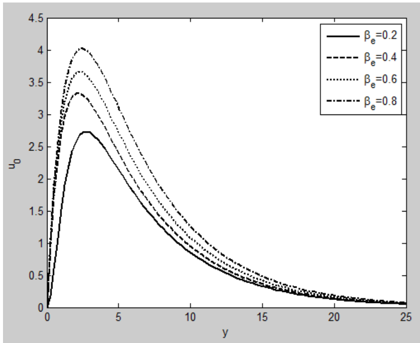

Figure 3: Mean velocity profile u_0 for different b_e
Pr = 0.71, Gm = 5,ω = 15, ε = 0.5, Sc=0.22, Gr=7, S =1, Ec = 0.001, M = 2, K_0= 10, b_i =0.1, Du=3

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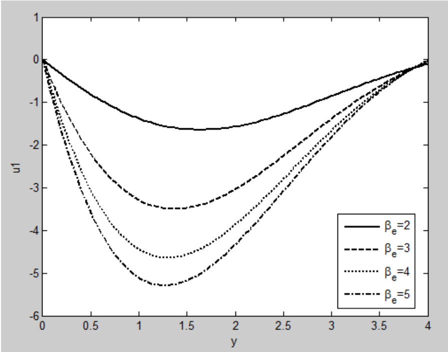

Figure 4: Transient velocity profile u_1 for different b_e

Pr = 0.71, Gm=2, ω = 10, ε = 0.5, Sc = 0.22, S =1, Gr=5, Ec=0.001, M=1, K_0=10, b_i =0.1, Du=4

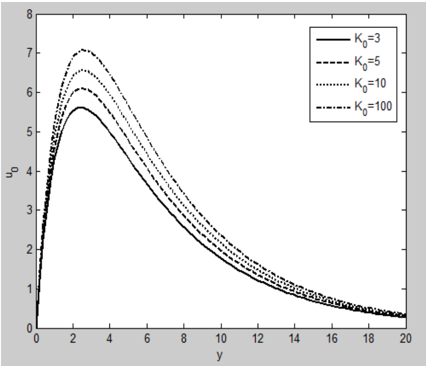

Figure 5: Mean velocity profile u_0 for different K_0

Pr = 0.71, Gm=5,ω = 10, ε = 0.05, Sc = 0.22, S =1, Ec = 0.001, Gr = 10, M = 1, b_e =0.2, b_i =0.1, Du=4

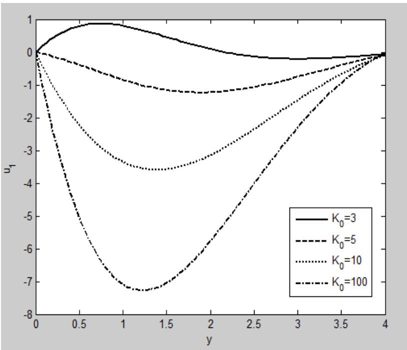

Figure 6: Transient velocity profile u_1 for different K_0

Pr=0.71, Gm=2, ω = 10, ε = 0.5, Sc = 0.22, S =1, Ec= 0.001, Gr = 5, M = 1, b_e =3, b_i =1, Du=5

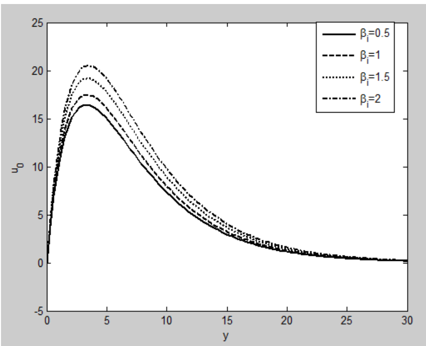

Figure 7: Mean velocity profile u_0 for different b_i

Pr = 0.71, Gm=5,ω =15, ε = 0.5, Sc = 0.22, S =1, Ec = 0.001, Gr = 7, M = 1, b_e =2, K_0 = 10, Du=4

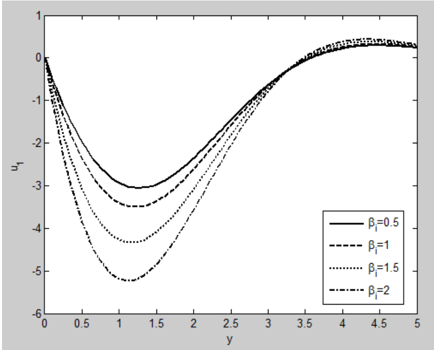

Figure 8: Transient velocity profile u_1 for different b_i

Pr = 0.71, Gm=5,ω = 15, ε = 0.5, Sc = 0.22, Gr = 7, Ec = 0.001, M = 1, b_e =2, S =1, K_0 = 10, Du=4

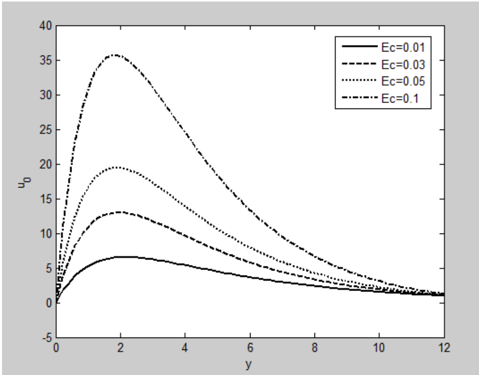

Figure 9: Mean velocity profile u_0 for different Ec

Pr = 0.71, Gm=2, ω = 10,ε = 0.05, Sc = 0.22, S =1, Gr = 7, M = 1, b_e =0.2, b_i =0.1, K_0 = 10, Du=4

