Quasi α- Local Functions In Ideal Bitopological Spaces

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Abstract: ***In this paper we extend the concept of α- local function due to W. Al-omeri, Mohd. Salmi, Md. Noorani and A. Al-omari [1] to ideal bitopological spaces and study some of its properties. Further the concepts of qα$\mathcal{I}$- open sets and qα$\mathcal{I}$- continuous mappings are introduced and studied.***

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1. Preliminaries

In 1965 Njastad [9] introduced the concept of α- open sets in topology. A subset A of a topological space (X, τ) is said to be α- open if A ⊂ int(Cl(int(A))). Every open set is α-open but the converse may not be true. Further in 1985, Maheshwari and Thakur introduced α- continuous mapping. A function f: (X,τ) → (Y,σ) is said to be α- continuous if for every open set V in Y, $f^{-1}(V)$ is α- open in X. [8]

In 1963 Kelly [5] introduced the concept of bitopological spaces as an extension of topological spaces. A bitopological space (X, τ_1, τ_2) is a nonempty set X equipped with two topologies τ_1 and τ_2 [5]. The study of quasi open sets in bitopological spaces was initiated by Datta [2] in 1971. In a bitopological space (X, τ_1, τ_2) a set A of X is said to be quasi open [2] if it is a union of a τ_1- open set and a τ_2-open set. Complement of a quasi open set is termed quasi closed. Every τ_1- open (resp. τ_2- open) set is quasi open but the converse may not be true. Any union of quasi open sets of X is quasi open in X. The intersection of all quasi closed sets which contains A is called quasi closure of A[7]. It is denoted by qcl(A). The union of quasi open subsets of A is called quasi interior of A. It is denoted by qInt(A) [7].

In 1985, Thakur and Paik [10] introduced the concept of quasi α- open sets in bitopological spaces. A set A in a bitopological space (X, τ_1, τ_2) is called quasi α- open [10] if it is a union of a $\tau_{1\alpha}$- open set and a $\tau_{2\alpha}$- open set. Complement of a quasi α- open set is called quasi α- closed. Every $\tau_{1\alpha}$- open ($\tau_{2\alpha}$- open, quasi open) set is quasi α- open but the converse may not be true. Any union of quasi α-open sets of X is a quasi α- open set in X. The intersection of all quasi α- closed sets which contains A is called quasi α- closure of A. It is denoted by qαcl(A). The union of quasi α- open subsets of A is called quasi α- interior of A. It is denoted by qαInt(A)[10].

The concept of ideal topological spaces was initiated Kuratowski [6] and Vaidyanathaswamy [11]. An Ideal $\mathcal{I}$ on a topological space (X, τ) is a non empty collection of subsets of X which satisfies: i) A ∈ $\mathcal{I}$ and B ⊂ A ⇒ B ∈ $\mathcal{I}$ and ii) A ∈ $\mathcal{I}$ and B ∈ $\mathcal{I}$ ⇒ A∪B ∈ $\mathcal{I}$ If $\mathcal{P}$(X) is the set of all subsets of X, in a topological space (X, τ) a set operator $(.)^*$:$\mathcal{P}$(X) → $\mathcal{P}$(X) called the local function [3] of A with respect to τ and $\mathcal{I}$ and is defined as follows:
$A^*(\tau, \mathcal{I}) = \{x \in X \mid U \cap A \notin \mathcal{I}, \forall U \in \tau(x)\}$, where $\tau(x) = U \in \tau \mid x \in U\}$. Given an ideal bitopological space (X,τ_1,τ_2,$\mathcal{I}$) the quasi local function [4] of A with respect to τ_1, τ_2 and $\mathcal{I}$ denoted by A^*_q (τ_1,τ_2,$\mathcal{I}$) (in short A^*_q) is defined as follows:
$A^*_q(\tau_1,\tau_2,\mathcal{I}) = \{x \in X \mid U \cap A \notin \mathcal{I}, \forall$ quasi open set U containing x$\}$.

A subset A of an ideal bitopological space (X, τ_1, τ_2) is said to be q$\mathcal{I}$- open [4] if A ⊂ qInt(A^*_q). A mapping f: (X,τ_1,τ_2,$\mathcal{I}$) → (Y,σ_1,σ_2) is called q$\mathcal{I}$- continuous [4] if $f^{-1}(V)$ is q$\mathcal{I}$-open in X for every quasi open set V of Y.

