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Prime Labeling in Context of Ring Sum of Graphs

A. H. Rokad[1], G. V. Ghodasara[2]

[1]PhD Scholar, School of Science, RK University, Rajkot - 360020, Gujarat, India

[2]H. & H. B. Kotak Institute of Science, Rajkot - 360001, Gujarat – India

Abstract: ***A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers 1, 2, . . . , |V | such that for each edge xy the labels assigned to x and y are relatively prime. In this paper we have derived prime labeling of ringsum of different graphs.***

Keywords: prime labeling, ring Sum, cycle with one chord, cycle with twin chord, cycle with triangle, shell

1. Introduction

Throughout this paper, a graph G = (V,E) is a undirected, finite, connected, and simple graph with vertex set V and edge set E. For different notations and terminology we follow Gross and Yellen[2]. A survey on different graph labeling techniques is given by Gallian[1]. The notion of a prime labeling originated with Entringer and was introduced in a paper by Tout, Dabboucy, and Howalla[3]. Around 1980, Entringer conjectured that all trees have a prime labeling. Little progress was made on this conjecture until 2011 when Haxell, Pikhurko, Taraz[4] proved that all large trees are prime. Seoud, Diab, and Elsakhawi[5] have shown the following graphs are prime: fans, helms, flowers, stars. Vaidya and Prajapati[6] proved that the graphs obtained by duplication of a vertex by a vertex in P_n and $K_{1,n}$ are prime graphs and the graphs obtained by duplication of a vertex by an edge, duplication of an edge by a vertex, duplication of an edge by an edge in P_n, $K_{1,n}$, and Cn are prime graphs. They also proved that graph obtained by duplication of every vertex by an edge in P_n, $K_{1,n}$, and Cn are not prime graphs. Youssef and Elsakhawi[7] have shown: the union of stars $S_m \cup S_n$ are prime; the union of cycles and stars $C_m \cup S_n$ are prime; $K_m \cup P_n$ is prime if and only if m is at most 3 or if m = 4 and n is odd.

2. Definition

Definition 1. Ring sum of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, denoted by $G_1 \oplus G_2$, is the graph, $G_1 \oplus G_2 = ((V_1 \cup V_2), ((E_1 \cup E_2) - (E_1 \cap E_2)))$.

Definition 2. Let G = (V (G), E (G)) be a graph with p vertices.

A bijection $f : V(G) \rightarrow 1, 2, \ldots, p$ is called a prime labeling if for each edge e = uv, gcd(f (u), f (v)) = 1.
A graph which admits prime labeling is called a prime graph.

3. Main Results

Theorem 1. $C_n \oplus K_{1,n}$ is a prime graph for all $n \in N$.

Proof. Let $V(G) = V_1 \cup V_2$, where $V_1 = \{u_1, u_2, \ldots, u_n\}$ be the vertex set of C_n and
$V_2 = \{v = u_1, v_1, v_2, \ldots, v_n\}$ be the vertex set of $K_{1,n}$. Here $v_1, v_2, \ldots, v_n$ are pendent vertices. Also |V (G)| = |E (G)| = 2n.
We define labeling $f: V(G) \rightarrow \{1, 2, 3, \ldots, 2n\}$ as follows.

$f(u_i) = 2i - 1 \; ; \; 1 \leq i \leq n$
$f(v_j) = 2j \; ; \; 1 \leq j \leq n$

According to this pattern the vertices are labeled such that for any edge $e = u_iu_j \in G$, $\gcd(f(u_i), f(u_j)) = 1$.

Thus we proved that the graph under consideration admits prime labeling.

That is, $C_n \oplus K_{1,n}$ is a prime graph.

Example 1. prime labeling of the graph $C_5 \oplus K_{1,5}$ is shown in Fig. 1 as an illustration for the Theorem 1.

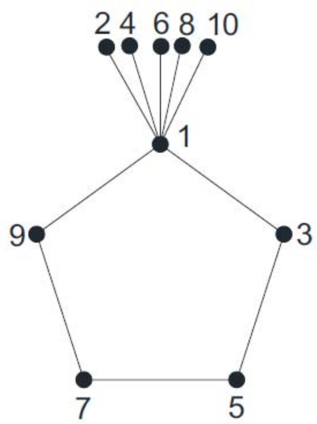

Figure 1: prime labeling of the graph $C_5 \oplus K_{1,5}$

Theorem 2. $G \oplus K_{1,n}$ is a prime graph, where G is cycle with one chord and chord forms a triangle with two edges of the cycle, for all $n \in N$.