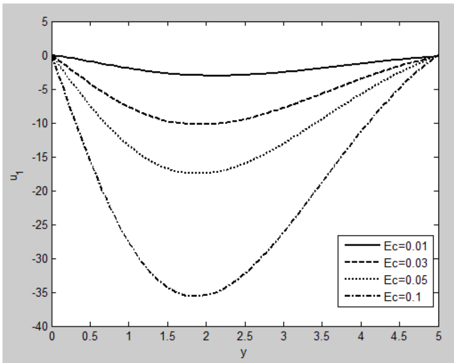

Figure 10: Transient velocity profile u_1 for different Ec

Pr = 0.71, Gm=2, ω = 6, ε = 0.5, Sc = 0.22, S =1, Gr = 5, M = 1, b_e =0.2, b_i =0.1, K_0 = 10, Du=4

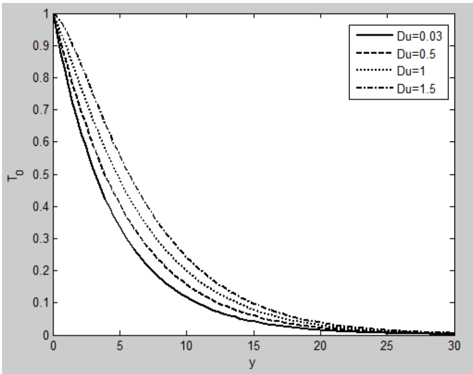

Figure 11: Mean temperature profile T_0 for various Du

Pr = 0.71, Gm=3,ω = 10, ε = 0.5, b_e = 0.2, S=1, Ec= 0.001, Gr =7, M=1, K_0=10, ,Sc = 0.30, b_i =0.1

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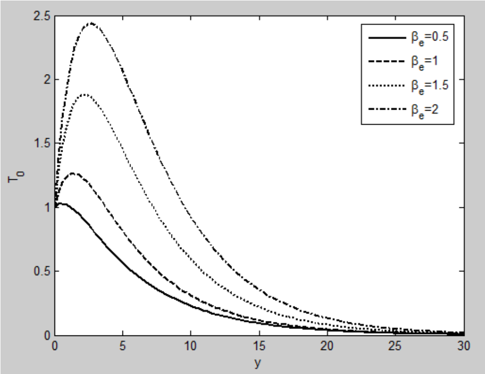

Figure 12: Mean temperature profile T_0 for various b_e

Pr=0.71, Gm=5,ω =10, ε =0.05, Ec =0.001, S=1, Gr = 8, M = 1, K_0 = 10, Sc = 0.22, b_i =0.1, Du=5

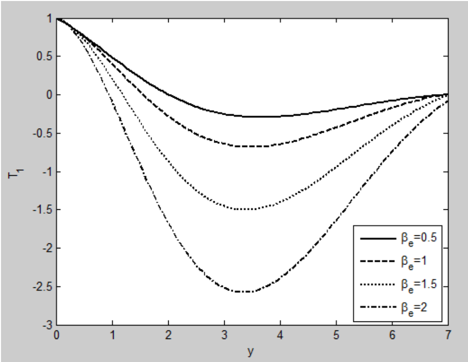

Figure 13: Transient temperature profile T_1 for various b_e

Pr = 0.71, Gm=3,ω = 5, ε = 0.5, Ec = 0.001, S=1, Gr = 5, M = 1, K_0 = 10, Sc = 0.22, b_i =0.1, Du=4

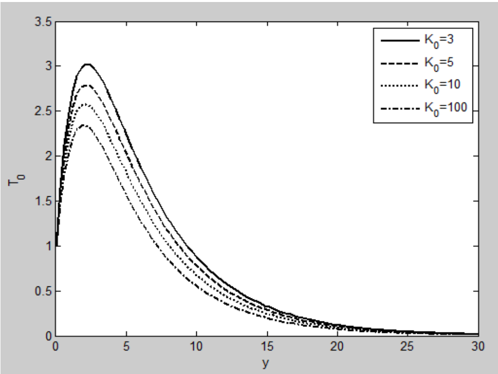

Figure 14: Mean temperature profile T_0 for various K_0

Pr = 0.71, Gm=5, ω = 10, ε = 0.05, Gr = 7, M = 1, b_e=0.2, Ec = 0.001, S=1, Sc = 0.22, b_i =0.1, Du=4

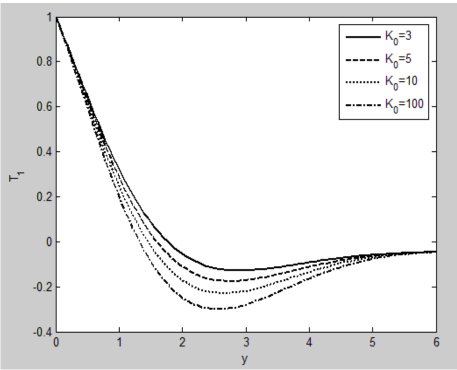

Figure 15: Transient temperature profile T_1 for various K_0

Pr = 0.71, Gm=5,ω = 10, ε=0.05, Ec = 0.001, S=1, Gr =7, M = 1, b_e =0.2, Sc = 0.22, b_i =0.1, Du=4

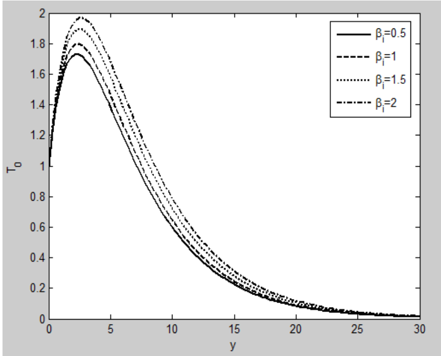

Figure 16: Mean temperature profile T_0 for various b_i

Pr= 0.71, Gm=5, ω = 1, ε= 0.5, Ec = 0.001, S=1, Gr = 7, M = 1, b_e =2, K_0 =10, Sc =0.22, Du=4

