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Prime Labeling in Context of Ring Sum of Graphs

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Abstract: ***A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers 1, 2, . . . , |V | such that for each edge xy the labels assigned to x and y are relatively prime. In this paper we have derived prime labeling of ringsum of different graphs.***

Keywords: prime labeling, ring Sum, cycle with one chord, cycle with twin chord, cycle with triangle, shell

1. Introduction

Throughout this paper, a graph G = (V,E) is a undirected, finite, connected, and simple graph with vertex set V and edge set E. For different notations and terminology we follow Gross and Yellen[2]. A survey on different graph labeling techniques is given by Gallian[1]. The notion of a prime labeling originated with Entringer and was introduced in a paper by Tout, Dabboucy, and Howalla[3]. Around 1980, Entringer conjectured that all trees have a prime labeling. Little progress was made on this conjecture until 2011 when Haxell, Pikhurko, Taraz[4] proved that all large trees are prime. Seoud, Diab, and Elsakhawi[5] have shown the following graphs are prime: fans, helms, flowers, stars. Vaidya and Prajapati[6] proved that the graphs obtained by duplication of a vertex by a vertex in P_n and $K_{1,n}$ are prime graphs and the graphs obtained by duplication of a vertex by an edge, duplication of an edge by a vertex, duplication of an edge by an edge in P_n, $K_{1,n}$, and Cn are prime graphs. They also proved that graph obtained by duplication of every vertex by an edge in P_n, $K_{1,n}$, and Cn are not prime graphs. Youssef and Elsakhawi[7] have shown: the union of stars $S_m \cup S_n$ are prime; the union of cycles and stars $C_m \cup S_n$ are prime; $K_m \cup P_n$ is prime if and only if m is at most 3 or if m = 4 and n is odd.

2. Definition

Definition 1. Ring sum of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, denoted by $G_1 \oplus G_2$, is the graph, $G_1 \oplus G_2 = ((V_1 \cup V_2), ((E_1 \cup E_2) - (E_1 \cap E_2)))$.

Definition 2. Let G = (V (G), E (G)) be a graph with p vertices.

A bijection f : V (G) → 1, 2, . . . , p is called a prime labeling if for each edge e = uv, gcd(f (u), f (v)) = 1.
A graph which admits prime labeling is called a prime graph.

3. Main Results

Theorem 1. $C_n \oplus K_{1,n}$ is a prime graph for all $n \in N$.

Proof. Let V (G) = $V_1 \cup V_2$, where $V_1 = \{u_1, u_2, . . . ,u_n\}$ be the vertex set of C_n and
$V_2 = \{v = u_1, v_1, v_2, . . . ,v_n\}$ be the vertex set of $K_{1,n}$. Here $v_1, v_2, . . . ,v_n$ are pendent vertices. Also |V (G)| = |E (G)| = 2n.
We define labeling f:V (G) → {1, 2, 3, . . . ,2n} as follows.

$f(u_i) = 2i - 1 ; 1 \le i \le n$
$f(v_j) = 2j ; 1 \le j \le n$

According to this pattern the vertices are labeled such that for any edge $e = u_iu_j \in G$, $\gcd(f(u_i), f(u_j)) = 1$.

Thus we proved that the graph under consideration admits prime labeling.

That is, $C_n \oplus K_{1,n}$ is a prime graph.

Example 1. prime labeling of the graph $C_5 \oplus K_{1,5}$ is shown in Fig. 1 as an illustration for the Theorem 1.

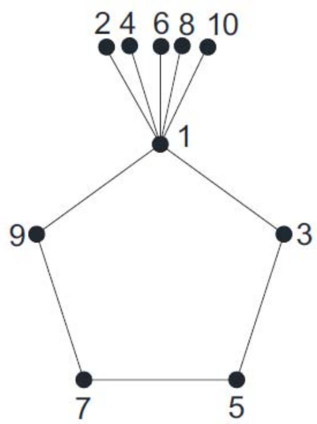

Figure 1: prime labeling of the graph $C_5 \oplus K_{1,5}$

Theorem 2. $G \oplus K_{1,n}$ is a prime graph, where G is cycle with one chord and chord forms a triangle with two edges of the cycle, for all $n \in N$.

