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Invariant Submanifold of $\tilde{\psi}(p,1)$ Structure Manifold

Lakhan Singh[1], Shailendra Kumar Gautam[2]

[1]Department of Mathematics, D.J. College, Baraut, Baghpat (U.P.), India

[2]Eshan College of Engineering, Mathura (UP), India

Abstract: ***In this paper, we have studied various properties of a $\tilde{\psi}(p,1)$ structure manifold and its invariant submanifold, where p is odd prime. Under two different assumptions, the nature of induced structure ψ, has also been discussed.***

Keywords: Invariant submanifold, Nijenhuis tensor, projection operators and complementary distributions

1. Introduction

Let V^m be a C^∞ m-dimensional Riemannian manifold imbedded in a C^∞ n-dimensional Riemannian manifold M^n, where $m<n$. The imbedding being denoted by

$$f: V^m \longrightarrow M^n$$

Let B be the mapping induced by *f* i.e. $B=df$

$$df: T(V) \longrightarrow T(M)$$

Let $T(V,M)$ be the set of all vectors tangent to the submanifold $f(V)$. It is well known that

$$B: T(V) \longrightarrow T(V,M)$$

Is an isomorphism. The set of all vectors normal to $f(V)$ forms a vector bundle over $f(V)$, which we shall denote by $N(V,M)$. We call $N(V,M)$ the normal bundle of V^m. The vector bundle induced by f from $N(V,M)$ is denoted by $N(V)$. We denote by $C: N(V) \longrightarrow N(V,M)$ the natural isomorphism and by $\eta_s^r(V)$ the space of all C^∞ tensor fields of type (r,s) associated with N (V). Thus $\zeta_0^0(V)=\eta_0^0(V)$ is the space of all C^∞ functions defined on V^m while an element of $\eta_0^1(V)$ is a C^∞ vector field normal to V^m and an element of $\zeta_0^1(V)$ is a C^∞ vector field tangential to V^m.

Let $\bar{X}$ and $\bar{Y}$ be vector fields defined along $f(V)$ and $\tilde{X},\tilde{Y}$ be the local extensions of $\bar{X}$ and $\bar{Y}$ respectively. Then $[\tilde{X},\tilde{Y}]$ is a vector field tangential to M^n and its restriction $[\tilde{X},\tilde{Y}]/f(V)$ to $f(V)$ is determined independently of the choice of these local extension $\tilde{X}$ and $\tilde{Y}$. Thus $[\bar{X},\bar{Y}]$ is defined as

(1.1) $$[\bar{X},\bar{Y}]=[\tilde{X},\tilde{Y}]/f(V)$$

Since B is an isomorphism

(1.2) $$[BX,BY]=B[X,Y] \text{ for all } X,Y\in\zeta_0^1(V)$$

Let $\tilde{G}$ be the Riemannain metric tensor of M^n, we define g and g^* on V^m and $N(V)$ respectively as

(1.3) $$g(X_1,X_2)=\tilde{G}(BX_1,BX_2)f, \text{ and}$$

(1.4) $$g^*(N_1,N_2)=\tilde{G}(CN_1,CN_2)$$

For all $X_1,X_2\in\zeta_0^1(V)$ and $N_1,N_2\in\eta_0^1(V)$

It can be verified that g and g^* are the induced metrics on V^m and $N(V)$ respectively.

Let $\tilde{\nabla}$ be the Riemannian connection determined by $\tilde{G}$ in M^n, then $\tilde{\nabla}$ induces a connection ∇ in $f(V)$ defined by

(1.5) $$\nabla_{\bar{X}}\bar{Y}=\tilde{\nabla}_{\tilde{X}}\tilde{Y}/f(V)$$

where $\bar{X}$ and $\bar{Y}$ are arbitrary C^∞ vector fields defined along $f(V)$ and tangential to $f(V)$.

Let us suppose that M^n is a C^∞ $\tilde{\psi}(p,1)$ structure manifold with structure tensor $\tilde{\psi}$ of type (1,1) satisfying

(1.6) $$\tilde{\psi}^p+\tilde{\psi}=0$$

Let $\tilde{L}$ and $\tilde{M}$ be the complementary distributions corresponding to the projection operators

(1.7) $$\tilde{l}=-\tilde{\psi}^{p-1}, \qquad \tilde{m}=I+\tilde{\psi}^{p-1}$$

where I denotes the identity operator.

