

# Quasi $\alpha$ - Local Functions In Ideal Bitopological Spaces

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**Abstract:** In this paper we extend the concept of  $\alpha$ - local function due to W. Al-omeri, Mohd. Salmi, Md. Noorani and A. Al-omari [1] to ideal bitopological spaces and study some of its properties. Further the concepts of  $q\alpha I$ - open sets and  $q\alpha I$ - continuous mappings are introduced and studied.

**Keywords:** Ideal bitopological spaces, quasi  $\alpha$ - local functions,  $q\alpha I$ - open sets and  $q\alpha I$ - continuous mappings.

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## 1. Preliminaries

In 1965 Njastad [9] introduced the concept of  $\alpha$ - open sets in topology. A subset  $A$  of a topological space  $(X, \tau)$  is said to be  $\alpha$ - open if  $A \subset \text{int}(\text{Cl}(\text{int}(A)))$ . Every open set is  $\alpha$ - open but the converse may not be true. Further in 1985, Maheshwari and Thakur introduced  $\alpha$ - continuous mapping. A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is said to be  $\alpha$ - continuous if for every open set  $V$  in  $Y$ ,  $f^{-1}(V)$  is  $\alpha$ - open in  $X$ . [8]

In 1963 Kelly [5] introduced the concept of bitopological spaces as an extension of topological spaces. A bitopological space  $(X, \tau_1, \tau_2)$  is a nonempty set  $X$  equipped with two topologies  $\tau_1$  and  $\tau_2$  [5]. The study of quasi open sets in bitopological spaces was initiated by Datta [2] in 1971. In a bitopological space  $(X, \tau_1, \tau_2)$  a set  $A$  of  $X$  is said to be quasi open [2] if it is a union of a  $\tau_1$ - open set and a  $\tau_2$ - open set. Complement of a quasi open set is termed quasi closed. Every  $\tau_1$ - open (resp.  $\tau_2$ - open) set is quasi open but the converse may not be true. Any union of quasi open sets of  $X$  is quasi open in  $X$ . The intersection of all quasi closed sets which contains  $A$  is called quasi closure of  $A$  [7]. It is denoted by  $qcl(A)$ . The union of quasi open subsets of  $A$  is called quasi interior of  $A$ . It is denoted by  $qInt(A)$  [7].

In 1985, Thakur and Paik [10] introduced the concept of quasi  $\alpha$ - open sets in bitopological spaces. A set  $A$  in a bitopological space  $(X, \tau_1, \tau_2)$  is called quasi  $\alpha$ - open [10] if it is a union of a  $\tau_{1\alpha}$ - open set and a  $\tau_{2\alpha}$ - open set. Complement of a quasi  $\alpha$ - open set is called quasi  $\alpha$ - closed. Every  $\tau_{1\alpha}$ - open ( $\tau_{2\alpha}$ - open, quasi open) set is quasi  $\alpha$ - open but the converse may not be true. Any union of quasi  $\alpha$ - open sets of  $X$  is a quasi  $\alpha$ - open set in  $X$ . The intersection of all quasi  $\alpha$ - closed sets which contains  $A$  is called quasi  $\alpha$ - closure of  $A$ . It is denoted by  $q\alpha cl(A)$ . The union of quasi  $\alpha$ - open subsets of  $A$  is called quasi  $\alpha$ - interior of  $A$ . It is denoted by  $q\alpha Int(A)$  [10].

The concept of ideal topological spaces was initiated Kuratowski [6] and Vaidyanathaswamy [11]. An Ideal  $I$  on a topological space  $(X, \tau)$  is a non empty collection of subsets of  $X$  which satisfies: i)  $A \in I$  and  $B \subset A \Rightarrow B \in I$

and ii)  $A \in I$  and  $B \in I \Rightarrow A \cup B \in I$  If  $\mathcal{P}(X)$  is the set of all subsets of  $X$ , in a topological space  $(X, \tau)$  a set operator  $(.)^*: \mathcal{P}(X) \rightarrow \mathcal{P}(X)$  called the local function [3] of  $A$  with respect to  $\tau$  and  $I$  and is defined as follows:

$A^*(\tau, I) = \{x \in X \mid U \cap A \notin I, \forall U \in \tau(x)\}$ , where  $\tau(x) = \{U \in \tau \mid x \in U\}$ . Given an ideal bitopological space  $(X, \tau_1, \tau_2, I)$  the quasi local function [4] of  $A$  with respect to  $\tau_1, \tau_2$  and  $I$  denoted by  $A_q^*(\tau_1, \tau_2, I)$  (in short  $A_q^*$ ) is defined as follows:  
 $A_q^*(\tau_1, \tau_2, I) = \{x \in X \mid U \cap A \notin I, \forall \text{ quasi open set } U \text{ containing } x\}$ .

