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A Displaced Probability Model for Child Mortality for Fixed Parity

Ashutosh Pandey[1], Himanshu Pandey[2], Kamal Kishore Sahu[3]

[1]Department of Applied Sciences, NCU[Formerly ITM University] Gurgaon(Haryana),India

[2]Department of Mathematics and Statistics, DDU Gorakhpur University Gorakhpur U.P., India

[3]Research Scholar, Department of Mathematics and Statistics DDU Gorakhpur University ,Gorakhpur U.P.

Abstract: ***In demography, child mortality is useful as a sensitive index of a nation's health conditions and as guided for the structuring of public health schemes. In the present study, we proposed a displaced Poisson probability model for the number of child death among females for a fixed parity. The model involved several parameters, which is estimated by method of maximum likelihood and fitted to observed survey data [National Family Health Survey-III (NFHS-III).] to draw important conclusions.***

Keywords: Displaced Probability Model, Fixed Parity, Method of Estimation. Chi-Square

1. Introduction

In demography child mortality are useful as a sensitive index of a nation's health conditions and as guided for the structuring of public health programmes. Child Mortality is interrelated to social, cultural, economic, physiological and other factor. The high rate of infant and child mortality shows a low level development of the health programme and also for the nation's. Infant and Child mortality has been of interest of researchers and demographers because of its apparent relationship with the acceptance of modern contraceptive means (Kabir et al, 1993).

Some effects have been made to estimate the current levels of child mortality by using data available from the different survey and other specific sources. Hill and Devid (1989) have suggested an approach for estimating child mortality from all births which have taken place in last five years before the survey. However, the estimates obtained though this method also suffer from the problem of under reporting Pathak, Pandey, and Mishra, (1991). In this Circumstances, some of the earlier studies about child mortality by using model (Chauhan, 1997; Gavrilova, 1991; Goldblatt, 1989; Heligman and Pollard, 1980; Krishna, 1993; Ronald and Carter, 1992; Thiele, 1972). Initially, Keyfitz (1977) used a hyperbolic function to study the infant and child mortality. Later Arnold (1983) used pareto distribution and Krishnan and Yin (1993) applied finite range model for the same. Pandey H. Pandey A and Shukla V.K., have been developed "A Displaced Probability Model for Child Mortality in a family" (2014).Sonam Maheshwari and et al. have been developed a new probability model in child mortality (2015).

Level of child mortality of any nation is a widely accepted and sensitive sign of the social and economic progress of that nation also it also helps in evaluating the impact of various intervention programs, which have aim to improve child survival. Child mortality, the probability of death between the first and the fifth birth-day has been a main concern for the Government of India over the past several decades. Many child survival programs have been commenced by Government of India. Due to these programs, child mortality has grasped remarkable improvement. However, at the current pace, is unable to attain the MillenniumDevelopment Goal (MDG) 4 -which aims to reduce Under-five Mortality (U5MR) by two thirds between 1990 and 2015. Six states, namely Kerala, Tamil Nadu, Maharashtra, Punjab, Himachal Pradesh and West Bengal are likely to achieve the goal by 2015.Data obtained from the three rounds of National Family Health Surveys conducted in the years 1992-93, 1998-99 and 2005-06 and the Sample Registration System (1978-2010) revealed drastic turn down in seventies, stand still in nineties and then started de-clining again in the last decade. During 1968-70, the level of child mortality rates in India was about 190 per 1000 live births; the child mortality rate started declining in the late 1970s and until 1993 the rate of decline was substantial. The decline, however, slowed during 1993-98. The country's goal to achieve child mor-tality rate less than 100 per 1000 live births by the year 2000, was not achieved despite improved interventions and an increase in the overall resources. The latest mortality estimates for 2009 in India indicates that 64 per 1000 live births died before reaching the age from five years (SRS 2011).

In the present study, we proposed a displaced Poisson probability model for the number of child death among females for a fixed parity. The model involved several parameters, which is estimated MLE and fitted to observed survey data [National Family Health Survey-III (NFHS-III).] to draw important conclusions.

2. Probability Model

Let x denote the number of child deaths in a family at the survey point. Then the distribution of x is derived under the following assumption.

1) Only those families are considered in which at least one death prior to the survey has occurred.
2) At the survey point, a family either has experienced a child loss or not. Let α and $(1-\alpha)$ be the respective proportions.

