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Invariant Submanifold of $\tilde{\psi}(p,1)$ Structure Manifold

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Abstract: ***In this paper, we have studied various properties of a $\tilde{\psi}(p,1)$ structure manifold and its invariant submanifold, where p is odd prime. Under two different assumptions, the nature of induced structure ψ, has also been discussed.***

Keywords: Invariant submanifold, Nijenhuis tensor, projection operators and complementary distributions

1. Introduction

Let V^m be a C^∞ m-dimensional Riemannian manifold imbedded in a C^∞ n-dimensional Riemannian manifold M^n, where $m<n$. The imbedding being denoted by

$$f : V^m \longrightarrow M^n$$

Let B be the mapping induced by *f* i.e. $B=df$

$$df : T(V) \longrightarrow T(M)$$

Let $T(V,M)$ be the set of all vectors tangent to the submanifold $f(V)$. It is well known that

$$B : T(V) \longrightarrow T(V,M)$$

Is an isomorphism. The set of all vectors normal to $f(V)$ forms a vector bundle over $f(V)$, which we shall denote by $N(V,M)$. We call $N(V,M)$ the normal bundle of V^m. The vector bundle induced by f from $N(V,M)$ is denoted by $N(V)$. We denote by $C : N(V) \longrightarrow N(V,M)$ the natural isomorphism and by $\eta_s^r(V)$ the space of all C^∞ tensor fields of type (r,s) associated with N (V). Thus $\zeta_0^0(V)=\eta_0^0(V)$ is the space of all C^∞ functions defined on V^m while an element of $\eta_0^1(V)$ is a C^∞ vector field normal to V^m and an element of $\zeta_0^1(V)$ is a C^∞ vector field tangential to V^m.

Let $\bar{X}$ and $\bar{Y}$ be vector fields defined along $f(V)$ and $\tilde{X},\tilde{Y}$ be the local extensions of $\bar{X}$ and $\bar{Y}$ respectively. Then $\left[\tilde{X},\tilde{Y}\right]$ is a vector field tangential to M^n and its restriction $\left[\tilde{X},\tilde{Y}\right]/f(V)$ to $f(V)$ is determined independently of the choice of these local extension $\tilde{X}$ and $\tilde{Y}$. Thus $\left[\bar{X},\bar{Y}\right]$ is defined as

(1.1) $\left[\bar{X},\bar{Y}\right]=\left[\tilde{X},\tilde{Y}\right]/f(V)$

Since B is an isomorphism

(1.2) $\left[BX,BY\right]=B\left[X,Y\right]$ for all $X,Y\in \zeta_0^1(V)$

Let $\bar{G}$ be the Riemannain metric tensor of M^n, we define g and g^* on V^m and $N(V)$ respectively as

(1.3) $g(X_1,X_2)=\tilde{G}(BX_1,BX_2)f,$ and

(1.4) $g^*(N_1,N_2)=\tilde{G}(CN_1,CN_2)$

For all $X_1,X_2\in \zeta_0^1(V)$ and $N_1,N_2\in \eta_0^1(V)$

It can be verified that g and g^* are the induced metrics on V^m and $N(V)$ respectively.

Let $\tilde{\nabla}$ be the Riemannian connection determined by $\tilde{G}$ in M^n, then $\tilde{\nabla}$ induces a connection ∇ in $f(V)$ defined by

(1.5) $\nabla_{\bar{X}}\bar{Y}=\tilde{\nabla}_{\tilde{X}}\tilde{Y}/f(V)$

where $\bar{X}$ and $\bar{Y}$ are arbitrary C^∞ vector fields defined along $f(V)$ and tangential to $f(V)$.

Let us suppose that M^n is a $C^\infty\ \tilde{\psi}(p,1)$ structure manifold with structure tensor $\tilde{\psi}$ of type (1,1) satisfying

(1.6) $\tilde{\psi}^p+\tilde{\psi}=0$

Let $\tilde{L}$ and $\tilde{M}$ be the complementary distributions corresponding to the projection operators

(1.7) $\tilde{l}=-\tilde{\psi}^{p-1},\qquad \tilde{m}=I+\tilde{\psi}^{p-1}$

where I denotes the identity operator.