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From (1.6) and (1.7), we have

(1.8) **(a)** $\tilde{l}+\tilde{m}=I$ (b) $\tilde{l}^2=\tilde{l}$

(c) $\tilde{m}^2=\tilde{m}$

(d) $\tilde{l}\,\tilde{m}=\tilde{m}\,\tilde{l}=0$

Let D_l and D_m be the subspaces inherited by complementary projection operators l and m respectively.

We define

$$D_l=\{X\in T_p(V):\ lX=X,\ mX=0\}$$

$$D_m=\{X\in T_p(V):\ mX=X,\ lX=0\}$$

Thus $T_p(V)=D_l+D_m$

Also $Ker\,l=\{X:lX=0\}=D_m$

$Ker\,m=\{X:mX=0\}=D_l$

at each point p of $f(V)$.

2. Invariant Submanifold of $\tilde{\psi}(p,1)$ Structure Manifold

We call V^m to be invariant submanifold of M^n if the tangent space $T^p(f(V))$ of $f(V)$ is invariant by the linear mapping $\tilde{\psi}$ at each point p of $f(V)$. Thus

(2.1) $\tilde{\psi}BX=B\psi X$, for all $X\in\zeta_0^1(V)$,, and ψ being a (1,1) tensor field in V^m.

Theorem (2.1): Let $\tilde{N}$ and N be the Nijenhuis tensors determined by $\tilde{\psi}$ and ψ in M^n and V^m respectively, then

(2.2) $\tilde{N}(BX,BY)=BN(X,Y)$, for all $X,Y\in\zeta_0^1(V)$

Proof: We have, by using (1.2) and (2.1)

(2.3) $\tilde{N}(BX,BY)=[\tilde{\psi}BX,\tilde{\psi}BY]$

$$+\tilde{\psi}^2[BX,BY]-\tilde{\psi}[\tilde{\psi}BX,BY]$$
$$-\tilde{\psi}[BX,\tilde{\psi}BY]$$
$$=[B\psi X,B\psi Y]+\tilde{\psi}^2B[X,Y]$$
$$-\tilde{\psi}[B\psi X,BY]-\tilde{\psi}[BX,B\psi Y]$$

$$=B[\psi X,\psi Y]+B\psi^2[X,Y]-\tilde{\psi}B[\psi X,Y]$$
$$-\tilde{\psi}B[X,\psi Y]$$

$$=B\{[\psi X,\psi Y]+\psi^2[X,Y]-\psi[\psi X,Y]$$
$$-\psi[X,\psi Y]\}$$
$$=BN(X,Y)$$

3. Distribution $\tilde{M}$ Never Being Tangential to $f(V)$

Theorem (3.1) if the distribution $\tilde{M}$ is never tangential to $f(V)$, then

(3.1) $\tilde{m}(BX)=0$ for all $X\in\zeta_0^1(V)$

and the induced structure ψ on V^m satisfies

(3.2) $\psi^{p-1}=-I$

Proof : if possible $\tilde{m}(BX)\neq 0$. From (2.1) We get

(3.3) $\tilde{\psi}^{p-1}BX=B\psi^{p-1}X$; from (1.7) and (3.3)

$$\tilde{m}(BX)=(I+\tilde{\psi}^{p-1})BX$$
$$=BX+B\psi^{p-1}X$$

(3.4) $\tilde{m}(BX)=B(X+\psi^{p-1}X)$

This relation shows that $\tilde{m}(BX)$ is tangential to $f(V)$ which contradicts the hypothesis. Thus $\tilde{m}(BX)$= 0. Using this result in (3.4) and remembering that B is an isomorphism, We get

(3.5) $\psi^{p-1}=-I$, which gives that $\psi^{(p-1)/2}$ acts as an almost complex structure on V^m. Thus V^m is even dimensional.

