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Coincidence and Common Fixed Point Theorems in D-Metric Spaces

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Abstract: ***In this paper we used the concept of compatible mappings of type (P) in D-metric space. Our result generalize the result of Parsai V. and Singh B., Fisher and Pathak.***

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1. Introduction

In 1992, a new structure of a generalized metric space was introduced by Dhage on the line of ordinary metric space defined as under:

Let R denoted the real line and X denoted a nonempty set. Let D : XxXxX → R be a function satisfying properties:
(D_1) $D(x, y, z) \geq 0$ for all x, y, z $\in$ X, equality holds if and only if $x = y = z$.
(D_2) $D(x, y, z) = D(x, z, y) = \ldots\ldots\ldots\ldots \; \forall\, x, y, z \in X$,
(D_3) $D(x, y, z) \leq D(x, y, u) + D(x, u, z) + D(u, y, z) \; \forall\, x, y, z, u \in X$,

The function D is called a D-metric for the space X and (X, D) denotes a D-metric space. Generally the usual ordinary metric is called the distance function. D-metric is called diameter function of the points of X (Daghe)

In the last three decades, a number of authors have studied the aspects of fixed point theory in the setting of D-metric spaces. They have been motivated by various concepts already known for metric space and have thus introduced analogous of various concepts in the framework of the D-metric spaces. Khan, Murthy-Chang-Cho-Sharma and Naidu-Prasad introduced the concepts of weakly commuting pairs of self mappings, compatible pairs of self mapping of type (A) in a D-metric space and notion of weak continuity of a D-metric, respectively, and they have proved several common fixed point theorems by using the weakly commuting pairs of self-mappings, compatible pairs of self-mappings of type (A) in a D-metric space and the weak continuity of a D-metric.

In this paper , we use the concept of compatible mappings of type (P) and compare these mappings with compatible mappings and compatible mappings of type (A) in D-metric spaces. In the sequel, we drive some relations between these mappings. Also, we prove a coincidence point a common fixed point theorem for compatible mappings of type (P) in D-metric spaces.

DEFINITIONS [1]: A sequence $\{x_n\}$ in a D-metric space (X, D) is said to be convergent to a point $x \in X$, denoted by $\lim_{n\to\infty} x_n = x$, if $\lim_{n\to\infty} D(x_n,x,z) = 0$ for all $z \in X$. The point x is said to be limit of sequence $\{x_n\}$ in X.

DEFINITION [2]: A sequence $\{x_n\}$ in a D-metric space (X,D) is called a Cauchy sequence if $D(x_m,x_n,z) \to 0$ as n, m $\to \infty$ for all $z \in X$.

DEFINITION [3]: A D-metric space in which every Cauchy sequence is convergent is called complete.

REMARK [1]: In a D-metric space (X, D) a convergent sequence need not be a Cauchy sequence, but every convergent sequence is a Cauchy sequence when the D-metric D is continuous on X.

DEFINITION [4]: Let S and T be mappings from a D-metric space (X,D) into itself. The mappings S and T are said to be compatible if $\lim_{n\to\infty} D(STx_n, TSx_n,z) = 0$ for all $z \in X$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$.

DEFINITION [5]: Let S and T be mappings from a D-metric space (X,D) into itself. The mappings S and T are said to be compatible of type (A) if $\lim_{n\to\infty} D(STx_n, TTx_n,z) = 0$ and $\lim_{n\to\infty} D(STx_n, SSx_n,z) = 0$ for all $z \in X$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$.

DEFINITION [6]: Let S and T be mappings from a D-metric space (X,D) into itself. The mappings S and T are said to be compatible of type (P) if $\lim_{n\to\infty} D(SSx_n, TTx_n,z) = 0$ for all $z \in X$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$.

The following propositions show that Definition [3.5] & [3.6] are equivalent under some conditions:

PROPOSITION [1]: Let S and T be compatible mappings of type(P) from a D-metric space (X, D) into itself. If St = Tt for some t in X, Then STt = SSt = TTt = TSt.

PROOF : Suppose that $\{x_n\}$ is a sequence in X defined by $x_n = t$, n = 1,2,3,... and St = Tt . Then we have $\lim_{n\to\infty}$ Sxn = $\lim_{n\to\infty}$ Txn = St. Since S and T are compatible mappings of type (P), we have
$D(SSt, TTt, z) = \lim_{n\to\infty} D(SSx_n, TTx_n, z) = 0$.