From (1.8) and (1.7), we have

(1.8) **(a)** $\tilde{l}+\tilde{m}=I$ (b) $\tilde{l}^2=\tilde{l}$

(c) $\tilde{m}^2=\tilde{m}$

(d) $\tilde{l}\,\tilde{m}=\tilde{m}\,\tilde{l}=0$

Let D_l and D_m be the subspaces inherited by complementary projection operators l and m respectively.

We define

$$D_l=\{X\in T_p(V):\ lX=X,\,mX=0\}$$

$$D_m=\{X\in T_p(V):\ mX=X,\,lX=0\}$$

Thus $T_p(V)=D_l+D_m$

Also $Ker\,l=\{X:lX=0\}=D_m$

$Ker\,m=\{X:mX=0\}=D_l$

at each point p of $f(V)$.

2. Invariant Submanifold of $\tilde{\psi}(p,1)$ Structure Manifold

We call V^m to be invariant submanifold of M^n if the tangent space $T^p(f(V))$ of $f(V)$ is invariant by the linear mapping $\tilde{\psi}$ at each point p of $f(V)$. Thus

(2.1) $\tilde{\psi}BX=B\psi X$, for all $X\in\zeta_0^1(V)$,, and ψ being a (1,1) tensor field in V^m.

Theorem (2.1): Let $\tilde{N}$ and N be the Nijenhuis tensors determined by $\tilde{\psi}$ and ψ in M^n and V^m respectively, then

(2.2) $\tilde{N}(BX,BY)=BN(X,Y)$, for all $X,Y\in\zeta_0^1(V)$

Proof: We have, by using (1.2) and (2.1)

(2.3) $\tilde{N}(BX,BY)=[\tilde{\psi}BX,\tilde{\psi}BY]$

$+\tilde{\psi}^2[BX,BY]-\tilde{\psi}[\tilde{\psi}BX,BY]$

$-\tilde{\psi}[BX,\tilde{\psi}BY]$

$=[B\psi X,B\psi Y]+\tilde{\psi}^2B[X,Y]$

$-\tilde{\psi}[B\psi X,BY]-\tilde{\psi}[BX,B\psi Y]$

$=B[\psi X,\psi Y]+B\psi^2[X,Y]-\tilde{\psi}B[\psi X,Y]$

$-\tilde{\psi}B[X,\psi Y]$

$=B\{[\psi X,\psi Y]+\psi^2[X,Y]-\psi[\psi X,Y]$

$-\psi[X,\psi Y]\}$

$=B\,N(X,Y)$

3. Distribution $\tilde{M}$ Never Being Tangential to $f(V)$

Theorem (3.1) if the distribution $\tilde{M}$ is never tangential to $f(V)$, then

(3.1) $\tilde{m}(BX)=0$ for all $X\in\zeta_0^1(V)$

and the induced structure ψ on V^m satisfies

(3.2) $\psi^{p-1}=-I$

Proof : if possible $\tilde{m}(BX)\neq 0$. From (2.1) We get

(3.3) $\tilde{\psi}^{p-1}BX=B\psi^{p-1}X$; from (1.7) and (3.3)

$\tilde{m}(BX)=(I+\tilde{\psi}^{p-1})BX$

$=BX+B\psi^{p-1}X$

(3.4) $\tilde{m}(BX)=B(X+\psi^{p-1}X)$

This relation shows that $\tilde{m}(BX)$ is tangential to $f(V)$ which contradicts the hypothesis. Thus $\tilde{m}(BX)=0$. Using this result in (3.4) and remembering that B is an isomorphism, We get

(3.5) $\psi^{p-1}=-I$, which gives that $\psi^{(p-1)/2}$ acts as an almost complex structure on V^m. Thus V^m is even dimensional.