Proof. Let G be the cycle C_n with one chord. Let $V = V_1 \cup V_2$, where V_1 is the vertex set of G and V_2 be the vertex set of $K_{1,n}$. And let $u_1, u_2, . . . ,u_n$ be consecutive vertices of cycle C_n and $e = u_1u_3$ be a chord of cycle C_n. The vertices u_1, u_2, u_3 forms a triangle with chord e.

Let $v_1, v_2, . . . ,v_n$ be the pendent vertices, v be the apex

vertex of $K_{1,n}$ and take $v = u_1$.
Also $|V(G)| = 2n$ and $|E(G)| = 2n+1$
We define labeling $f:V(G) \rightarrow \{1, 2, 3, \ldots, 2n\}$ as follows.

$f(u_1) = 1$, $f(u_2) = 3$, $f(u_n) = 5$
$f(u_i) = 2i + 1$; $3 \le i \le n - 1$
$f(v_j) = 2j$; $1 \le j \le n$

According to this pattern the vertices are labeled such that for any edge $e = u_iu_j \in G$,
$\gcd(f(u_i), f(u_j)) = 1$.

Thus we proved that the graph under consideration admits prime labeling.
That is, $G \oplus K_{1,n}$ is a prime graph, G is the cycle C_n with one chord.

Example 2. prime labeling of ring sum of the graph cycle C_6 with one chord and $K_{1,6}$ is shown in Fig. 2 as an illustration for the Theorem 2.

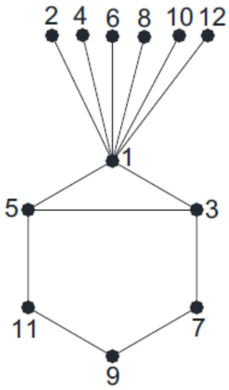

Figure 2: prime labeling of ring sum of the graph cycle C_6 with one chord and $K_{1,6}$

Theorem 3. $G \oplus K_{1,n}$ is a prime graph, where G is cycle with twin chord and chords forms two triangles and one cycle C_{n-2}, for all $n \in N$.

Proof. Let G be the cycle C_n with twin chords, where chords form two triangles and one cycle C_{n-2}. Let $V = V_1 \cup V_2$, where V_1 is the vertex set of G and V_2 be the vertex set of $K_{1,n}$. And let $u_1, u_2, \ldots, u_n$ be successive vertices of G and $e_1 = u_nu_2$ and $e_2 = u_nu_3$ be the chords of cycle C_n. Let $v_1, v_2, \ldots, v_n$ be the pendent vertices, v be the apex vertex of $K_{1,n}$ and take $v = u_1$.
Also $|V(G)| = 2n$ and $|E(G)| = 2n+2$
We define labeling $f:V(G) \rightarrow \{1, 2, 3, \ldots, 2n\}$ as follows.

$f(u_1) = 1$, $f(u_2) = 3$, $f(u_n) = 5$
$f(u_i) = 2i + 1$; $3 \le i \le n - 1$
$f(v_j) = 2j$; $1 \le j \le n$

According to this pattern the vertices are labeled such that for any edge $e = u_iu_j \in G$,
$\gcd(f(u_i), f(u_j)) = 1$.

Thus we proved that the graph under consideration admits prime labeling.
That is, $G \oplus K_{1,n}$ is a prime graph, where G is the cycle C_n with twin chord .

Example 3: prime labeling of ring sum of the graph the graph cycle C_7 with twin chords and $K_{1,7}$ is shown in Fig. 3 as an illustration for the Theorem 3.

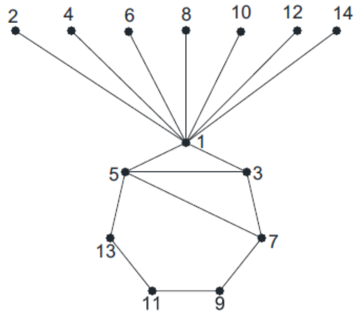

Figure 3: prime labeling of ring sum of the graph cycle C_7 with twin chords and $K_{1,7}$

Theorem 4. $G \oplus K_{1,n}$ is a prime graph, where G is cycle with triangle and chords forms three triangles and one cycle C_{n-5}, for all $n \in N$.