A subset  $A$  of an ideal bitopological space  $(X, \tau_1, \tau_2)$  is said to be  $qI$ - open [4] if  $A \subset qInt(A_q^*)$ . A mapping  $f: (X, \tau_1, \tau_2, I) \rightarrow (Y, \sigma_1, \sigma_2)$  is called  $qI$ - continuous [4] if  $f^{-1}(V)$  is  $qI$ - open in  $X$  for every quasi open set  $V$  of  $Y$ .

## 2. Quasi $\alpha$ - Local Functions

**Definition 2.1.** Given an ideal bitopological space  $(X, \tau_1, \tau_2, I)$  the quasi  $\alpha$ - local function of  $A$  with respect to  $\tau_1, \tau_2$  and  $I$  denoted by  $A_{q\alpha}^*(\tau_1, \tau_2, I)$  is defined as follows:

$A_{q\alpha}^*(\tau_1, \tau_2, I) = \{x \in X \mid U \cap A \notin I, \forall \text{ quasi } \alpha\text{- open set } U \text{ containing } x\}$ .

When there is no ambiguity  $A_{q\alpha}^*$  shall be written for  $A_{q\alpha}^*(\tau_1, \tau_2, I)$ .

**Theorem 2.1** Let  $(X, \tau_1, \tau_2, I)$  be an ideal bitopological space and  $A \subset X$  then:

- $A_{q\alpha}^* \subset A_q \subset A^*(\tau_1, I)$  and  $A_{q\alpha}^* \subset A_q \subset A^*(\tau_2, I)$
- $A_{q\alpha}^* \subset A_\alpha(\tau_1, I)$  and  $A_{q\alpha}^* \subset A_\alpha(\tau_2, I)$
- $A_{q\alpha}^*(\tau_1, \tau_2, \{\emptyset\}) = q\alpha cl(A)$
- $A_{q\alpha}^*(\tau_1, \tau_2, \mathcal{P}(X)) = \emptyset$
- If  $A \in I$ , then  $A_{q\alpha}^* = \emptyset$
- Neither  $A \subset A_{q\alpha}^*$  nor  $A_{q\alpha}^* \subset A$

**Proof:** Obvious.

**Theorem 2.2** Let  $(X, \tau_1, \tau_2, I)$  be an ideal bitopological space and  $A, B$  be subsets of  $X$  then,

- a) If  $A \subset B$ , then  $A_{q\alpha}^* \subset B_{q\alpha}^*$
- b)  $A_{q\alpha}^* = q\alpha cl A_{q\alpha}^* \subset q\alpha cl(A)$  and  $A_{q\alpha}^*$  is a quasi  $\alpha$ -
- c) closed set in  $(X, \tau_1, \tau_2)$
- d)  $(A_{q\alpha}^*)_{q\alpha}^* \subset A_{q\alpha}^*$
- e)  $(A \cup B)_{q\alpha}^* = A_{q\alpha}^* \cup B_{q\alpha}^*$
- f)  $A_{q\alpha}^* - B_{q\alpha}^* = (A - B)_{q\alpha}^* - B_{q\alpha}^* \subset (A - B)_{q\alpha}^*$
- g) If  $C \in I$ , then  $(A - C)_{q\alpha}^* \subset A_{q\alpha}^* = (A \cup C)_{q\alpha}^*$

**Proof:** (a) Suppose  $A \subset B$  and  $x \notin B_{q\alpha}^*$  then there exists a quasi  $\alpha$ - open set  $U$  containing  $x$  such that  $U \cap B \in I$ . Since  $A \subset B$ ,  $U \cap A \in I$  and so  $x \notin A_{q\alpha}^*$ . Hence  $A_{q\alpha}^* \subset B_{q\alpha}^*$

(b) We have  $A_{q\alpha}^* \subset q\alpha cl(A_{q\alpha}^*)$ , in general. Let  $x \in q\alpha cl(A_{q\alpha}^*)$ , then  $A_{q\alpha}^* \cap U \neq \emptyset$  for every quasi  $\alpha$ - open set  $U$  containing  $x$ . Therefore  $\exists y \in A_{q\alpha}^* \cap U$  and quasi  $\alpha$ - open set  $U$  containing  $y$ . Since  $y \in A_{q\alpha}^*$  and  $U \cap A \notin I$ , therefore  $x \in A_{q\alpha}^*$ . Hence  $q\alpha cl(A_{q\alpha}^*) \subset A_{q\alpha}^*$ . Consequently,  $A_{q\alpha}^* = q\alpha cl(A_{q\alpha}^*)$ . Again let  $x \in q\alpha cl(A_{q\alpha}^*) = A_{q\alpha}^*$ . Then  $U \cap A \notin I$  for every quasi  $\alpha$ - open set containing  $x$ . Therefore  $x \in q\alpha cl(A)$ . This proves  $A_{q\alpha}^* = q\alpha cl(A_{q\alpha}^*) \subset q\alpha cl(A)$