Hence we have SSt = TTt. Therefore, STt = SSt = TTt = TSt.

Let R^+ denote the set of all non-negative real numbers and F be the family of mappings $\phi : (R^+)^5 \to R^+$ such that each ϕ is upper-semi-continuous, non-decreasing in each coordinate variable, and for any $t > 0$, $\gamma(t) = \phi(t,t,a_1t,a_2t,t) < t$, where $\gamma : R^+ \to R^+$ is a mapping with $\gamma(0) = 0$ and $a_1 + a_2 = 3$.

We have prove the following theorems:

THEOREM [1.1]: Let A, B, S and T be mappings from a complete D-metric space (X, D) into itself , satisfying the following conditions:

[1.1] $A(X) \subset T(X)$ and $B(X) \subset S(X)$,
[1.2] $S(X) \cap T(X)$ is a complete subspace of X.
[1.3] $[1+p\{D(Ax,Sx,z) + D(By,Ty,z)\}]\, D(Ax,By,z)$
$\leq p[D^2(Ax,Sx,z) + D^2(By,Ty,z)] + \phi(D(Sx,Ty,z), D(Ax,Sx,z), D(By,Ty,z), D(Ax,Ty,z), D(By,Sx,z))$
for all $x,y,z \in X$, where $\phi \in F$. Then the pairs A, S and B, T have a coincidence point in X.
For our theorems, we need the following LEMMAS:

LEMMA [1]: For every $t > 0$, $\gamma(t) < t$ if and only if $\lim_{n\to\infty} \gamma^n(t) = 0$, where γ^n denotes the n-times composition of γ.

LEMMA [2] : Let A, B, S and T be mappings from a complete D-metric space (X, D) into itself , satisfying the conditions [3.1.1], [3.4.3]. Then we have the following :
(a) For every $n \in N0$, $D(y_n,y_{n+1},y_{n+2}) = 0$,
(b) For every $i, j, k \in N0$, $D(y_i, y_j, y_k) = 0$, where $\{y_n\}$ is the sequence in X defined by [1.4].