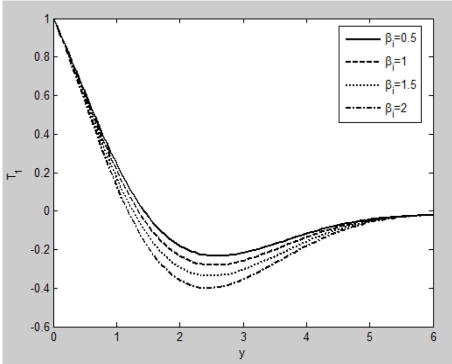

Figure 17: Transient temperature profile T_1 for various b_i

Pr = 0.71, Gm=5, ω =10, ε= 0.5, Ec=0.001, S=1, Gr = 7, M = 1, b_e =0.2, K_0 =10, Sc= 0.22, Du=3

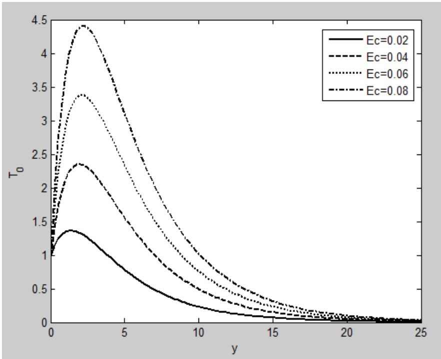

Figure 18: Mean temperature profile T_0 for various Ec

Pr = 0.71, Gm=5, ω = 10, ε = 0.5, b_i =0.1, Gr = 7, M = 1, b_e =2, K_0 = 10, S=3, Sc = 0.22, Du=5

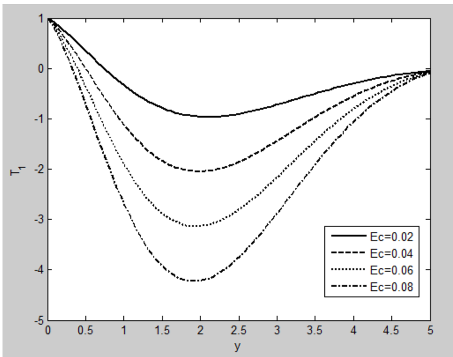

Figure 19: Transient temperature profile T_1 for various Ec

Pr = 0.71, Gm=4, ω = 10, ε = 0.5, b_i =0.1, Gr = 3, M = 1, b_e =0.2, K_0 = 10, S=1, Sc = 0.22, Du=4

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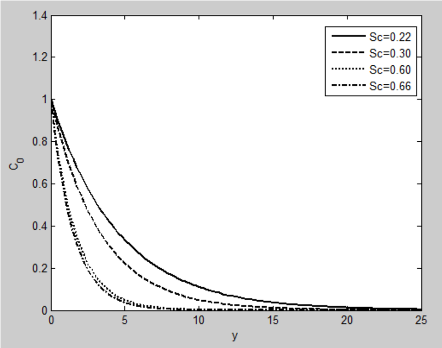

Figure 20: Mean concentration profile C_0 for various Sc
Pr = 0.71, Gm=2, ω = 10, ε = 0.05, b_i =0.1, S=1,
Ec=0.001, Gr = 5, M = 1, b_e =0.2, K_0 = 10, Du=4

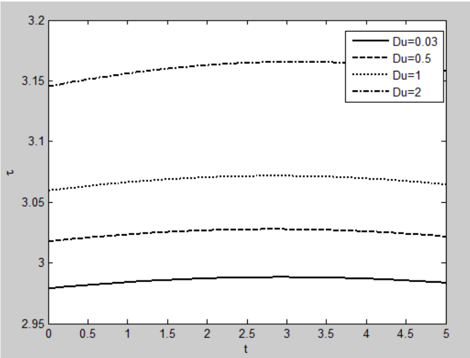

Figure 21: Effect of Du on Skin Friction
Gr=5, ω = 0.1, ε = 0.05, b_i =0.1, Gm=3, Ec=0.001,
Pr = 0.71, M = 1, b_e =1, K_0= 1, Sc=0.22, S=1

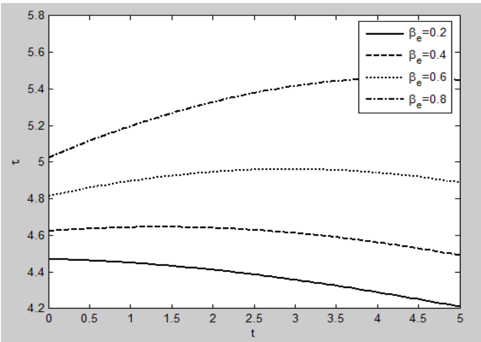

Figure 22: Effect of b_e on Skin Friction
Gr=5, ω = 0.1, ε = 0.005, b_i =0.2, Gm=5, M = 1,
Pr = 0.71, Ec=0.001, K_0 = 10, Sc=0.22, S=1, Du=4

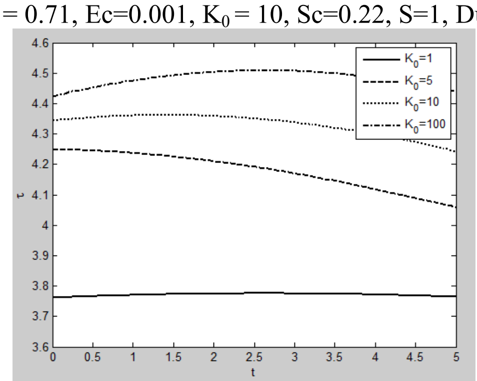

Figure 23: Effect of K_0 on Skin Friction
Gr=5, ω = 0.1, ε = 0.005, b_i =1, Gm=5, Ec=0.001,
b_e =0.2, Pr = 0.71, M = 1, Sc=0.22, S=1, Du=3