(c) Let  $x \in (A_{q\alpha}^*)_{q\alpha}^*$ , then for every quasi  $\alpha$ - open set  $U$  containing  $x$ ,  $U \cap A_{q\alpha}^* \notin I$  and hence  $\neq \emptyset$ . Let  $y \in A_{q\alpha}^* \cap U$ . Then  $\exists$  a quasi  $\alpha$ - open set  $U$  containing  $y$  and  $y \in A_{q\alpha}^*$ . Hence we have  $U \cap A \notin I$ , and  $x \in A_{q\alpha}^*$ . Therefore  $(A_{q\alpha}^*)_{q\alpha}^* \subset A_{q\alpha}^*$

(d) By (1)  $A_{q\alpha}^* \cup B_{q\alpha}^* \subset (A \cup B)_{q\alpha}^*$ . Let  $x \in (A \cup B)_{q\alpha}^*$  then for every quasi  $\alpha$ - open set  $U$  containing  $x$ ,  $(U \cap A) \cup (U \cap B) = U \cap (A \cup B) \notin I$ . This implies  $x \in A_{q\alpha}^*$  or  $x \in B_{q\alpha}^*$ . Hence,  $x \in A_{q\alpha}^* \cup B_{q\alpha}^*$ .

(e) We have  $A_{q\alpha}^* = (A - B)_{q\alpha}^* \cup (A \cap B)_{q\alpha}^*$ . Thus  $A_{q\alpha}^* - B_{q\alpha}^* = A_{q\alpha}^* \cap (X - B)_{q\alpha}^* = (A - B)_{q\alpha}^* \cup (A \cap B)_{q\alpha}^* \cap (X - B)_{q\alpha}^* = (A - B)_{q\alpha}^* \cap (X - B)_{q\alpha}^* \cup (A \cap B)_{q\alpha}^* \cap (X - B)_{q\alpha}^* = ((A - B)_{q\alpha}^* - B_{q\alpha}^*) \cup \emptyset \subset (A - B)_{q\alpha}^*$ .

(f) Since  $A - C \subset A$ , by (a)  $(A - C)_{q\alpha}^* \subset A_{q\alpha}^*$ . From Theorem 2.2 (d) and Theorem 2.1 (e), we get  $(A \cup C)_{q\alpha}^* = A_{q\alpha}^* \cup C_{q\alpha}^* = A_{q\alpha}^* \cup \emptyset = A_{q\alpha}^*$ . Hence,  $(A - C)_{q\alpha}^* \subset A_{q\alpha}^* = (A \cup C)_{q\alpha}^*$

**Theorem 2.3.** Let  $(X, \tau_1, \tau_2)$  be a bitopological space with Ideals  $I_1$  and  $I_2$  on  $X$  and  $A$  is a subset of  $X$ . Then:

- (a) If  $I_1 \subset I_2$ , then  $A_{q\alpha}^*(I_2) \subset A_{q\alpha}^*(I_1)$
- (b)  $A_{q\alpha}^*(I_1 \cap I_2) = A_{q\alpha}^*(I_1) \cup A_{q\alpha}^*(I_2)$

**Proof:** (a) Let  $I_1 \subset I_2$  and  $x \in A_{q\alpha}^*(I_2)$ , then  $A \cap U \notin I_2$  for every quasi  $\alpha$ - open set  $U$  containing  $x$ . From given  $A \cap U \notin I_1$  hence  $x \in A_{q\alpha}^*(I_1)$ . Therefore, we have  $A_{q\alpha}^*(I_2) \subset A_{q\alpha}^*(I_1)$

(b) Let  $x \in A_{q\alpha}^*(I_1 \cap I_2)$ , then for every quasi  $\alpha$ - set  $U$  containing  $x$ ,  $A \cap U \notin (I_1 \cap I_2)$ , hence  $A \cap U \notin I_1$  and  $A \cap U \notin I_2$ . This shows,  $x \in A_{q\alpha}^*(I_1)$  or  $x \in A_{q\alpha}^*(I_2)$  that is  $x \in A_{q\alpha}^*(I_1) \cup A_{q\alpha}^*(I_2)$ . Thus,  $A_{q\alpha}^*(I_1 \cap I_2) \subset A_{q\alpha}^*(I_1) \cup A_{q\alpha}^*(I_2)$ . But  $A_{q\alpha}^*(I_1) \cup A_{q\alpha}^*(I_2) \subset A_{q\alpha}^*(I_1 \cap I_2)$ . Therefore,  $A_{q\alpha}^*(I_1 \cap I_2) = A_{q\alpha}^*(I_1) \cup A_{q\alpha}^*(I_2)$ .