3) Let b be the proportion of families in which only one child death has occurred.
4) Remaining $(1-b)a$ proportion of families, experiencing multiple child deaths, follows a displaced Poisson distribution with parameter t according to the number of child death is

$$P(X=0)=(1-a)$$

$$P(X=1)=a.b$$

$$P(X=x)=a(1-b)e^{-t}.\frac{t^{x-2}}{(x-2)!}$$

$$x=2,3,4,5.....$$

3. Estimation

Maximum Likelihood Method:

The likelihood function of the probability model can be expressed as:

$$L=(1-a)^{f_0}(a.b)^{f_1}[(1-b)a.b]^{f_2}[1-\{(1-a)+a.b+(1-b)a.e^{-t}\}]^{(f-f_o-f_1-f_2)}$$

Taking log both sides, we get

$$\ln L = f_0\ln 1-a)+f_1\ln a+f_1\ln b+f_2\ln(1-b)+f_2\ln a-tf_2+(f-f_0-f_1-f_2)\ln(1-b)+$$
$$(f-f_0-f_1-f_2)\ln a+(f-f_0-f_1-f_2)\ln(1-e^{-t})$$

Partially differentiating with respect to a, b and t and equating to zero. This results with following equations:

$$\frac{\partial}{\partial a}\ln L = f_0\left[\frac{-1}{1-a}\right]+\frac{f_1}{a}+\frac{f_2}{a}+\frac{(f-f_0-f_1-f_2)}{a}=0$$

This gives

$$\hat{a}=\frac{(f-f_0)}{f}$$

$$\frac{\partial}{\partial b}\ln L=\frac{f_1}{b}+f_2\left[\frac{-1}{1-b}\right]+(f-f_0-f_1-f_2)\left[\frac{-1}{1-b}\right]=0$$

This gives

$$\hat{b}=\frac{f_1}{(f-f_0)}$$

$$\frac{\partial}{\partial t}\ln L=-f_2+(f-f_0-f_1-f_2).\frac{1}{(1-e^{-t})}e^{-t}=0$$

This gives

$$\hat{t}=(-1)\left[\ln\frac{f_2}{(f-f_0-f_1)}\right]$$

$\hat{a}$, $\hat{b}$ and $\hat{t}$ are maximum likelihood estimators of a, b and t respectively.

Now

$$\frac{\partial^2}{\partial a^2}\ln L=-\left[\frac{f_0}{(1-a)^2}+\frac{(f-f_0)}{a^2}\right]$$

$$\frac{\partial^2}{\partial b^2}\ln L=-\left[\frac{f_1}{b^2}+\frac{(f-f_0-f_1}{(1-b)^2}\right]$$

$$\frac{\partial^2}{\partial t^2}\ln L=-\left[\frac{e^{-t}(f-f_0-f_1-f_2)}{(1-e^{-t})^2}\right]$$

Now

$$-\frac{E\left(-\frac{\partial^2}{\partial a^2}\ln L\right)}{f}=\left[\frac{1}{1-a}+\frac{1}{a}\right]$$

$$-\frac{E\left(-\frac{\partial^2}{\partial b^2}\ln L\right)}{f}=\left[\frac{a}{b(1-b)}\right]$$

$$\frac{E\left(-\frac{\partial^2}{\partial t^2}\ln L\right)}{f}=\left[\frac{ae^{-t}(1-b)(1-t)}{(1-e^{-t})^2}\right]$$

Using the fact we get,

$$E(f_0)=f(1-a)$$
$$E(f_1)=f.a.b$$
$$E(f_2)=f(1-b)ae^{-t}$$
$$E(f-f_0-f_1-f_2)=f[a\{(1-b)(1-e^{-t})\}]$$

Case1: When a is known, The Fisher Information matrix is given by

$$\text{FIM (b,t)} = \begin{bmatrix} \frac{-E\left[\frac{\partial^2}{\partial b^2}\ln L\right]}{f} & 0 \\ 0 & \frac{-E\left[\frac{\partial^2}{\partial t^2}\ln L\right]}{f}\end{bmatrix}$$

$$=\begin{bmatrix}\left[\frac{a}{b(1-b)}\right] & 0 \\ 0 & \left[\frac{ae^{-t}(1-b)(1-t)}{(1-e^{-t})^2}\right]\end{bmatrix}=\begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{21} & \psi_{22}\end{bmatrix}$$

$$\psi_{12}=\left[\frac{\partial^2}{\partial b\partial t}\ln L\right]=0$$

$$\psi_{21}=\left[\frac{\partial^2}{\partial t\partial b}\ln L\right]=0$$

$$V(\hat{t})=\frac{1}{f}\left[\frac{\psi_{11}}{\psi_{11}.\psi_{22}}\right]$$

$$V(\hat{b})=\frac{1}{f}\left[\frac{\psi_{22}}{\psi_{11}.\psi_{22}}\right]$$

Case2: when b is known, The Fisher Information matrix is given by

$$\text{FIM (a, t)}=\begin{bmatrix} \dfrac{-E\left[\dfrac{\partial^2}{\partial a^2}\ln L\right]}{f} & 0 \\ 0 & \dfrac{-E\left[\dfrac{\partial^2}{\partial t^2}\ln L\right]}{f} \end{bmatrix}$$