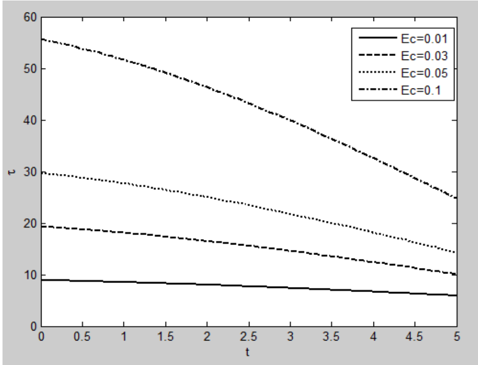

Figure 24: Effect of Ec on Skin Friction
Gr=5, ω = 0.1, ε = 0.005, b_i =0.1, Gm=5, b_e =0.2,
Pr = 0.71, M = 1, K_0= 10, Sc=0.22, S=1, Du=3

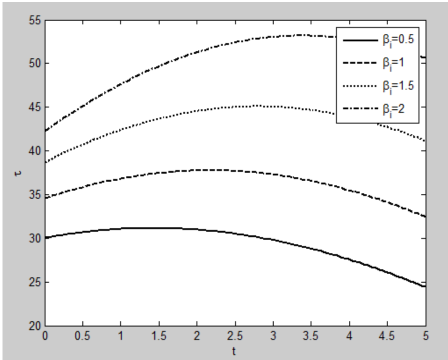

Figure 25: Effect of b_i on Skin Friction
Gr=5, ω = 0.1, ε = 0.5, Gm=5, Ec=0.001, b_e =0.2,
Pr = 0.71, M = 1, K_0 =10, Sc=0.22, S=1, Du=1

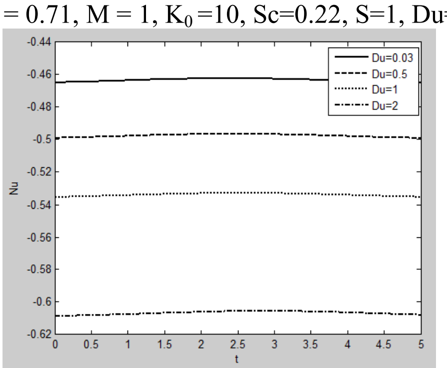

Figure 26: Effect of Du on Rate of Heat Transfer
Gr=5, Gm=5, ω =0. 5, ε =0.005, b_i =0.1, Sc=0.22,
Ec=0.001, Pr = 0.71, M = 1, b_e =0.2, K_0 = 1, S=1

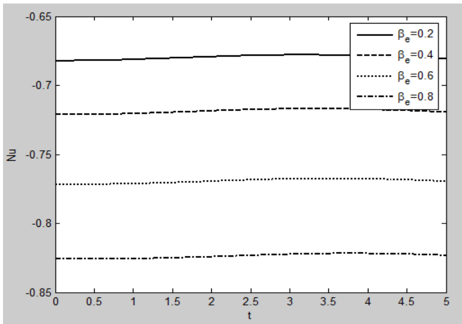

Figure 27: Effect of b_e on Rate of Heat Transfer
Gr=5, Gm=5, ω =0. 5, ε = 0.005, b_i =0.1, Sc=0.22,
Ec=0.001, Pr = 0.71, M = 1, K_0= 1, S=1, Du=3

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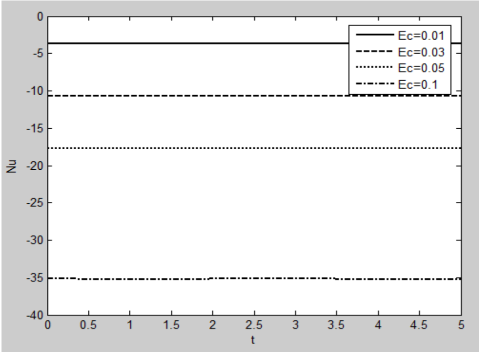

Figure 28: Effect of Ec on Rate of Heat Transfer
Gr=5, Gm=5, ω = 1, ε = 0.005, b_i =0.1, Sc=0.22, b_e =0.2, Pr = 0.71, M = 1, K_0= 1, S=1, Du=1

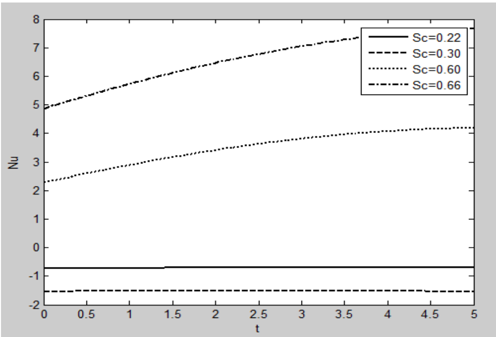

Figure 29: Effect of Sc on Rate of Heat Transfer
Pr=0.71, Gm=5, ω = 0.1, ε = 0.05, b_i =0.1, S=1, Gr = 5, Ec=0.001, M = 2, b_e =0.2, K_0 = 1, Du=10

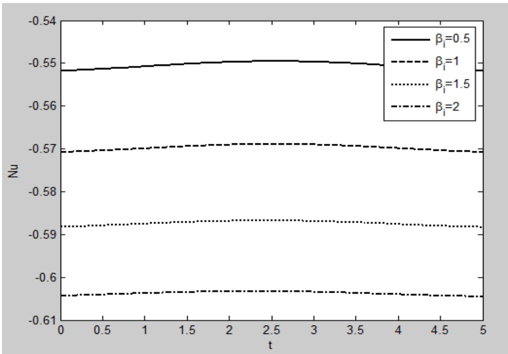

Figure 30: Effect of b_i on Rate of Heat Transfer
Pr=0.71, Gm=5, ω=0.5, ε=0.005, Sc=0.22, S=1, Ec=0.001, Gr = 5, M = 1, b_e =0.2, K_0= 1, Du=1