PROOF OF THE LEMMA: (a) By(3.1.1) since $A(X) \subset T(X)$, for any arbitrary point $x_0 \in X$, there exists a point $x_1 \in X$ such that $Ax_0 = Tx_1$. Since $B(X) \subset S(X)$, for any arbitrary point $x_1 \in X$, there exists a point $x_2 \in X$ such that $Bx_1 = Sx_2$ and so on. Inductively, we can define a sequence $\{y_n\}$ in X such that
[1.4] $y_{2n} = Tx_{2n+1} = Ax_{2n}$ and $y_{2n+1} = Sx_{2n+2} = Bx_{2n+1}$ for n = 0,1,2, …
In [1.3], taking $x = x_{2n+2}$, $y = x_{2n+1}$, $z = x_{2n}$ we have,
$[1+p\{D(Ax_{2n+2},Sx_{2n+2},y_{2n}) + D(Bx_{2n+1},Tx_{2n+1},y_{2n})\}]\, D(Ax_{2n+2},Bx_{2n+1},y_{2n})$
$\leq p[D^2(Ax_{2n+2},Sx_{2n+2},y_{2n}) + D^2(Bx_{2n+1},Tx_{2n+1},y_{2n})]$
$+ \phi(D(Sx_{2n+2},Tx_{2n+1},y_{2n}), D(Ax_{2n+2},Sx_{2n+2},y_{2n}), D(Bx_{2n+1},Tx_{2n+1},y_{2n}), D(Ax_{2n+2},Tx_{2n+1},y_{2n}), D(Bx_{2n+1},Sx_{2n+2},y_{2n}))$
$[1+p\{D(y_{2n+2},y_{2n+1},y_{2n}) + D(y_{2n+1},y_{2n},y_{2n})\}]\, D(y_{2n+2},y_{2n+1},y_{2n})$
$\leq p[D^2(y_{2n+2},y_{2n+1},y_{2n}) + D^2(y_{2n+1},y_{2n},y_{2n})]$
$+ \phi(D(y_{2n+1},y_{2n},y_{2n}), D(y_{2n+2},y_{2n+1},y_{2n}), D(y_{2n+1},y_{2n},y_{2n}), D(y_{2n+2},y_{2n},y_{2n}), D(y_{2n+1},y_{2n+1},y_{2n}))$
$[1+p\{D(y_{2n+2},y_{2n+1},y_{2n}) + 0\}]\, D(y_{2n+2},y_{2n+1},y_{2n})$
$\leq p[D^2(y_{2n+2},y_{2n+1},y_{2n}) + 0] + \phi(0, D(y_{2n+2},y_{2n+1},y_{2n}), 0, 0, 0)$
$D(y_{2n+2},y_{2n+1},y_{2n}) \leq \phi(0, D(y_{2n+2},y_{2n+1},y_{2n}), 0, 0, 0)$
$< D(y_{2n+2},y_{2n+1},y_{2n})$.
which is a contradiction. Thus we have $D(y_{2n+2},y_{2n+1},y_{2n}) = 0$, similarly , we have $D(y_{2n+1},y_{2n},y_{2n-1}) = 0$.
Hence , for n = 0,1,2 …., we have [1.4] $D(y_{n+2},y_{n+1},y_n) = 0$.
(b) For all $z \in X$, let $d_n(z) = D(y_n, y_{n+1},z)$ for n = 0,1,2,….. .
By (a), we have
$D(y_n, y_{n+2},z) \leq D(y_n, y_{n+2}, y_{n+1}) + D(y_n, y_{n+1},z) + D(y_{n+1}, y_{n+2},z)$
$D(y_n, y_{n+2},z) \leq D(y_n, y_{n+1},z) + D(y_{n+1}, y_{n+2},z)$
$D(y_n, y_{n+2},z) \leq d_n(z) + d_{n+1}(z)$
Taking $x = x_{2n+2}$ and $y = x_{2n+1}$ in [3.1.3], we have
$[1+p\{D(Ax_{2n+2},Sx_{2n+2},z) + D(Bx_{2n+1},Tx_{2n+1},z)\}]\, D(Ax_{2n+2},Bx_{2n+1},z) \leq p[D^2(Ax_{2n+2},Sx_{2n+2},z) + D^2(Bx_{2n+1},Tx_{2n+1},z)] + \phi(D(Sx_{2n+2},Tx_{2n+1},z), D(Ax_{2n+2},Sx_{2n+2},z), D(Bx_{2n+1},Tx_{2n+1},z), D(Ax_{2n+2},Tx_{2n+1},z), D(Bx_{2n+1},Sx_{2n+2},z))$
$[1+p\{D(y_{2n+2},y_{2n+1},z) + D(y_{2n+1},y_{2n},z)\}]\, D(y_{2n+2},y_{2n+1},z)$
$\leq p[D^2(y_{2n+2},y_{2n+1},z) + D^2(y_{2n+1},y_{2n},z)] + \phi(D(y_{2n+1},y_{2n},z), D(y_{2n+2},y_{2n+1},z), D(y_{2n+1},y_{2n},z), D(y_{2n+2},y_{2n},z), D(y_{2n+1},y_{2n+1},z))$
[1.5] $[1+p\{d_{2n+1}(z) + d_{2n}(z)\}]\, d_{2n+1}(z)$
$\leq p[D^2{}_{2n+1}(z) + D^2{}_{2n}(z)] + \phi(d_{2n}(z),d_{2n+1}(z), d_{2n}(z), \{d_{2n}(z)+d_{2n+1}(z)\}, 0)$
Now, we shall show that $\{ d_n(z)\}$ is a non increasing sequence in R^+ . In fact, let $d_{n+1}(z) > d_n(z)$ for some n.
By [1.5] we have, $d_{2n+1}(z) < d_{2n+1}(z)$, which is a contradiction in R^+.
Now, we claim that $d_n(y_m) = 0$ for all non negative integers m, n.
Case 1. $n \geq m$. Then we have $0 = d_m(y_m) \geq d_n(y_m)$.
Case 2. $n < m$. By (M_4), we have
$d_n(y_m) \leq d_n(y_{m-1}) + d_{m-1}(y_n) \leq d_n(y_{m-1}) + d_n(y_n) = d_n(y_{m-1})$
By using the above inequality repeatedly, we have
$d_n(y_m) \leq d_n(y_{m-1}) \leq d_n(y_{m-2}) \leq \ldots\ldots \leq d_n(y_n) = 0$,
which completes the proof of our claim.
Finally, let i, j, and k be arbitrary non-negative integers. We may assume that $i < j$. By (M_4), we have
$D(y_i,y_j,y_k) \leq d_i(y_j) + d_i(y_k) + D(y_{i+1}, y_j, y_k) = D(y_{i+1},y_j, y_k)$.
Therefore, by repeatition of the above inequality, we have
$D(y_i,y_j,y_k) \leq D(y_{i+1},y_j, y_k) \leq \ldots\ldots \leq D(y_i,y_j, y_k) = 0$.
This completes the proof.