2. Quasi α- Local Functions

Definition 2.1. Given an ideal bitopological space (X,τ_1,τ_2,$\mathcal{I}$) the quasi α- local function of A with respect to τ_1, τ_2 and $\mathcal{I}$ denoted by $A^*_{q\alpha}$ (τ_1,τ_2,$\mathcal{I}$) is defined as follows:
$A^*_{q\alpha}(\tau_1,\tau_2,\mathcal{I}) = \{x \in X \mid U \cap A \notin \mathcal{I}, \forall$ quasi α- open set U containing x$\}$.
When there is no ambiguity $A^*_{q\alpha}$ shall be written for $A^*_{q\alpha}(\tau_1,\tau_2,\mathcal{I})$.

Theorem 2.1 Let (X,τ_1, τ_2, $\mathcal{I}$) be an ideal bitopological space and A ⊂ X then:

a) $A^*_{q\alpha} \subset A_q \subset A^*(\tau_1,\mathcal{I})$ and $A^*_{q\alpha} \subset A_q \subset A^*(\tau_2, \mathcal{I})$
b) $A^*_{q\alpha} \subset A_\alpha(\tau_1,\mathcal{I})$ and $A^*_{q\alpha} \subset A_\alpha(\tau_2,\mathcal{I})$
c) $A^*_{q\alpha}(\tau_1, \tau_2, \{\emptyset\}) =$ qαcl(A)
d) $A^*_{q\alpha}(\tau_1,\tau_2, \mathcal{P}(X)) = \emptyset$
e) If A ∈ $\mathcal{I}$, then $A^*_{q\alpha} = \emptyset$
f) Neither A ⊂ $A^*_{q\alpha}$ nor $A^*_{q\alpha}$ ⊂ A

Proof: Obvious.

Theorem 2.2 Let $(X,\tau_1,\tau_2, \mathcal{I})$ be an ideal bitopological space and A, B be subsets of X then,

a) If $A \subset B$, then $A^*_{q\alpha} \subset B^*_{q\alpha}$
b) $A^*_{q\alpha} = q\alpha cl A^*_{q\alpha} \subset q\alpha cl(A)$ and $A^*_{q\alpha}$ is a quasi α-
c) closed set in (X,τ_1,τ_2)
d) $(A^*_{q\alpha})^*_{q\alpha} \subset A^*_{q\alpha}$
e) $(A\cup B)^*_{q\alpha} = A^*_{q\alpha} \cup B^*_{q\alpha}$
f) $A^*_{q\alpha} - B^*_{q\alpha} = (A - B)^*_{q\alpha} - B^*_{q\alpha} \subset (A - B)^*_{q\alpha}$
g) If $C \in \mathcal{I}$, then $(A - C)^*_{q\alpha} \subset A^*_{q\alpha} = (A\cup C)^*_{q\alpha}$

Proof: (a) Suppose $A \subset B$ and $x \notin B^*_{q\alpha}$ then there exists a quasi α- open set U containing x such that $U \cap B \in \mathcal{I}$. Since $A \subset B$, $U \cap A \in \mathcal{I}$ and so $x \notin A^*_{q\alpha}$. Hence $A^*_{q\alpha} \subset B^*_{q\alpha}$

(b) We have $A^*_{q\alpha} \subset q\alpha cl(A^*_{q\alpha})$, in general. Let $x \in q\alpha cl(A^*_{q\alpha})$, then $A^*_{q\alpha} \cap U \neq \emptyset$ for every quasi α- open set U containing x. Therefore $\exists\, y \in A^*_{q\alpha} \cap U$ and quasi α- open set U containing y. Since $y \in A^*_{q\alpha}$ and $U \cap A \notin \mathcal{I}$, therefore $x \in A^*_{q\alpha}$. Hence $q\alpha cl(A^*_{q\alpha}) \subset A^*_{q\alpha}$. Consequently, $A^*_{q\alpha} = q\alpha cl(A^*_{q\alpha})$. Again let $x \in q\alpha cl(A^*_{q\alpha}) = A^*_{q\alpha}$ Then $U \cap A \notin \mathcal{I}$ for every quasi α- open set containing x. Therefore $x \in q\alpha cl(A)$. This proves $A^*_{q\alpha} = q\alpha cl(A^*_{q\alpha}) \subset q\alpha cl(A)$

(c) Let $x \in (A^*_{q\alpha})^*_{q\alpha}$, then for every quasi α- open set U containing x, $U \cap A^*_{q\alpha} \notin \mathcal{I}$ and hence $\neq \emptyset$. Let $y \in A^*_{q\alpha} \cap U$. Then ∃ a quasi α- open set U containing y and $y \in A^*_{q\alpha}$. Hence we have $U \cap A \notin \mathcal{I}$, and $x \in A^*_{q\alpha}$. Therefore $(A^*_{q\alpha})^*_{q\alpha} \subset A^*_{q\alpha}$

(d) By (1) $A^*_{q\alpha} \cup B^*_{q\alpha} \subset (A\cup B)^*_{q\alpha}$. Let $x \in (A\cup B)^*_{q\alpha}$ then for every quasi α- open set U containing x, $(U \cap A) \cup (U \cap B) = U \cap (A\cup B) \notin \mathcal{I}$. This implies $x \in A^*_{q\alpha}$ or $x \in B^*_{q\alpha}$. Hence, $x \in A^*_{q\alpha} \cup B^*_{q\alpha}$.