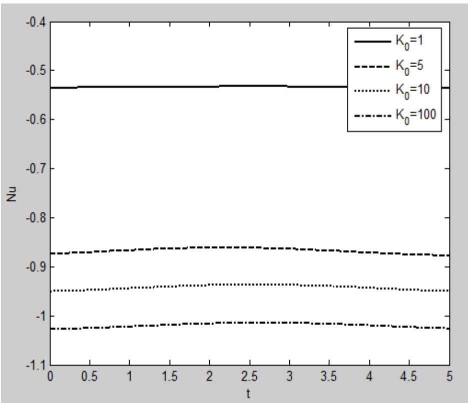

Figure 31: Effect of K_0 on Rate of Heat Transfer
Pr=0.71, Gm=5, ω=0.5, ε=0.005, Sc=0.22, S=1, Ec=0.001, Gr = 5, M = 1, b_e =0.2, b_i = 0.1, Du=1

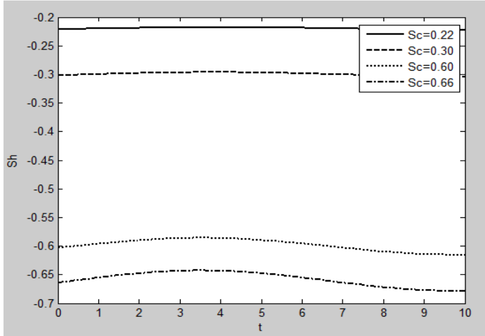

Figure 32: Effect of Sc on Rate of Concentration
Pr=0.71, Gm=5, ω=0.5, ε=0.005, b_i =0.1, S=1, Ec=0.001, Gr = 5, M = 1, b_e =0.2, K_0 = 10, Du=1

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Appendix

$A_1 = M_1 + \frac{1}{K_c}$, $\quad A_2 = A_1 + \frac{i\omega}{4}$

$a_2 = \frac{-Sc+\sqrt{Sc^2+i\omega Sc}}{2}$ $\quad a_4 = \frac{-Pr+\sqrt{Pr^2-Prs}}{2}$

$a_6 = \frac{-Pr+\sqrt{Pr^2+Pr(i\omega-S)}}{2}$ $\quad a_8 = \frac{-1+\sqrt{1+4A_1}}{2}$

$a_{10} = \frac{-1+\sqrt{1+4A_2}}{2}$ $\quad D_1 = \frac{Gr(1+D_0)}{a_4^2-a_4-A_1}$

$D_2 = \frac{GrD_0 - Gm}{Sc^2 - Sc - A_1}$ $\quad D_0 = \frac{PrDuSc^2}{Sc^2 - PrSc + \frac{Prs}{4}}$

$D_3 = \frac{Pr(D_1-D_2)^2 a_8^2}{4a_8^2 - 2Pra_8 + \frac{Prs}{4}}$ $\quad D_4 = \frac{PrD_1^2 a_4^2}{4a_4^2 - 2P_r a_4 + \frac{P_r s}{4}}$

$$D_5 = \frac{PrD_2^2 Sc^2}{4Sc^2 - 2PrSc + \frac{Prs}{4}}$$

$$D_6 = \frac{2(D_1-D_2)D_1 a_4 a_8 Pr}{(a_4+a_8)^2 - Pr(a_4+a_8) + \frac{Prs}{4}}$$

$$D_7 = \frac{2D_1 D_2 a_4 ScPr}{(a_4+Sc)^2 - Pr(a_4+S_c) + \frac{Prs}{4}}$$

$$D_8 = \frac{2(D_1-D_2)D_1 a_8 ScPr}{(Sc+a_8)^2 - Pr(Sc+a_8) + \frac{Prs}{4}}$$

$$D_9 = \frac{2(D_1-D_2)D_1 a_8 ScPr}{a_8{}^2 - Pra_8 + \frac{Prs}{4}}$$

$$D_{10} = \frac{2(D_1-D_2)D_1 a_8 ScPr}{Sc^2 - PrSc + \frac{Prs}{4}}$$

$$D_{11} = \frac{PrSc^2 Du}{Sc^2 - PrSc + \frac{Prs}{4}}$$

$$D_{12} = -(D_3 - D_4 - D_5 + D_6 + D_7 - D_8 - D_9 - D_{10} + D_{11})$$

$D_{13} = \frac{GrD_{12}}{a_4^2 - a_4 - A_1}$, $\quad D_{14} = \frac{GrD_3}{4a_8^2 - 2a_8 - A_1}$

$D_{15} = \frac{GrD_4}{4a_4^2 - 2a_4 - A_1}$, $\quad D_{16} = \frac{GrD_5}{4S_c^2 - 2S_c - A_1}$

$$D_{17} = \frac{GrD_6}{(a_4+a_8)^2 - (a_4+a_8) - A_1}$$

$$D_{18} = \frac{GrD_7}{(a_4+S_c)^2 - (a_4+S_c) - A_1}$$

$D_{19} = \frac{GrD_8}{4S_c^2 - 2S_c - A_1}$, $\quad D_{20} = \frac{GrD_9}{a_8^2 - a_8 - A_1}$

$$D_{21} = \frac{GrD_{10} - GrD_{11} + Gm}{Sc^2 - S_c - A_1}$$

$$D_{00} = D_{13} + D_{14} + D_{15} + D_{16} - D_{17} - D_{18} + D_{19} + D_{20} + D_{21}$$

$$D_{22} = \frac{Pra_4(D_0+1)}{a_4^2 - Pra_4 - \frac{Pr}{4}(i\omega - s)}$$

$$D_{23} = \frac{PrSc\left(D_0 - \frac{i4DuSc^2}{\omega}\right)}{Sc^2 - PrSc - \frac{P_r}{4}(i\omega - s)}$$