**Definition 2.2.** In an ideal bitopological space  $(X, \tau_1, \tau_2, I)$  the quasi  $\alpha$ - closure of  $A$  of  $X$  denoted by  $q\alpha cl^*(A)$  is defined by  $q\alpha cl^*(A) = A \cup A_{q\alpha}^*$ .

**Theorem 2.4.** Let  $(X, \tau_1, \tau_2, I)$  be an ideal bitopological space and  $A, B$  be the subsets of  $X$ . Then:

- (a)  $A \subset q\alpha cl^*(A)$
- (b)  $q\alpha cl^*(\emptyset) = \emptyset$  and  $q\alpha cl^*(X) = X$
- (c) If  $A \subset B$ , then  $q\alpha cl^*(A) \subset q\alpha cl^*(B)$
- (d)  $q\alpha cl^*(A) \cup q\alpha cl^*(B) \subset q\alpha cl^*(A \cup B)$
- (e) If  $I = \emptyset$ , then  $q\alpha cl^*(A) = q\alpha cl(A)$

**Proof:** Follows from Definition 2.2.

**Definition 2.3.** A subset  $A$  of an ideal bitopological space  $(X, \tau_1, \tau_2, I)$  is said to be:

- (a)  $q\alpha I$ - open if  $A \subset q\alpha Int(A_{q\alpha}^*)$ .
- (b)  $q\alpha I$ - closed if its complement is  $q\alpha I$ - open.

The family of all  $q\alpha I$ - open (respectively  $q\alpha I$ - closed) sets of an ideal bitopological space  $(X, \tau_1, \tau_2, I)$  is denoted by  $QAIO(X)$  (respectively  $QAIC(X)$ ).

The family of all  $q\alpha I$ - open sets of  $(X, \tau_1, \tau_2, I)$  containing a point  $x$  is denoted by  $QAIO(X, x)$ .

**Remark 2.1.** Every  $qI$ - open set is  $q\alpha I$ - open but the converse is not true. For,

**Example 2.1.** Let  $X = \{a, b, c, d\}$  and  $\tau_1 = \{X, \emptyset, \{b\}, \{a, c\}, \{a, b, c\}\}$ ,  $\tau_2 = \{X, \emptyset, \{d\}, \{a, b\}, \{a, b, d\}\}$  be topologies on  $X$  and  $I = \{\emptyset, \{a\}\}$  be an ideal on  $X$ . Then the set  $A = \{c, d\}$  is  $q\alpha I$ - open but not  $qI$ - open in  $(X, \tau_1, \tau_2, I)$ .

**Remark 2.2.** The concepts of  $q\alpha I$ - open sets and quasi  $\alpha$ - open sets are independent. For, in an ideal bitopological space  $(X, \tau_1, \tau_2, I)$  of Example 2.1, the set  $\{b, c\}$  is  $q\alpha I$ - open but not quasi  $\alpha$ - open and the set  $\{a, b, d\}$  is quasi  $\alpha$ - open but not  $q\alpha I$ - open.

**Remark 2.3.** For an ideal bitopological space  $(X, \tau_1, \tau_2, I)$  we have the following:

- (a)  $X$  need not be a  $q\alpha I$ - open set.
- (b) If  $I = \mathcal{P}(X)$ , then only the empty set is  $q\alpha I$ - open.
- (c) If  $I = \emptyset$ ,  $q\alpha I$ - openness and quasi  $\alpha$ - openness are equivalent.

**Theorem 2.5.** If  $A$  is  $q\alpha I$ - open, then  $A_{q\alpha}^* = (q\alpha Int(A_{q\alpha}^*))_{q\alpha}^*$

**Proof:** Since  $A$  is  $qaI$ - open,  $A \subset qaInt(A_{qa}^*)$ . Therefore,  $A_{qa}^* \subset (qaInt(A_{qa}^*))_{qa}^*$ . Also we have  $qaInt(A_{qa}^*) \subset A_{qa}^*(qaInt(A_{qa}^*))^* \subset (A_{qa}^*)_{qa}^* \subset (A_{qa}^*)$ . Hence,  $A_{qa}^* = (qaInt(A_{qa}^*))_{qa}^*$

**Theorem 2.6.** Any union of a family of  $qaI$ - open sets in an ideal bitopological space  $(X, \tau_1, \tau_2, I)$  is  $qaI$ - open in  $X$ .

**Proof:** Let  $\{U_\delta : \delta \in \Delta\}$  be a family of  $qaI$ - open sets of an ideal bitopological space  $(X, \tau_1, \tau_2, I)$ . Then  $U_\delta \subset qaInt((U_\delta)_{qa}^*) \forall \delta \in \Delta$ . It follows that  $\cup_{\delta \in \Delta} U_\delta \subset \cup_{\delta \in \Delta} (qaInt((U_\delta)_{qa}^*)) \subset qaInt(\cup_{\delta \in \Delta} (U_\delta)) \subset qaInt(\cup_{\delta \in \Delta} (U_\delta)_{qa}^*)$ . Hence  $\cup_{\delta \in \Delta} U_\delta$  is  $qaI$ - open set in  $X$ .