Proof. Let G be cycle with triangle $C_n(1, 1, n - 5)$. Let $V = V_1 \cup V_2$, where V_1 is the vertex set of G and V_2 be the vertex set of $K_{1,n}$. And Let $u_1, u_2, \ldots, u_n$ be successive vertices of G.

Let u_1, u_3 and u_5 be the vertices of triangle formed by edges $e_1 = u_1u_3$, $e_2 = u_3u_5$ and $e_3 = u_1u_5$.

Let $v_1, v_2, \ldots, v_n$ be the pendent vertices, v be the apex vertex of $K_{1,n}$ and take $v = u_1$.
Also $|V(G)| = 2n$ and $|E(G)| = 2n+3$
We define labeling $f: V(G) \rightarrow \{1, 2, 3, \ldots, 2n\}$ as follows.

$f(u_1) = 1$, $f(u_2) = 3$, $f(u_3) = 5$, $f(u_{n-1}) = 7$, $f(u_n) = n$
$f(u_i) = 2i + 1$; $4 \le i \le n - 2$
$f(v_j) = 2j$; $1 \le j \le n$
According to this pattern the vertices are labeled such that for any edge $e = u_iu_j \in G$, $\gcd(f(u_i), f(u_j)) = 1$.

Thus we proved that the graph under consideration admits prime labeling. That is, $G \oplus K_{1,n}$ is a prime graph, where G is the cycle C_n with triangle.

Example 4. prime labeling of ring sum of the graph the graph cycle C_8 with twin chords and $K_{1,8}$ is shown in Fig. 4 as an illustration for the Theorem 4.

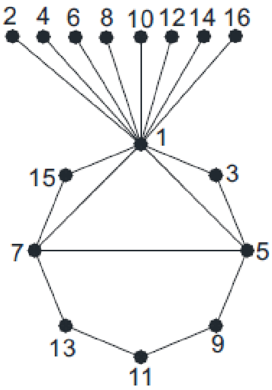

Figure 4: prime labeling of ring sum of the graph cycle C_8 with triangle and $K_{1,8}$

Theorem 5. $S_n \oplus K_{1,n}$ is prime graph, for all $n \in N$.
Proof. Let $V(G) = V_1 \cup V_2$, where $V_1 = \{u_1, u_2, \ldots, u_n\}$ with apex u_1 and $u_2, u_3, \ldots, u_n$ are other vertices of S_n and $V_2 = \{v = u_1, v_1, v_2, \ldots, v_n\}$ be the vertex set of $K_{1,n}$. Here $v_1, v_2, \ldots, v_n$ are pendent vertices.
We define labeling $f: V(G) \rightarrow \{1, 2, 3, \ldots, 2n\}$ as follows.

$f(u_i) = 2i - 1$; $1 \leq i \leq n$
$f(v_j) = 2j$; $1 \leq j \leq n$

According to this pattern the vertices are labeled such that for any edge $e = u_iu_j \in G$,
$\gcd(f(u_i), f(u_j)) = 1$.

Thus we proved that the graph under consideration admits prime labeling.
That is, $S_n \oplus K_{1,n}$ is a prime graph.

Example 5. prime labeling of the graph $S_7 \oplus K_{1,7}$ is shown in Fig. 5 as an illustration for the Theorem 5.

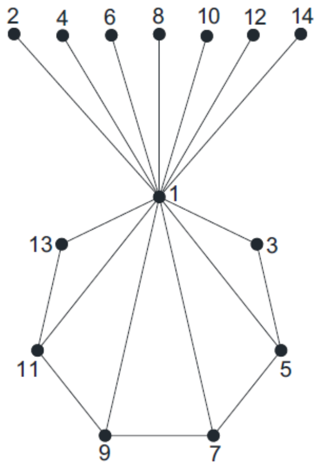

Figure 5: prime labeling of the graph $S_7 \oplus K_{1,7}$

4. Conclusion

We have proved that the graph $C_n \oplus K_{1,n}$, $G \oplus K_{1,n}$, where G is cycle with one chord, $G \oplus K_{1,n}$, where G is cycle with twin chord, $G \oplus K_{1,n}$, where G is cycle with triangle, $S_n \oplus K_{1,n}$ are prime graphs.

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