$$D_{24} = \frac{PrDua_2^2\left(1 - \frac{i4Sc}{\omega}\right)}{a_2^2 - Pra_2 - \frac{P_r}{4}(i\omega - s)}$$

$$D_{25} = 1 - D_{22} + D_{23} + D_{24}$$

$$D_{26} = \frac{GrD_{25}}{a_6^2 - a_6 - A_2}$$

$$D_{27} = \frac{-GrD_{22} + \frac{D_1}{K_0} - D_1 a_4}{a_4^2 - a_4 - A_2}$$

$$D_{28} = \frac{GrD_{23} - \frac{i4GmSc}{\omega} - \frac{D_2}{K_0} + D_2 Sc}{Sc^2 - Sc - A_2}$$

$$D_{29} = \frac{GrD_{24} - Gm\left(1 - \frac{i4Sc}{\omega}\right)}{a_2^2 - a_2 - A_2}$$

$$D_{30} = \frac{(D_1-D_2)a_8 - \frac{(D_1-D_2)}{K_0}}{a_8^2 - a_8 - A_2}$$

$$D_{31} = D_{26} - D_{27} - D_{28} - D_{29} - D_{30}$$

$$D_{32} = \frac{2Pr((D_1-D_2)D_{31}a_8 a_{10} + D_{31}D_{30})}{(a_8+a_{10})^2 - Pr(a_8+a_{10}) - \frac{Pr}{4}(i\omega - s)}$$

$$D_{33} = \frac{2Pr(D_1-D_2)D_{26}a_6 a_8}{(a_6+a_8)^2 - Pr(a_6+a_8) - \frac{Pr}{4}(i\omega - s)}$$

$$D_{34} = \frac{Pr(2(D_1-D_2)D_{29}a_4 a_8 - 2D_{30}D_1 a_4 a_8 + (a_4+a_8)D_6)}{(a_4+a_8)^2 - Pr(a_4+a_8) - \frac{Pr}{4}(i\omega - s)}$$

$$D_{35} = \frac{Pr(2(D_1-D_2)D_{28}a_8 S_c + 2D_2 a_4 a_8 ScD_{30} - D_8(a_8+S_c))}{(a_8+S_c)^2 - P_r(a_8+S_c) - \frac{P_r}{4}(i\omega - s)}$$

$$D_{36} = \frac{2Pr((D_1-D_2)D_{29}a_2 a_8 + D_{30}D_{29})}{(a_2+a_8)^2 - Pr(a_2+a_8) - \frac{Pr}{4}(i\omega - s)}$$

$$D_{37} = \frac{Pr(2(D_1-D_2)D_{30}a_8^2 - 2D_3 a_8 + D_{30}^2)}{4a_8{}^2 - 2P_r a_8 - \frac{P_r}{4}(i\omega - s)}$$

$$D_{38} = \frac{2PrD_{31}D_1 a_{10}a_4}{(a_4+a_{10})^2 - Pr(a_4+a_{10}) - \frac{Pr}{4}(i\omega - s)}$$

$$D_{39} = \frac{2Pr(D_1 D_{26}a_4 a_6 - D_{26}D_{27})}{(a_4+a_6)^2 - Pr(a_4+a_6) - \frac{Pr}{4}(i\omega - s)}$$

$$D_{40} = \frac{Pr(2D_{27}D_1 a_4^2 + 2a_4 D_4 - D_{27}^2)}{4a_4^2 - 2Pra_4 - \frac{Pr}{4}(i\omega - s)}$$

$$D_{41} = \frac{Pr(2D_{28}D_1 a_4 S_c - 2D_{27}D_2 a_4 S_c - D_7(a_4+S_c) - 2D_{27}D_{28})}{(a_4+S_c)^2 - Pr(a_4+S_c) - \frac{Pr}{4}(i\omega - s)}$$

$$D_{42} = \frac{2PrD_{29}D_1 a_2 a_4}{(a_2+a_4)^2 - Pr(a_2+a_4) - \frac{Pr}{4}(i\omega - s)}$$

$$D_{43} = \frac{2PrD_{31}D_2 a_{10}Sc}{(Sc+a_{10})^2 - Pr(Sc+a_{10}) - \frac{Pr}{4}(i\omega - s)}$$

$$D_{44} = \frac{2PrD_{26}D_2 a_6 Sc}{(Sc+a_6)^2 - Pr(Sc+a_6) - \frac{Pr}{4}(i\omega - s)}$$

$$D_{45} = \frac{Pr(2D_{28}D_2 Sc^2 - 2D_5 Sc + D_{28}{}^2)}{4Sc^2 - 2PrSc - \frac{Pr}{4}(i\omega - s)}$$

$$D_{46} = \frac{2Pr(D_{29}D_2 Sc\, a_2 + D_{29}D_{28})}{(Sc+a_2)^2 - Pr(Sc+a_2) - \frac{Pr}{4}(i\omega - s)}$$

$$D_{47} = \frac{PrD_{12}a_4}{a_4{}^2 - Pra_4 - \frac{Pr}{4}(i\omega - s)}$$

$$D_{48} = \frac{PrD_9 a_8}{a_8{}^2 - Pra_8 - \frac{Pr}{4}(i\omega - s)}$$

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$$D_{49}=\frac{PrSc\left(D_{10}-D_{11}-\frac{i4DuSc^{2}}{\omega}\right)}{Sc^{2}-PrSc-\frac{Pr}{4}(i\omega-s)}$$

$$D_{27}=\frac{-GrD_{22}+\frac{D_{1}}{K_{0}}-D_{1}a_{4}}{a_{4}^{2}-a_{4}-A_{2}}$$