Theorem (3.2) Let $\tilde{M}$ be never tangential to $f(V)$, then

(3.6) $\underset{\tilde{m}}{\tilde{N}}(BX,BY)=0$

Proof : We have

(3.7)

$$\underset{\tilde{m}}{\tilde{N}}(BX,BY)=[\tilde{m}BX,\tilde{m}BY]+\tilde{m}^2[BX,BY]$$
$$-\tilde{m}[\tilde{m}BX,BY]-\tilde{m}[BX,\tilde{m}BY]$$

Using (1.2), (1.8) (c) and (3.1), we get (3.6).

Theorem (3.3) Let $\tilde{M}$ be never tangential to $f(V)$, then

(3.8) $\underset{\tilde{l}}{\tilde{N}}(BX,BY)=0$

Proof: We have

(3.9)

$$\underset{\tilde{l}}{\tilde{N}}\left(BX,BY\right)=\left[\tilde{l}\ BX,\tilde{l}\ BY\right]+\tilde{l}^{2}\left[BX,BY\right]$$

$$-\tilde{l}\left[\tilde{l}\ BX,BY\right]-\tilde{l}\left[BX,\tilde{l}\ BY\right]$$

Using (1.2), (1.8) (a), (b) and (3.1) in (3.9); we get (3.8)

Theoren (3.4) Let $\tilde{M}$ be never tangential to $f\left(V\right)$. Define

(3.10) $\tilde{H}\left(\tilde{X},\tilde{Y}\right)=\tilde{N}\left(\tilde{X},\tilde{Y}\right)-\tilde{N}\left(\tilde{m}\tilde{X},\tilde{Y}\right)$

$-\tilde{N}\left(\tilde{X},\tilde{m}\tilde{Y}\right)+\tilde{N}\left(\tilde{m}\tilde{X},\tilde{m}\tilde{Y}\right)$

For all $\tilde{X},\tilde{Y}\in\zeta_{0}^{1}\left(M\right)$, then

(3.11) $\tilde{H}\left(\tilde{X},\tilde{Y}\right)=BN\left(X,Y\right)$

Proof : Using $\tilde{X}=BX,\ \tilde{Y}=BY$ and (2.2), (3.1) in (3.10) We get (3.11).

4. Distribution $\tilde{M}$ Always Being Tangential to $f\left(V\right)$

Theorem (4.1) Let $\tilde{M}$ be always tangential to $f\left(V\right)$, then

(4.1) (a) $\tilde{m}\left(BX\right)=Bm\ X$ (b) $\tilde{l}\left(BX\right)=Bl\ X$

Proof : from (3.4), We get (4.1) (a). Also

(4.2) $l=-\psi^{p-1}$

$lX=-\psi^{p-1}\ X$

(4.3) $BlX=-B\,\psi^{p-1}\ X$

Using (2.1) in (4.3)

(4.4) $BlX=-\ \tilde{\psi}^{p-1}\ BX=\tilde{l}\left(BX\right)$, which is (4.1) (b).

Theorem (4.2) Let $\tilde{M}$ be always tangential to $f\left(V\right)$, then *l* and *m* satisfy

(4.5) (a) $l+m=I$ (b) $lm=ml=0$ (c) $l^2=l$ (d) $m^2=m$.

Proof : Using (1.8) and (4.1) We get the results.

Theorem (4.3) If $\tilde{M}$ is always tangential to $f\left(V\right)$, then

(4.6) $\psi^{p}+\ \psi=0$

Proof: From (2.1)

(4.7) $\tilde{\psi}^{p}\ BX=B\,\psi^{p}\ X$ Using (1.6) in (4.7)

$-\tilde{\psi}\ BX=B\,\psi^{p}\ X-B\psi X=B\,\psi^{p}\ X$

Or $\psi^{p}+\psi=0$ which is (4.6)

Theorem (4.4) : If $\tilde{M}$ Is always tangential to $f\left(V\right)$ then as in (3.10)

(4.8) $\tilde{H}\left(BX,BY\right)=BH\left(X,Y\right)$

Proof: from (3.10) we get

(4.9)

$$\tilde{H}\left(BX,BY\right)=\tilde{N}\left(BX,BY\right)-\tilde{N}\left(\tilde{m}BX,BY\right)$$

$-\tilde{N}\left(BX,\tilde{m}BY\right)+\tilde{N}\left(\tilde{m}BX,\tilde{m}BY\right)$ Using (4.1) (a) and (2.2) in (4.9) we get (4.8).

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