$$=\begin{bmatrix} \left[\dfrac{1}{1-a}+\dfrac{1}{a}\right] & 0 \\ 0 & \left[\dfrac{ae^{-t}(1-b)(1-t)}{(1-e^{-t})^2}\right] \end{bmatrix}=\begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix}$$

$$\psi_{12}=\left[\frac{\partial^2}{\partial a \partial t}\ln L\right]=0$$

$$\psi_{21}=\left[\frac{\partial^2}{\partial t \partial a}\ln L\right]=0$$

$$V(\hat{t})=\frac{1}{f}\left[\frac{\psi_{11}}{\psi_{11}.\psi_{22}}\right]$$

$$V(\hat{a})=\frac{1}{f}\left[\frac{\psi_{22}}{\psi_{11}.\psi_{22}}\right]$$

Case3: when t is known, The Fisher Information matrix is given by

$$\text{FIM (a,b)}=\begin{bmatrix} \dfrac{-E\left[\dfrac{\partial^2}{\partial a^2}\ln L\right]}{f} & 0 \\ 0 & \dfrac{-E\left[\dfrac{\partial^2}{\partial b^2}\ln L\right]}{f} \end{bmatrix}$$

$$=\begin{bmatrix} \left[\dfrac{1}{1-a}+\dfrac{1}{a}\right] & 0 \\ 0 & \left[\dfrac{a}{b(1-b)}\right] \end{bmatrix}=\begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix}$$

$$\psi_{12}=\left[\frac{\partial^2}{\partial a \partial b}\ln L\right]=0$$

$$\psi_{21}=\left[\frac{\partial^2}{\partial b \partial a}\ln L\right]=0$$

$$V(\hat{b})=\frac{1}{f}\left[\frac{\psi_{11}}{\psi_{11}.\psi_{22}}\right] \quad V(\hat{a})=\frac{1}{f}\left[\frac{\psi_{22}}{\psi_{11}.\psi_{22}}\right]$$

Table 1: Expected and Observed Distribution of Child death to the females in Madhya Pradesh with Parity 5

No. of Child Death	Observed females	Numbers of Expected Frequency
0	159	159
1	111	111
2	30	30
3	10	9.371240551
4	1 11	
5	0	
Total	311	
$\hat{a}$	0.488745981	
$\hat{b}$	0.730263158	
$\hat{t}$	0.312374685	
χ^2	0.283084969	

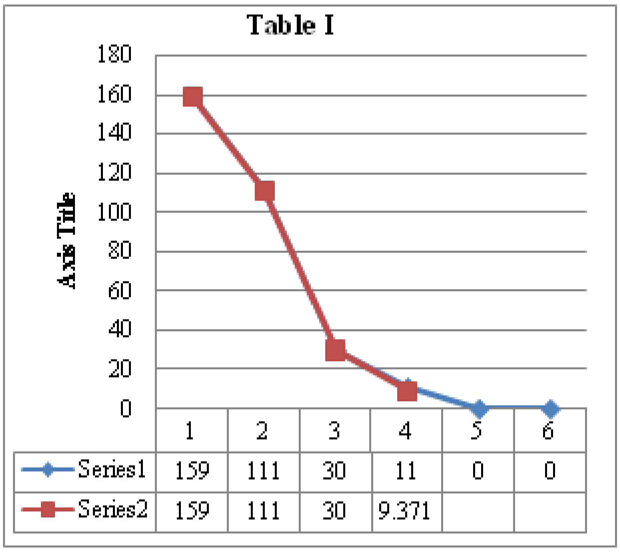
Table 2: Expected and Observed Distribution of Child death to the females in Madhya Pradesh with Parity 6

No. of Child Death	Observed females	Numbers of Expected Frequency
0	59	59
1	49	49
2	28	28
3	12	12.01187695
4	3	
5	0 15	
6	0	
Total	151	
$\hat{a}$	0.609271523	
$\hat{b}$	0.532608696	
$\hat{t}$	0.428995606	
χ^2	0.743337562	