LEMMA [3]: Let A, B, S and T be mappings from a D-metric space (X, D) into itself satisfying the following conditions [1.1] and [1.3]. Then the sequence $\{y_n\}$ defined by [1.4] is a Cauchy sequence in X.

PROOF OF THE LEMMA : In the proof of LEMMA [2], since $d_n(z)$ is a non increasing sequence in R^+ , by [1.3], we have ,
$[1+p\{D(Ax_2,Sx_2,z) + D(Bx_1,Tx_1,z)\}]\, D(Ax_2,Bx_1,z)$
$\leq p[d^2(Ax_2,Sx_2,z) + d^2(Bx_1,Tx_1,z)] + \phi(D(Sx_2,Tx_1,z), D(Ax_2,Sx_2,z),D(Bx_1,Tx_1,z), D(Ax_2,Tx_1,z), D(Bx_1,Sx_2,z))$
$[1+p\{D(y_2,y_1,z) + D(y_1,y_0,z)\}]\, D(y_2,y_1,z)$
$\leq p[d^2(y_2,y_1,z) + d^2(y_1,y_0,z)] + \phi(D(y_1,y_0,z), D(y_2,y_1,z), D(y_1,y_0,z), D(y_2,y_0,z),D(y_1,y_1,z))$
$[1+p\{d_1(z) + d_0(z)\}]\, d_1(z) \leq p[d^2{}_1(z) + d^2{}_0(z)] + \phi(d_0(z), d_1(z), d_0(z), \{d_0(z)+d_1(z)\}, 0)$
$d_1(z) \leq \phi(d_0(z), d_0(z), d_0(z), \{d_0(z)+d_0(z)\}, 0)$

$d_1(z) \leq \gamma(d_0(z))$
and $d_2(z) \leq \gamma(d_1(z)) \leq \gamma(\gamma(d_0(z)) = \gamma^2(d_0(z))$.
In general, we have $d_n(z) \leq \gamma^n(d_0(z))$.
Thus, if $d_0(z) > 0$, by LEMMA [3.1] $\lim_{n\to\infty} d_n(z) = 0$. If $d_0(z) = 0$, we have clearly $\lim_{n\to\infty} d_n(z) = 0$ since $d_n(z) = 0$ for n = 1,2, …