**Definition 2.4.** Let  $A$  be a subset of an ideal bitopological space  $(X, \tau_1, \tau_2, I)$  and  $x \in X$ . Then:

- (a)  $x$  is called a  $qaI$ - interior point of  $A$  if  $\exists V \in QAIO(X)$  such that  $x \in V \subset A$ .
- (b) Set of all  $qaI$ - interior points of  $A$  denoted by  $qaIInt(A)$  is called the  $qaI$ - interior of  $A$ .

The following theorem summarizes the properties of  $qaI$ - interior of subsets in ideal bitopological spaces.

**Theorem 2.7.** Let  $A, B$  be subsets of an ideal bitopological space  $(X, \tau_1, \tau_2, I)$ . Then:

- (a)  $qaIInt(A) = \cup \{T : T \subset A \text{ and } A \in QAIO(X)\}$
- (b)  $qaIInt(A)$  is the largest  $qaI$ - open subset of  $X$  contained in  $A$ .
- (c)  $A$  is  $qaI$ - open if and only if  $A = qaIInt(A)$
- (d)  $qaIInt(qaIInt(A)) = qaIInt(A)$
- (e) If  $A \subset B$ , then  $qaIInt(A) \subset qaIInt(B)$
- (f)  $qaIInt(A) \cup qaIInt(B) \subset qaIInt(A \cup B)$
- (g)  $qaIInt(A \cap B) \subset qaIInt(A) \cap qaIInt(B)$

**Proof:** (a) Let  $x \in \cup \{T : T \subset A \text{ and } A \in QAIO(X)\}$ . Then, there exists  $T \in QAIO(X, x)$  such that  $x \in T \subset A$  and hence  $x \in qaIInt(A)$ . This shows that  $\cup \{T : T \subset A \text{ and } A \in QAIO(X)\} \subset qaIInt(A)$ . For the reverse inclusion, let  $x \in qaIInt(A)$ , then there exists  $T \in QAIO(X, x)$ , such that  $x \in T \subset A$  and we obtain  $x \in \cup \{T : T \subset A \text{ and } A \in QAIO(X)\}$ . This shows that  $qaIInt(A) \subset \cup \{T : T \subset A \text{ and } A \in QAIO(X)\}$ . Therefore  $\cup \{T : T \subset A \text{ and } A \in QAIO(X)\} = qaIInt(A)$ .

The proof of properties (b) - (e) are obvious.

- (f) Clearly  $qaIInt(A) \subset qaIInt(A \cup B)$  and  $qaIInt(B) \subset qaIInt(A \cup B)$ . Thus  $qaIInt(A) \cup qaIInt(B) \subset qaIInt(A \cup B)$
- (g) Since  $A \cap B \subset A$  and  $A \cap B \subset B$ , by (e) we have  $qaIInt(A \cap B) \subset qaIInt(A)$  and  $qaIInt(A \cap B) \subset qaIInt(B)$ . Then  $qaIInt(A \cap B) \subset qaIInt(A) \cap qaIInt(B)$

**Definition 2.5.** Let  $A$  be a subset of an ideal bitopological space  $(X, \tau_1, \tau_2, I)$  and  $x \in X$ . Then:

- (a)  $x$  is called a  $qaI$ - cluster point of  $A$ , if  $V \cap A \neq \emptyset$ . for every  $V \in QAIO(X, x)$
- (b) The set of all  $qaI$ - cluster points of  $A$  denoted by  $qaICl(A)$  is called the  $qaI$ - closure of  $A$ . The following theorem summarizes the properties of  $qaI$ - closure of subsets in an ideal bitopological spaces.

**Theorem 2.8.** Let  $A$  and  $B$  be subsets of an ideal bitopological space  $(X, \tau_1, \tau_2, I)$ , Then:

- (a)  $qaICl(A) = \cap \{F : A \subset F \text{ and } F \in QAIC(X)\}$
- (b)  $qaICl(A)$  is the smallest  $qaI$ - closed subset of  $X$  containing  $A$ .
- (c)  $A$  is  $qaI$ - closed if and only if  $A = qaICl(A)$ .
- (d)  $qaICl(qaIInt(A)) = qaICl(A)$
- (e) If  $A \subset B$ , then  $qaICl(A) \subset qaICl(B)$
- (f)  $qaICl(A) \cup qaICl(B) = qaICl(A \cup B)$
- (g)  $qaICl(A \cap B) \subset qaICl(A) \cap qaICl(B)$

**Proof:** (a) Suppose  $x \notin qaICl(A)$ . Then, there exists  $F \in QAIO(X)$  such that  $F \cap A = \emptyset$ . Since  $X - F$  is  $qaI$ - closed set containing  $A$  and  $x \notin X - F$ , we obtain  $x \notin \cap \{F : A \subset F \text{ and } F \in QAIC(X)\}$ . For the reverse, there exists  $F \in QAIO(X)$  such that  $A \subset F$  and  $x \notin F$ . Since  $X - F$  is  $qaI$ - closed set containing  $x$ , we get  $(X - F) \cap A = \emptyset$ . This shows that  $x \notin qaICl(A)$ . Therefore  $qaICl(A) = \cap \{F : A \subset F \text{ and } F \in QAIC(X)\}$ .