$$D_{28}=\frac{GrD_{23}-\frac{i4GmSc}{\omega}-\frac{D_{2}}{K_{0}}+D_{2}Sc}{Sc^{2}-Sc-A_{2}}$$

$$D_{29}=\frac{GrD_{24}-Gm\left(1-\frac{i4Sc}{\omega}\right)}{a_{2}^{2}-a_{2}-A_{2}}$$

$$D_{30}=\frac{(D_{1}-D_{2})a_{8}-\frac{(D_{1}-D_{2})}{K_{0}}}{a_{8}^{2}-a_{8}-A_{2}}$$

$$D_{31}=D_{26}-D_{27}-D_{28}-D_{29}-D_{30}$$

$$D_{32}=\frac{2\Pr\left((D_{1}-D_{2})D_{31}a_{8}a_{10}+D_{31}D_{30}\right)}{(a_{8}+a_{10})^{2}-Pr(a_{8}+a_{10})-\frac{Pr}{4}(i\omega-s)}$$

$$D_{33}=\frac{2Pr(D_{1}-D_{2})D_{26}a_{6}a_{8}}{(a_{6}+a_{8})^{2}-Pr(a_{6}+a_{8})-\frac{Pr}{4}(i\omega-s)}$$

$$D_{34}=\frac{\Pr\left(2(D_{1}-D_{2})D_{29}a_{4}a_{8}-2D_{30}D_{1}a_{4}a_{8}+(a_{4}+a_{8})D_{6}\right)}{(a_{4}+a_{8})^{2}-Pr(a_{4}+a_{8})-\frac{Pr}{4}(i\omega-s)}$$

$$D_{35}=\frac{Pr\left(2(D_{1}-D_{2})D_{28}a_{8}S_{c}+2D_{2}a_{4}a_{8}ScD_{30}-D_{8}(a_{8}+S_{c})\right)}{(a_{8}+S_{c})^{2}-P_{r}(a_{8}+S_{c})-\frac{P_{r}}{4}(i\omega-s)}$$

$$D_{36}=\frac{2\Pr\left((D_{1}-D_{2})D_{29}a_{2}a_{8}+D_{30}D_{29}\right)}{(a_{2}+a_{8})^{2}-Pr(a_{2}+a_{8})-\frac{Pr}{4}(i\omega-s)}$$

$$D_{37}=\frac{\Pr\left(2(D_{1}-D_{2})D_{30}a_{8}^{2}-2D_{3}a_{8}+D_{30}^{2}\right)}{4a_{8}^{2}-2P_{r}a_{8}-\frac{P_{r}}{4}(i\omega-s)}$$

$$D_{38}=\frac{2PrD_{31}D_{1}a_{10}a_{4}}{(a_{4}+a_{10})^{2}-Pr(a_{4}+a_{10})-\frac{Pr}{4}(i\omega-s)}$$

$$D_{39}=\frac{2\Pr\left(D_{1}D_{26}a_{4}a_{6}-D_{26}D_{27}\right)}{(a_{4}+a_{6})^{2}-Pr(a_{4}+a_{6})-\frac{Pr}{4}(i\omega-s)}$$

$$D_{40}=\frac{\Pr\left(2D_{27}D_{1}a_{4}^{2}+2a_{4}D_{4}-D_{27}^{2}\right)}{4a_{4}^{2}-2Pra_{4}-\frac{Pr}{4}(i\omega-s)}$$

$$D_{41}=\frac{Pr\left(2D_{28}D_{1}a_{4}S_{c}-2D_{27}D_{2}a_{4}S_{c}-D_{7}(a_{4}+S_{c})-2D_{27}D_{28}\right)}{(a_{4}+S_{c})^{2}-Pr(a_{4}+S_{c})-\frac{Pr}{4}(i\omega-s)}$$

$$D_{42}=\frac{2PrD_{29}D_{1}a_{2}a_{4}}{(a_{2}+a_{4})^{2}-Pr(a_{2}+a_{4})-\frac{Pr}{4}(i\omega-s)}$$

$$D_{43}=\frac{2PrD_{31}D_{2}a_{10}Sc}{(Sc+a_{10})^{2}-Pr(Sc+a_{10})-\frac{Pr}{4}(i\omega-s)}$$

$$D_{44}=\frac{2PrD_{26}D_{2}a_{6}Sc}{(Sc+a_{6})^{2}-Pr(Sc+a_{6})-\frac{Pr}{4}(i\omega-s)}$$

$$D_{45}=\frac{Pr\left(2D_{28}D_{2}Sc^{2}-2D_{5}Sc+D_{28}{}^{2}\right)}{4Sc^{2}-2PrSc-\frac{Pr}{4}(i\omega-s)}$$

$$D_{46}=\frac{2\Pr\left(D_{29}D_{2}Sc\,a_{2}+D_{29}D_{28}\right)}{(Sc+a_{2})^{2}-\Pr(Sc+a_{2})-\frac{Pr}{4}(i\omega-s)}$$

$$D_{47}=\frac{PrD_{12}a_{4}}{a_{4}{}^{2}-Pra_{4}-\frac{Pr}{4}(i\omega-s)}$$

$$D_{48}=\frac{PrD_{9}a_{8}}{a_{8}{}^{2}-Pra_{8}-\frac{Pr}{4}(i\omega-s)}$$

$$D_{49}=\frac{PrSc\left(D_{10}-D_{11}-\frac{i4DuSc^{2}}{\omega}\right)}{Sc^{2}-PrSc-\frac{Pr}{4}(i\omega-s)}$$

$$D_{50}=\frac{-PrDu\,a_{2}{}^{2}\left(1-\frac{i4Sc}{\omega}\right)}{a_{2}{}^{2}-Pr\,a_{2}-\frac{Pr}{4}(i\omega-s)}$$