Proof. Let G be the cycle C_n with one chord. Let $V = V_1 \cup V_2$, where V_1 is the vertex set of G and V_2 be the vertex set of $K_{1,n}$. And let $u_1, u_2, \ldots, u_n$ be consecutive vertices of cycle C_n and $e = u_1u_3$ be a chord of cycle C_n. The vertices u_1, u_2, u_3 forms a triangle with chord e.

Let $v_1, v_2, \ldots, v_n$ be the pendent vertices, v be the apex

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vertex of $K_{1,n}$ and take $v = u_1$.
Also $|V(G)| = 2n$ and $|E(G)| = 2n+1$
We define labeling $f: V(G) \rightarrow \{1, 2, 3, \ldots, 2n\}$ as follows.

$f(u_1) = 1,\ f(u_2) = 3,\ f(u_n) = 5$
$f(u_i) = 2i + 1\ ;\ 3 \leq i \leq n - 1$
$f(v_j) = 2j\ ;\ 1 \leq j \leq n$

According to this pattern the vertices are labeled such that for any edge $e = u_iu_j \in G$,
$\gcd(f(u_i), f(u_j)) = 1$.

Thus we proved that the graph under consideration admits prime labeling.
That is, $G \oplus K_{1,n}$ is a prime graph, G is the cycle C_n with one chord.

Example 2. prime labeling of ring sum of the graph cycle C_6 with one chord and $K_{1,6}$ is shown in Fig. 2 as an illustration for the Theorem 2.

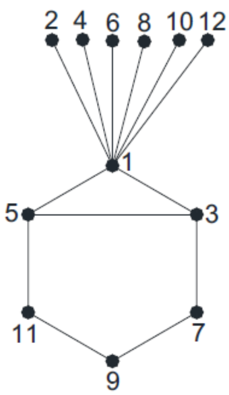

Figure 2: prime labeling of ring sum of the graph cycle C_6 with one chord and $K_{1,6}$

Theorem 3. $G \oplus K_{1,n}$ is a prime graph, where G is cycle with twin chord and chords forms two triangles and one cycle C_{n-2}, for all $n \in N$.

Proof. Let G be the cycle C_n with twin chords, where chords form two triangles and one cycle C_{n-2}. Let $V = V_1 \cup V_2$, where V_1 is the vertex set of G and V_2 be the vertex set of $K_{1,n}$. And let $u_1, u_2, \ldots, u_n$ be successive vertices of G and $e_1 = u_nu_2$ and $e_2 = u_nu_3$ be the chords of cycle C_n. Let $v_1, v_2, \ldots, v_n$ be the pendent vertices, v be the apex vertex of $K_{1,n}$ and take $v = u_1$.
Also $|V(G)| = 2n$ and $|E(G)| = 2n+2$
We define labeling $f: V(G) \rightarrow \{1, 2, 3, \ldots, 2n\}$ as follows.

$f(u_1) = 1,\ f(u_2) = 3,\ f(u_n) = 5$
$f(u_i) = 2i + 1\ ;\ 3 \leq i \leq n - 1$
$f(v_j) = 2j\ ;\ 1 \leq j \leq n$

According to this pattern the vertices are labeled such that for any edge $e = u_iu_j \in G$,
$\gcd(f(u_i), f(u_j)) = 1$.

Thus we proved that the graph under consideration admits prime labeling.
That is, $G \oplus K_{1,n}$ is a prime graph, where G is the cycle C_n with twin chord .

Example 3: prime labeling of ring sum of the graph the graph cycle C_7 with twin chords and $K_{1,7}$ is shown in Fig. 3 as an illustration for the Theorem 3.

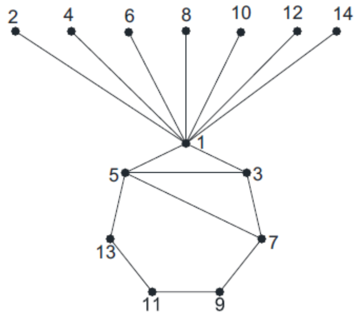

Figure 3: prime labeling of ring sum of the graph cycle C_7 with twin chords and $K_{1,7}$

Theorem 4. $G \oplus K_{1,n}$ is a prime graph, where G is cycle with triangle and chords forms three triangles and one cycle C_{n-5}, for all $n \in N$.

Proof. Let G be cycle with triangle $C_n(1, 1, n - 5)$. Let $V = V_1 \cup V_2$, where V_1 is the vertex set of G and V_2 be the vertex set of $K_{1,n}$. And Let $u_1, u_2, \ldots, u_n$ be successive vertices of G.

Let u_1, u_3 and u_5 be the vertices of triangle formed by edges $e_1 = u_1u_3$, $e_2 = u_3u_5$ and $e_3 = u_1u_5$.