Statements (b) - (g) have obvious proofs.

**Theorem 2.9.** Let  $(X, \tau_1, \tau_2, I)$  be an ideal bitopological space and  $A \subset X$ . Then the following properties hold:

- (a)  $qaICl(X - A) = X - qaIInt(A)$
- (b)  $qaIInt(X - A) = X - qaICl(A)$

**Proof:** (a) Let  $W$  be a subset of  $X$ .  $W \subset A$  if and only if  $(X - A) \subset (X - W)$ ,  $W$  is  $qaI$ - open if and only if  $(X - W)$  is  $qaI$ - closed. Thus,  $qaICl(X - A) = \cap \{(X - W) : W \subset A \text{ and } W \in QAIO(X)\} = X - \cup \{W \subset A \text{ and } W \in QAIO(X)\} = X - qaIInt(A)$ .

(b) Follows from (a).

**Definition 2.6.** A subset  $B_x$  of an ideal bitopological space  $(X, \tau_1, \tau_2, I)$  is said to be a  $qaI$ -neighbourhood of a point  $x \in X$  if there exists a  $qaI$ - open set  $U$  of  $X$  such that  $x \in U \subset B_x$ .

**Theorem 2.10.** A subset of an ideal bitopological space  $(X, \tau_1, \tau_2, I)$  is  $qaI$ - open if and only if it is a  $qaI$ -neighbourhood of each of its points.

**Proof: Necessary:** Let  $G$  be a  $qaI$ - open set of  $X$ . Then by definition, it is clear that  $G$  is a  $qaI$ - neighbourhood of each of its points, since  $\forall x \in G, x \in G \subset G$  and  $G$  is  $qaI$ - open. **Sufficient:** Suppose  $G$  is a  $qaI$ - neighbourhood of each of its points. Then for each  $x \in G$  there exists  $S_x \in QAIO(X)$  such that  $S_x \subset G$ . Therefore  $G = \cup \{S_x : x \in G\}$ . Since each

$S_x$  is  $q\alpha I$ - open and arbitrary union of  $q\alpha I$ - open sets is  $q\alpha I$ - open,  $G$  is  $q\alpha I$ - open in  $(X, \tau_1, \tau_2, I)$ .

### 3. $q\alpha I$ - Continuous Mappings

**Definition 3.1.** A mapping  $f: (X, \tau_1, \tau_2, I) \rightarrow (Y, \sigma_1, \sigma_2)$  is called a  $q\alpha I$ - continuous if  $f^{-1}(V)$  is a  $q\alpha I$ - open set in  $X$  for every quasi open set  $V$  of  $Y$ .

**Remark 3.1.** Every  $qI$ - continuous mapping is  $q\alpha I$ - continuous but the converse is not true. For,

**Example 3.1.** Let  $X = \{a, b, c, d\}$  and  $\tau_1 = \{X, \emptyset, \{b\}, \{a, c\}, \{a, b, c\}\}$ ,  $\tau_2 = \{X, \emptyset, \{d\}, \{a, b\}, \{a, b, d\}\}$  be topologies on  $X$  and  $I = \{\emptyset, \{a\}\}$  be an ideal on  $X$ .  $\sigma_1 = \{X, \emptyset, \{b\}, \{c, d\}, \{b, c, d\}\}$ , and  $\sigma_2 = \{X, \emptyset\}$  be topologies on  $X$ . Then the identity mapping  $f: (X, \tau_1, \tau_2, I) \rightarrow (X, \sigma_1, \sigma_2)$  is  $q\alpha I$ - continuous but not  $qI$ - continuous.

**Theorem 3.1.** Let  $f: (X, \tau_1, \tau_2, I) \rightarrow (Y, \sigma_1, \sigma_2)$  be a mapping. Then the following statements are equivalent:

- (a)  $f$  is  $q\alpha I$ - continuous.
- (b)  $f^{-1}(V)$  is  $q\alpha I$ - closed in  $X$  for every quasi closed set  $V$  of  $Y$ .
- (c) for each  $x \in X$  and every quasi open set  $V$  of  $Y$  containing  $f(x)$ ,  $\exists W \in Q\mathcal{A}IO(X, x)$  such that  $f(W) \subset V$ .
- (d) for each  $x \in X$  and every quasi open set  $V$  of  $Y$  containing  $f(x)$ ,  $f^{-1}(V)_{q\alpha}^*$  is a  $q\alpha I$ - neighbourhood of  $x$ .