Now, we shall prove that $\{ y_n\}$ is a Cauchy sequence in X. Since $\lim_{n\to\infty} d_n(z) = 0$, it is sufficient to show that a subsequence $\{ y_{2n}\}$ of $\{ y_n \}$ is a Cauchy sequence in X. Suppose that the sequence $\{ y_{2n}\}$ is not a Cauchy sequence in X. Then there exist a point $z \in X$, an $\varepsilon > 0$ and strictly increasing sequences $\{m(k)\}$, $\{n(k)\}$ of positive integers such that $k \leq n(k) < m(k)$,
[1.6] $(y_{2n(k)},y_{2m(k)},z) \geq \varepsilon$ and $D(y_{2n(k)},y_{(2m-2)(k)}, z) < \varepsilon$
for all k = 1,2,….. By LEMMA[3.2] anD(M_4), we have
$D(y_{2n(k)}, y_{2m(k)}, z) - D(y_{2n(k)}, y_{2m(k-2)}, z) \leq D(y_{2m(k-2)},y_{2m(k)}, z) \leq d_{2m(k-2)}(z) + d_{2m(k-1)}(z)$
Since $\{D(y_{2n(k)}, y_{2m(k)}, z) - \varepsilon \}$ and $\{ \varepsilon - D(y_{2n(k)}, y_{2m(k-2)}, z)\}$ are sequences in R^+ and $\lim_{n\to\infty} d_n(z) = 0$, we have
[1.7] $\lim_{k\to\infty} D(y_{2n(k)},y_{2m(k)}, z) = \varepsilon$ and $\lim_{k\to\infty} D(y_{2n(k)}, y_{2mk-2}, z) = \varepsilon$
Note that, by (M_4), we have
[1.8] $| D(x,y,a) - D(x,y,b)| \leq D(a,b,x) + D(a,b,y)$
for all $x, y, a, b \in X$. Taking $x = y_{2n(k)}$, $y = a$, $a = y_{2m(k-1)}$ and $b = y_{2m(k)}$ in [1..8] and using lemma [2] and [1.7], we have
[1.9] $\lim_{k\to\infty} D(y_{2n(k)}, y_{2m(k-1)}, z) = \varepsilon$.
Once again, by using lemma [2], [1..7] and [1.8], we have
[1.10] $\lim_{k\to\infty} D(y_{2n(k)+1}, y_{2m(k)}, z) = \varepsilon$ and $\lim_{k\to\infty} D(y_{2n(k-1)}, y_{2m(k-1)}, z) = \varepsilon$.
Thus, by [1.3], we have,
[1.11]
$[1+p\{D(Ax_{2m(k)},Sx_{2m(k)},z)+D(Bx_{2n(k+1)},Tx_{2n(k+1)},z)\}]D(Ax_{2m(k)},Bx_{2n(k+1)},z)$
$\leq p[d^2(Ax_{2m(k)},Sx_{2m(k)},z) + d^2(Bx_{2n(k+1)},Tx_{2n(k+1)},z)] + \phi(D(Sx_{2m(k)},Tx_{2n(k+1)},z), D(Ax_{2m(k)},Sx_{2m(k)},z), D(Bx_{2n(k+1)},Tx_{2n(k+1)},z),$
$D(Ax_{2m(k)},Tx_{2n(k+1)},z), D(Bx_{2n(k+1)},Sx_{2m(k)},z))$
$[1+p\{D(y_{2m(k)},y_{2m(k-1)},z) + D(y_{2n(k+1)},y_{2n(k)},z)\}] D(y_{2m(k)},y_{2n(k+1)},z)$
$\leq p[d^2(y_{2m(k)},y_{2m(k-1)},z) + d^2(y_{2n(k+1)},y_{2n(k)},z)] + \phi(D(y_{2m(k-1)},y_{2n(k)},z),$
$D(y_{2m(k)},y_{2m(k-1)},z), D(y_{2n(k+1)},y_{2n(k)},z),D(y_{2m(k)},y_{2n(k)},z),$
$D(y_{2n(k+1)},y_{2m(k-1)},z))$
As $k \to\infty$ in [1.11] and noting that d is continuous, we have
$\varepsilon \leq \phi(\varepsilon, 0, 0, \varepsilon, \varepsilon) < \gamma(\varepsilon) < \varepsilon$
which is a contradiction. Therefore, $\{ y_{2n}\}$ is a Cauchy sequence in X and so the sequence $\{ y_n\}$ is a Cauchy sequence in X. This completes the proof.