Theorem (3.2) Let $\tilde{M}$ be never tangential to $f(V)$, then

(3.6) $\underset{\tilde{m}}{\tilde{N}}(BX,BY)=0$

Proof : We have

(3.7)

$\underset{\tilde{m}}{\tilde{N}}(BX,BY)=[\tilde{m}BX,\tilde{m}BY]+\tilde{m}^2[BX,BY]$

$-\tilde{m}[\tilde{m}BX,BY]-\tilde{m}[BX,\tilde{m}BY]$

Using (1.2), (1.8) (c) and (3.1), we get (3.6).

Theorem (3.3) Let $\tilde{M}$ be never tangential to $f(V)$, then

(3.8) $\underset{\tilde{l}}{\tilde{N}}(BX,BY)=0$

Proof: We have

(3.9)

$$\underset{\tilde{l}}{\tilde{N}}(BX, BY) = [\tilde{l}\, BX, \tilde{l}\, BY] + \tilde{l}^2 [BX, BY]$$

$$-\tilde{l}[\tilde{l}\, BX, BY] - \tilde{l}[BX, \tilde{l}\, BY]$$

Using (1.2), (1.8) (a), (b) and (3.1) in (3.9); we get (3.8)

Theoren (3.4) Let $\tilde{M}$ be never tangential to $f(V)$. Define

(3.10) $\tilde{H}(\tilde{X}, \tilde{Y}) = \tilde{N}(\tilde{X}, \tilde{Y}) - \tilde{N}(\tilde{m}\tilde{X}, \tilde{Y})$

$-\tilde{N}(\tilde{X}, \tilde{m}\tilde{Y}) + \tilde{N}(\tilde{m}\tilde{X}, \tilde{m}\tilde{Y})$

For all $\tilde{X}, \tilde{Y} \in \zeta_0^1(M)$, then

(3.11) $\tilde{H}(\tilde{X}, \tilde{Y}) = BN(X, Y)$

Proof : Using $\tilde{X} = BX$, $\tilde{Y} = BY$ and (2.2), (3.1) in (3.10) We get (3.11).

4. Distribution $\tilde{M}$ Always Being Tangential to $f(V)$

Theorem (4.1) Let $\tilde{M}$ be always tangential to $f(V)$, then

(4.1) (a) $\tilde{m}(BX) = Bm\, X$ (b) $\tilde{l}(BX) = Bl\, X$

Proof : from (3.4), We get (4.1) (a). Also

(4.2) $l = -\psi^{p-1}$

$lX = -\psi^{p-1} X$

(4.3) $BlX = -B\psi^{p-1} X$

Using (2.1) in (4.3)

(4.4) $BlX = -\tilde{\psi}^{p-1} BX = \tilde{l}(BX)$, which is (4.1) (b).

Theorem (4.2) Let $\tilde{M}$ be always tangential to $f(V)$, then *l* and *m* satisfy

(4.5) (a) $l + m = I$ (b) $lm = ml = 0$ (c) $l^2 = l$ (d) $m^2 = m$.

Proof : Using (1.8) and (4.1) We get the results.

Theorem (4.3) If $\tilde{M}$ is always tangential to $f(V)$, then

(4.6) $\psi^p + \psi = 0$

Proof: From (2.1)

(4.7) $\tilde{\psi}^p BX = B\psi^p X$ Using (1.6) in (4.7)

$-\tilde{\psi} BX = B\psi^p X - B\psi X = B\psi^p X$

Or $\psi^p + \psi = 0$ which is (4.6)

Theorem (4.4) : If $\tilde{M}$ Is always tangential to $f(V)$ then as in (3.10)

(4.8) $\tilde{H}(BX, BY) = BH(X, Y)$

Proof: from (3.10) we get

(4.9)

$$\tilde{H}(BX, BY) = \tilde{N}(BX, BY) - \tilde{N}(\tilde{m}BX, BY)$$

$-\tilde{N}(BX, \tilde{m}BY) + \tilde{N}(\tilde{m}BX, \tilde{m}BY)$ Using (4.1) (a) and (2.2) in (4.9) we get (4.8).

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