$$D_{51}=\frac{PrD_{31}{}^{2}}{4a_{10}^{2}-2Pra_{10}-\frac{Pr}{4}(i\omega-s)}$$

$$D_{52}=\frac{PrD_{26}{}^{2}}{4a_{6}^{2}-2Pra_{6}-\frac{Pr}{4}(i\omega-s)}$$

$$D_{53}=\frac{PrD_{29}{}^{2}}{4a_{2}^{2}-2Pra_{2}-\frac{Pr}{4}(i\omega-s)}$$

$$D_{54}=\frac{2PrD_{31}D_{26}}{(a_{10}+a_{6})^{2}-Pr(a_{10}+a_{6})-\frac{Pr}{4}(i\omega-s)}$$

$$D_{55}=D_{23}-D_{33}+D_{34}+D_{35}+D_{36}+D_{37}-D_{38}+D_{39}-D_{40}-D_{41}-D_{42}+D_{43}-D_{44}+D_{45}+D_{46}-D_{47}-D_{48}-D_{49}-D_{50}+D_{51}+D_{52}+D_{53}-D_{54}$$

$$D_{56}=\frac{GrD_{55}}{a_{6}^{2}-a_{6}-A_{2}}$$

$$D_{57}=\frac{GrD_{32}}{(a_{8}+a_{10})^{2}-(a_{8}+a_{10})-A_{2}}$$

$$D_{58}=\frac{GrD_{33}}{(a_{6}+a_{8})^{2}-(a_{6}+a_{8})-A_{2}}$$

$$D_{59}=\frac{GrD_{34}+D_{17}(a_{4}+a_{8})-\left(\frac{1}{K_{0}}\right)D_{17}}{(a_{4}+a_{8})^{2}-(a_{4}+a_{8})-A_{2}}$$

$$D_{60}=\frac{GrD_{35}-D_{19}(a_{8}+Sc)+\left(\frac{1}{K_{0}}\right)D_{19}}{(a_{8}+Sc)^{2}-(a_{8}+Sc)-A_{2}}$$

$$D_{61}=\frac{GrD_{36}}{(a_{2}+a_{8})^{2}-(a_{2}+a_{8})-A_{2}}$$

$$D_{62}=\frac{GrD_{37}-2D_{14}a_{8}+\left(\frac{1}{K_{0}}\right)D_{14}}{4a_{8}{}^{2}-2a_{8}-A_{2}}$$

$$D_{63}=\frac{GrD_{38}}{(a_{4}+a_{10})^{2}-(a_{4}+a_{10})-A_{2}}$$

$$D_{64}=\frac{GrD_{39}}{(a_{4}+a_{6})^{2}-(a_{4}+a_{6})-A_{2}}$$

$$D_{65}=\frac{-GrD_{40}-2D_{15}a_{4}+\left(\frac{1}{K_{0}}\right)D_{15}}{4a_{4}{}^{2}-2a_{4}-A_{2}}$$

$$D_{66}=\frac{-GrD_{41}+D_{18}(a_{4}+Sc)-\left(\frac{1}{K_{0}}\right)D_{18}}{(a_{4}+Sc)^{2}-(a_{4}+Sc)-A_{2}}$$

$$D_{67}=\frac{GrD_{42}}{(a_{2}+a_{4})^{2}-(a_{2}+a_{4})-A_{2}}$$

$$D_{68}=\frac{GrD_{43}}{(a_{10}+Sc)^{2}-(a_{10}+Sc)-A_{2}}$$

$$D_{69}=\frac{GrD_{44}}{(a_{6}+Sc)^{2}-(a_{6}+Sc)-A_{2}}$$

$$D_{70}=\frac{GrD_{45}-2D_{16}Sc-\left(\frac{1}{K_{0}}\right)D_{16}}{4Sc^{2}-2Sc-A_{2}}$$

$$D_{71}=\frac{GrD_{46}}{(a_{2}+Sc)^{2}-(a_{2}+Sc)-A_{2}}$$

$$D_{72}=\frac{-GrD_{47}-D_{13}a_{4}+\left(\frac{1}{K_{0}}\right)D_{13}}{a_{4}{}^{2}-a_{4}-A_{2}}$$

$$D_{73}=\frac{-GrD_{48}+a_{8}D_{00}-a_{8}D_{20}-\left(\frac{1}{K_{0}}\right)D_{00}+\left(\frac{1}{K_{0}}\right)D_{20}}{a_{8}{}^{2}-a_{8}-A_{2}}$$

$$D_{74}=\frac{-Gr\,D_{49}-\frac{i4GmSc}{\omega}-ScD_{21}+\left(\frac{1}{K_{0}}\right)D_{21}}{Sc^{2}-Sc-A_{2}}$$

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$$D_{75} = \frac{-GrD_{50} - Gm(1 - \frac{i4Sc}{\omega})}{a_2^2 - a_2 - A_2}$$

$$D_{76} = \frac{GrD_{51}}{4a_{10}^2 - 2a_{10} - A_2}$$

$$D_{77} = \frac{GrD_{52}}{4a_6^2 - 2a_6 - A_2}$$

$$D_{78} = \frac{GrD_{53}}{4a_2^2 - 2a_2 - A_2}$$

$$D_{79} = \frac{GrD_{54}}{(a_6 + a_{10})^2 - (a_6 + a_{10}) - A_2}$$

$$D_{80} = D_{56} - D_{57} + D_{58} - D_{59} - D_{60} - D_{61} - D_{62} - D_{63} - D_{64} - D_{65} - D_{66} + D_{67} - D_{68} + D_{69} - D_{70} - D_{71} - D_{72} - D_{73} - D_{74} - D_{75} - D_{76} - D_{77} - D_{78} + D_{79}$$

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