PROOF OF THE THEOREM : By lemma[3], the sequence $\{ y_n\}$ defined by [1.2] is a Cauchy sequence in $S(X) \cap T(X)$. Since $S(X) \cap T(X)$ is a complete subspace of X, $\{y_n\}$ converges to a point w in $S(X) \cap T(X)$. On the other hand, since the subsequences $\{ y_{2n}\}$ and $\{ y_{2n+1}\}$ of $\{y_n\}$ are also Cauchy sequences in $S(X) \cap T(X)$, they also converge to the same limit w. Hence there exist two points u, v in X such that Su = w and Tv = w, respectively.
By [1.3], we have
$[1+p\{D(Au,Su,z) + D(Bx_{2n+1},Tx_{2n+1},z)\}] D(Au,Bx_{2n+1},z)$
$\leq p[d^2(Au,Su,z) + d^2(Bx_{2n+1},Tx_{2n+1},z)] + \phi(D(Su,Tx_{2n+1},z), D(Au,Su,z),$
$D(Bx_{2n+1},Tx_{2n+1},z),D(Au,Tx_{2n+1},z),D(Bx_{2n+1},Su,z))$
$[1+p\{D(Au,Su,z) + D(y_{2n+1},y_{2n},z)\}] D(Au,y_{2n+1},z)$
$\leq p[d^2(Au,Su,z) + d^2(y_{2n+1},y_{2n},z)] + \phi(D(Su,y_{2n},z), D(Au,Su,z), D(y_{2n+1},y_{2n},z),$
$D(Au,y_{2n},z),D(y_{2n+1},Su,z))$
Since $\lim_{n\to\infty} d_n(z) = 0$ in the proof of Lemma2, letting $n\to\infty$, we have
$[1+p\{D(Au, w,z) + D(w, w,z)\}] D(Au, w,z)$
$\leq p[d^2(Au, w,z) + d^2(w, w, z)] + \phi(D(w, w,z), D(Au, w,z),D(w ,w,z),$
$D(Au, w,z), D(w, w,z))$
$D(Au, w,z) \leq \phi(0, D(Au, w,z), 0,D(Au, w,z),0) < \gamma (D(Au, w,z)) < D(Au, w,z)$
which is contradiction . Hence Au = w = Sw, that is u is a coincidence of A and S.
Similarly, we can show that v is a coincidence point of B and T.

THEOREM [2] : Let A, B, S and T be mappings from a D-metric spaces (X,D) into itself satisfying the conditions (1.1), (3.1.3), (3.1..10) and the following:
[3.2.1] the pairs A, S and B,T are compatible mappings of type (P).
[3.2.2] the pairs A,S and B, T are sequentially continuous at their coincidence points.
Then A, B, S and T have a unique common fixed point in X.

PROOF OF THEOREM : By theorem [3.1], there exist two points u, v in X such that Au = Su = w and Bv = Tv = w, respectively , since A and S are compatible mappings of type(P), by Proposition 1 ASu = SSu = SAu = AAu, which implies that Aw = Sw, Similarly B and T are compatible mapping of type(P) we have Bw = Tw. Now, we prove that Aw = w. If $Aw \neq w$, then by (4.2), we have
$[1+p\{D(Aw,Sw,z) + D(Bx_{2n+1},Tx_{2n+1},z)\}] D(Aw,Bx_{2n+1},z)$
$\leq p[d^2(Aw,Sw,z) + d^2(Bx_{2n+1},Tx_{2n+1},z)] + \phi(D(Sw,Tx_{2n+1},z),D(Aw,Sw,z),$
$D(Bx_{2n+1},Tx_{2n+1},z), D(Aw,Tx_{2n+1},z),D(Bx_{2n+1},Sw,z))$
$[1+p\{D(Aw,Sw,z) + D(y_{2n+1},y_{2n},z)\}] D(Aw,y_{2n+1},z)$
$\leq p[d^2(Aw,Sw,z)+d^2(y_{2n+1}, y_{2n},z)]+\phi(D(Sw,y_{2n},z), D(Aw,Sw,z),$
$D(y_{2n+1},y_{2n},z), D(Aw,y_{2n},z), D(y_{2n+1},Sw,z))$
Since $\lim_{n\to\infty} d_n(z) = 0$ in the proof of Lemma2, letting $n\to\infty$, we have
$[1+p\{D(Aw, w,z) + D(w, w,z)\}] D(Aw, w,z)$
$\leq p[d^2(Aw, w,z) + d^2(w, w, z)] + \phi(D(w, w,z), D(Aw, w,z), D(w ,w,z),$
$D(Aw, w,z), D(w, w,z))$
$D(Aw, w,z) \leq \phi(0, D(Aw, w,z), 0,D(Aw, w,z),0) < \gamma (D(Aw, w,z))$
$< D(Aw, w,z)$
which is contradiction .
Hence Aw = w = Sw.
Similarly, we have Bw = Tw = w.
This means that w is a common fixed point of A, B, S and T.
The uniqueness of the fixed point w follows from [1.3].
This complete the proof.

Reference

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