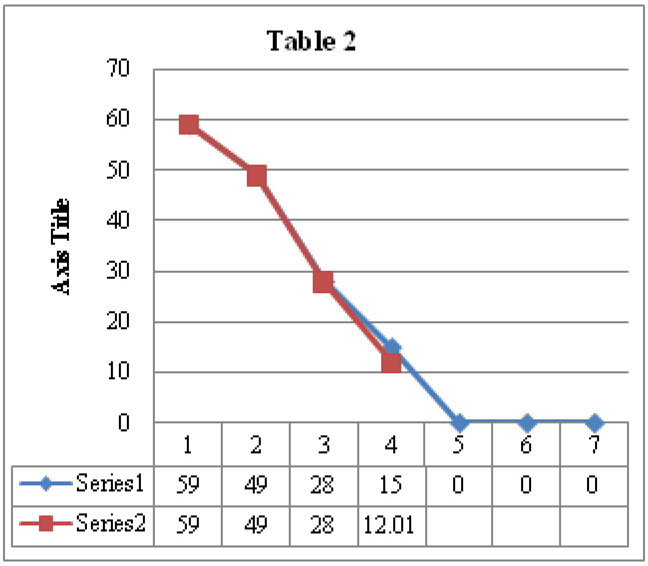
Table 3: Expected and Observed Distribution of Child death to the females in Madhya Pradesh with Parity 7

No. of Child Death	Observed females	Numbers of Expected Frequency
0	23	23
1	27	27
2	30	30
3	12	13.46851
4	3	
5	2 5	3.531493
6	0	
7	0	
Total	97	
$\hat{a}$	0.762886598	
$\hat{b}$	0.364864865	
$\hat{t}$	0.44895022	
χ^2	0.770766597	

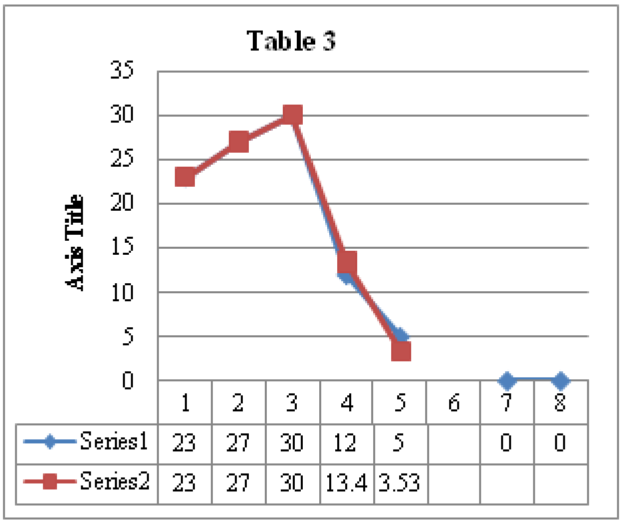
Case1: When a is known, The Fisher Information matrix is given by		
Table 1	Table 2	Table 3
$V(\hat{b})$=0.017811297 $V(\hat{t})$=0.022522169	$V(\hat{b})$=0.002705833 $V(\hat{t})$=0.007611112	$V(\hat{b})$=0.003131601 $V(\hat{t})$=0.035845269
Case2: when b is known, The Fisher Information matrix is given by		
Table 1	Table 2	Table 3
$V(\hat{a})$=0.010927 $V(\hat{t})$=0.047456	$V(\hat{a})$=0.001576555 $V(\hat{t})$=0.052474861	$V(\hat{a})$=0.001864852 $V(\hat{t})$=0.007913891
Case3: when t is known, The Fisher Information matrix is given by		
Table 1	Table 2	Table 3
$V(\hat{b})$=1.29591E-03 $V(\hat{a})$=0.000803451	$V(\hat{b})$=0.002705833 $V(\hat{a})$=0.001576555	$V(\hat{b})$=0.003131601 $V(\hat{a})$=0.001864852



Graph: The line chart for observed and expected data for table 1.



Graph: The line chart for observed and expected data for table 2.



Graph: The line chart for observed and expected data for table 3.

4. Application

The suitability of the proposed model is examined to the sample data collected from Madhya Pradesh from National Family Health Survey-III (NFHS-III), 2005-06 for Madhya Pradesh. The parameters of the proposed model have been estimated by method of maximum likelihood. The estimated values of different parameters are given in tables 1 to 3 for the child deaths for fixed parity.

The estimated value of a are 0 .4889, 0.6093 and 0.7629 for fixed parity of 5 , 6 and 7 by Method of MLE . This shows the proportion of families experienced a child loss for fixed parity was found slightly increasing for fixed parity of 6 and 7. The estimate of b are 0.7303, 0.5326 and 0.3649 for fixed parity for 5, 6 and 7 by method of MLE. This shows the proportion of families was found slightly decreasing for fixed parity of 6 and 7, and the estimated values for t are 0.3124, 0.4289 and 0.4489 by MLE respectively for the above mentioned fixed parity. The graph 1 to 3 shows the deviation of expected values from the observed values in the three types of data. From tables 1 to 3 it is found that the value of χ^2 are insignificant at 5% level of significance for all set of data. The proposed model suitably described the pattern of child mortality for different data for fixed parity.

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