**Proof:** (a)  $\Leftrightarrow$  (b). Obvious.

(a)  $\Rightarrow$  (c). Let  $x \in X$  and  $V$  be a quasi open set of  $Y$  containing  $f(x)$ . Since  $f$  is  $q\alpha I$  continuous,  $f^{-1}(V)$  is a  $q\alpha I$  open set. Putting  $W = f^{-1}(V)$ , we get  $f(W) \subset V$ .

(c)  $\Rightarrow$  (a). Let  $A$  be a quasi open set in  $Y$ . If  $f^{-1}(A) = \emptyset$ , then  $f^{-1}(A)$  is clearly a  $q\alpha I$ - open set. Assume that  $f^{-1}(A) \neq \emptyset$  and  $x \in f^{-1}(A)$ , then  $f(x) \in A \Rightarrow \exists$  a  $q\alpha I$ - open set  $W$  containing  $x$  such that  $f(W) \subset A$ . Thus  $W \subset f^{-1}(A)$ . Since  $W$  is  $q\alpha I$ - open,  $x \in W \subset q\alpha \text{Int}(W_{q\alpha}^*) \subset q\alpha \text{Int}(f^{-1}(A)_{q\alpha}^*)$  and so  $f^{-1}(A) \subset q\alpha \text{Int}(f^{-1}(A)_{q\alpha}^*)$ . Hence  $f^{-1}(A)$  is a  $q\alpha I$ - open set and therefore  $f: (X, \tau_1, \tau_2, I) \rightarrow (Y, \sigma_1, \sigma_2)$  is  $q\alpha I$ - continuous.

(c)  $\Rightarrow$  (d). Let  $x \in X$  and  $V$  be a quasi open set of  $Y$  containing  $f(x)$  then  $\exists$  a  $q\alpha I$ - open set  $W$  containing  $x$  such that  $f(W) \subset V$ . It follows that  $W \subset f^{-1}(f(W)_{q\alpha}^*) \subset f^{-1}(V)$ . Since  $W$  is a  $q\alpha I$ - open set,  $x \in W \subset q\alpha \text{Int}(W^*) \subset q\alpha \text{Int}(f^{-1}(V)_{q\alpha}^*) \subset f^{-1}(V)^*$ . Hence  $f^{-1}(V)_{q\alpha}^*$  is a  $q\alpha I$ - neighbourhood of  $x$ .

(d)  $\Rightarrow$  (c). Obvious.

**Definition 3.2.** A mapping  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2, I)$  is said to be :

- (a)  $q\alpha I$ - open if  $f(U)$  is a  $q\alpha I$ - open set of  $Y$  for every quasi open set  $U$  of  $X$ .
- (b)  $q\alpha I$ - closed if  $f(U)$  is a  $q\alpha I$ - closed set of  $Y$  for every quasi closed set  $U$  of  $X$ .

**Theorem 3.2.** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2, I)$  be a mapping. Then the following statements are equivalent:

- (a)  $f$  is  $q\alpha I$ - open
- (b)  $f(q\text{Int}(U)) \subset q\alpha \text{Int}(f(U))$  for each subset  $U$  of  $X$ .
- (c)  $q\text{Int}(f^{-1}(V)) \subset f^{-1}(q\alpha \text{Int}(V))$  for each subset  $V$  of  $Y$ .

**Proof:** (a)  $\Rightarrow$  (b). Let  $U$  be any subset of  $X$ . Then  $q\text{Int}(U)$  is a quasi open set of  $X$ . Then  $f(q\text{Int}(U))$  is a  $q\alpha I$ - open set of  $Y$ . Since  $f(q\text{Int}(U)) \subset f(U)$ ,  $f(q\text{Int}(U)) = q\alpha \text{Int}(f(q\text{Int}(U))) \subset q\alpha \text{Int}(f(U))$ .

(b)  $\Rightarrow$  (c). Let  $V$  be any subset of  $Y$ . Obviously  $f^{-1}(V)$  is a subset of  $X$ . Therefore by (b),  $f(q\text{Int}(f^{-1}(V))) \subset q\alpha \text{Int}(f(f^{-1}(V))) \subset q\alpha \text{Int}(V)$ . Hence,  $q\text{Int}(f^{-1}(V)) \subset f^{-1}(f(q\text{Int}(f^{-1}(V)))) \subset f^{-1}(q\alpha \text{Int}(V))$ .