Let $v_1, v_2, \ldots, v_n$ be the pendent vertices, v be the apex vertex of $K_{1,n}$ and take $v = u_1$.
Also $|V(G)| = 2n$ and $|E(G)| = 2n+3$
We define labeling $f: V(G) \rightarrow \{1, 2, 3, \ldots, 2n\}$ as follows.

$f(u_1) = 1,\ f(u_2) = 3,\ f(u_3) = 5,\ f(u_{n-1}) = 7,\ f(u_n) = n$
$f(u_i) = 2i + 1\ ;\ 4 \leq i \leq n - 2$
$f(v_j) = 2j\ ;\ 1 \leq j \leq n$
According to this pattern the vertices are labeled such that for any edge $e = u_iu_j \in G$, $\gcd(f(u_i), f(u_j)) = 1$.

Thus we proved that the graph under consideration admits prime labeling. That is, $G \oplus K_{1,n}$ is a prime graph, where G is the cycle C_n with triangle.

Example 4. prime labeling of ring sum of the graph the graph cycle C_8 with twin chords and $K_{1,8}$ is shown in Fig. 4 as an illustration for the Theorem 4.

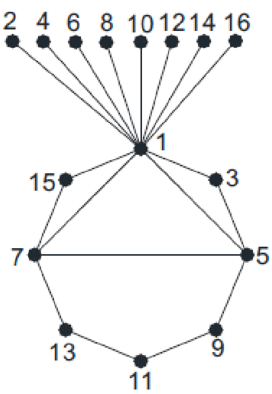

Figure 4: prime labeling of ring sum of the graph cycle C_8 with triangle and $K_{1,8}$

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Theorem 5. $S_n \oplus K_{1,n}$ is prime graph, for all $n \in N$.

Proof. Let $V(G) = V_1 \cup V_2$, where $V_1 = \{u_1, u_2, \ldots, u_n\}$ with apex u_1 and $u_2, u_3, \ldots, u_n$ are other vertices of S_n and $V_2 = \{v = u_1, v_1, v_2, \ldots, v_n\}$ be the vertex set of $K_{1,n}$. Here $v_1, v_2, \ldots, v_n$ are pendent vertices.
We define labeling $f: V(G) \to \{1, 2, 3, \ldots, 2n\}$ as follows.

$f(u_i) = 2i - 1$; $1 \le i \le n$
$f(v_j) = 2j$; $1 \le j \le n$

According to this pattern the vertices are labeled such that for any edge $e = u_iu_j \in G$,
$\gcd(f(u_i), f(u_j)) = 1$.

Thus we proved that the graph under consideration admits prime labeling.
That is, $S_n \oplus K_{1,n}$ is a prime graph.

Example 5. prime labeling of the graph $S_7 \oplus K_{1,7}$ is shown in Fig. 5 as an illustration for the Theorem 5.

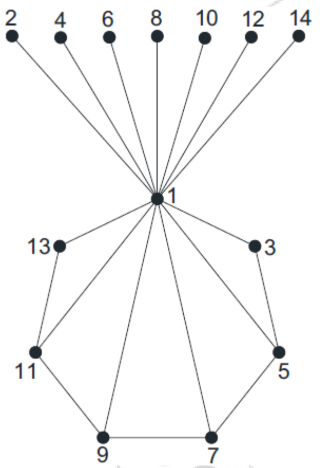

Figure 5: prime labeling of the graph $S_7 \oplus K_{1,7}$

4. Conclusion

We have proved that the graph $C_n \oplus K_{1,n}$, $G \oplus K_{1,n}$, where G is cycle with one chord, $G \oplus K_{1,n}$, where G is cycle with twin chord, $G \oplus K_{1,n}$, where G is cycle with triangle, $S_n \oplus K_{1,n}$ are prime graphs.

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Author Profile

Mr. Amit H. Rokad received M.Sc. degree from M. S. University, Baroda, (Gujarat) in 2009. Currently he is working as an assistant professor in Mathematics at School of Technology, R.K. University. He is doing his Ph.D. under the guidance of Dr. G. V. Ghodasara at R.K. University, Rajkot.

Dr. Gaurang V. Ghodasara received M.Sc. degree from Sardar Patel University, Vallabh Vidyanagar, (Gujarat). He received M. Phil and Ph. D. both degree from Saurashtra University, Rajkot(Gujarat) in 2005 and 2009 respectively. He worked as an assistant professor in Mathematics Department at Atmiya Institute of Technology & Science, Rajkot and L. E. College Morbi. Currently he is working as an assistant professor in Mathematics Department at H. & H. B. Kotak Institute of Science, Rajkot(Gujarat). He is registered guide at R.K. University, Rajkot.

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