(c)  $\Rightarrow$  (a). Let  $V$  be any quasi open set of  $X$ . Then  $q\text{Int}(V) = V$  and  $f(V)$  is a subset of  $Y$ . So  $V = q\text{Int}(V) \subset q\text{Int}(f^{-1}(f(V))) \subset f^{-1}(q\alpha \text{Int}(f(V)))$ . Then  $f(V) \subset f(f^{-1}(q\alpha \text{Int}(f(V)))) \subset q\alpha \text{Int}(f(V))$  and  $q\alpha \text{Int}(f(V)) \subset f(V)$ . Hence,  $f(V)$  is a  $q\alpha I$ - open set of  $Y$  and  $f$  is  $q\alpha I$ - open.

**Theorem 3.3.** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2, I)$  be a  $q\alpha I$ - open mapping. If  $V$  is a subset of  $Y$  and  $U$  is a quasi closed subset of  $X$  containing  $f^{-1}(V)$ , then there exists a  $q\alpha I$ - closed set  $F$  of  $Y$  containing  $V$  such that  $f^{-1}(F) \subset U$ .

**Proof:** Let  $V$  be any subset of  $Y$  and  $U$  a quasi closed subset of  $X$  containing  $f^{-1}(V)$ , and let  $F = (Y - (f(X - U)))$ . Then  $f(X - U) \subset f(f^{-1}(X - U)) \subset (X - U)$  and  $X - U$  is a quasi open set of  $X$ . Since  $f$  is  $q\alpha I$ - open,  $f(X - U)$  is a  $q\alpha I$ - open set of  $Y$ . Hence  $F$  is a quasi closed subset of  $Y$  and  $f^{-1}(F) = f^{-1}(Y - (f(X - U))) \subset U$ .

**Theorem 3.4.** A mapping  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2, I)$  is  $q\alpha I$ - closed if and only if  $q\alpha \text{Icl}(f(V)) \subset f(q\text{cl}(V))$  for each subset  $V$  of  $X$ .

**Proof:** Necessary: Let  $f$  be a  $q\alpha I$ - closed mapping and  $V$  be any subset of  $X$ . Then  $f(V) \subset f(q\text{cl}(V))$  and  $f(q\text{cl}(V))$  is a  $q\alpha I$ - closed set of  $Y$ . Thus  $q\alpha \text{Icl}(f(V)) \subset q\alpha \text{Icl}(f(q\text{cl}(V))) = f(q\text{cl}(V))$ .

Sufficient: Let  $V$  be a quasi closed set of  $X$ . Then by hypothesis  $f(V) \subset q\alpha \text{Icl}(f(V)) \subset f(q\alpha \text{cl}(V)) = f(V)$ . And so,  $f(V)$  is a  $q\alpha I$ - closed subset of  $Y$ . Hence,  $f$  is  $q\alpha I$ - closed.

**Theorem 3.5.** A mapping  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2, I)$   $q\alpha I$ - closed if and only if  $f^{-1}(q\alpha \text{Icl}(V)) \subset q\text{cl}(f^{-1}(V))$  for each subset  $V$  of  $Y$ .

**Proof:** Obvious.

**Theorem 3.6.** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2, I)$  be a  $q\alpha I$ - closed mapping. If  $V$  is a subset of  $Y$  and  $U$  is a quasi open subset of  $X$  containing  $f^{-1}(V)$ , then there exists a  $q\alpha I$ - open set  $F$  of  $Y$  containing  $V$  such that  $f^{-1}(F) \subset U$ .

**Proof:** Obvious.

## References

- [1] Al-omeri W., Noorani Mohd. Salmi Md., and Al-omari A.,  $\alpha$ - local function and its properties in ideal topological spaces, Fasciculi Mathematici Nr 53., (2014), 5-15
- [2] Datta M.C., Contributions to the theory of bitopological spaces, Ph.D. Thesis, B.I.T.S. Pilani India (1971)
- [3] Hayashi E., Topologies defined by local properties, Math. Ann., 156 (1964), 205-215
- [4] Jafari S. and Rajesh N., On  $qI$  open sets in ideal bitopological spaces, University of Bacau, Faculty of Sciences, Scientific Studies and Research, Series Mathematics and Informatics, Vol. 20 (2010), No.2, 29-38
- [5] Kelly J.C., Bitopological Spaces, Proc. London Math. Soc. 13(1963)71-89
- [6] Kuratowski K., Topology, Vol. I, Academic press, New York, (1966)
- [7] Maheshwari S.N., Chae G.I., and Jain P.C., On quasi open sets, U.I.T. Report, 11 (1980) 291-292.
- [8] Maheshwari S.N. and Thakur S.S. On  $\alpha$ -continuous mapping, J. Indian Acad. Math. 7 (1985) 46-50
- [9] Njastad O., On some classes of nearly open sets, Pacific, J. Math., 15(1965), 961-970
- [10] Thakur S.S. and Paik P., Quasi  $\alpha$ - sets, J. Indian Acad. Math., 7(1985), 91-95.
- [11] Vaidyanathaswamy R., The localization theory in set topology, Proc. Indian Acad. Sci., 20(1945),51-61

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