(e) We have $A^*_{q\alpha} = (A - B)^*_{q\alpha} \cup (A \cap B)^*_{q\alpha}$ Thus $A^*_{q\alpha} - B^*_{q\alpha} = A^*_{q\alpha} \cap (X - B)^*_{q\alpha} = (A - B)^*_{q\alpha} \cup (A \cap B)^*_{q\alpha} \cap (X - B)^*_{q\alpha} = (A - B)^*_{q\alpha} \cap (X - B)^*_{q\alpha} \cup (A \cap B)^*_{q\alpha} \cap (X - B)^*_{q\alpha} = ((A - B)^*_{q\alpha} - B^*_{q\alpha}) \cup \emptyset \subset (A - B)^*_{q\alpha}$.

(f) Since A- C ⊂ A, by (a) $(A - C)^*_{q\alpha} \subset A^*_{q\alpha}$. From Theorem 2.2 (d) and Theorem 2.1 (e), we get $(A\cup C)^*_{q\alpha} = A^*_{q\alpha} \cup C^*_{q\alpha} = A^*_{q\alpha} \cup \emptyset = A^*_{q\alpha}$. Hence, $(A - C)^*_{q\alpha} \subset A^*_{q\alpha} = (A\cup C)^*_{q\alpha}$

Theorem 2.3. Let (X, τ_1, τ_2) be a bitopological space with Ideals $\mathcal{I}_1$ and $\mathcal{I}_2$ on X and A is a subset of X. Then:

(a) If $\mathcal{I}_1 \subset \mathcal{I}_2$, then $A^*_{q\alpha}(\mathcal{I}_2) \subset A^*_{q\alpha}(\mathcal{I}_1)$
(b) $A^*_{q\alpha}(\mathcal{I}_1 \cap \mathcal{I}_2) = A^*_{q\alpha}(\mathcal{I}_1) \cup A^*_{q\alpha}(\mathcal{I}_2)$

Proof: (a) Let $\mathcal{I}_1 \subset \mathcal{I}_2$ and $x \in A^*_{q\alpha}(\mathcal{I}_2)$, then $A \cap U \notin \mathcal{I}_2$ for every quasi α- open set U containing x. From given $A \cap U \notin \mathcal{I}_1$ hence $x \in A^*_{q\alpha}(\mathcal{I}_1)$ Therefore, we have $A^*_{q\alpha}(\mathcal{I}_2) \subset A^*_{q\alpha}(\mathcal{I}_1)$

(b) Let $x \in A^*_{q\alpha}(\mathcal{I}_1 \cap \mathcal{I}_2)$, then for every quasi α- set U containing x, $A \cap U \notin (\mathcal{I}_1 \cap \mathcal{I}_2)$, hence $A \cap U \notin \mathcal{I}_1$ and $A \cap U \notin \mathcal{I}_2$. This shows, $x \in A^*_{q\alpha}(\mathcal{I}_1)$ or $x \in A^*_{q\alpha}(\mathcal{I}_2)$ that is $x \in A^*_{q\alpha}(\mathcal{I}_1) \cup A^*_{q\alpha}(\mathcal{I}_2)$ Thus, $A^*_{q\alpha}(\mathcal{I}_1 \cap \mathcal{I}_2) \subset A^*_{q\alpha}(\mathcal{I}_1) \cup A^*_{q\alpha}(\mathcal{I}_2)$. But $A^*_{q\alpha}(\mathcal{I}_1) \cup A^*_{q\alpha}(\mathcal{I}_2) \subset A^*_{q\alpha}(\mathcal{I}_1 \cap \mathcal{I}_2)$. Therefore, $A^*_{q\alpha}(\mathcal{I}_1 \cap \mathcal{I}_2) = A^*_{q\alpha}(\mathcal{I}_1) \cup A^*_{q\alpha}(\mathcal{I}_2)$.

Definition 2.2. In an ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ the quasi *- α closure of A of X denoted by $q\alpha cl^*(A)$ is defined by $q\alpha cl^*(A) = A \cup A^*_{q\alpha}$.

Theorem 2.4. Let $(X,\tau_1,\tau_2,\mathcal{I})$ be an ideal bitopological space and A , B be the subsets of X. Then:

(a) $A \subset q\alpha cl^*(A)$
(b) $q\alpha cl^*(\emptyset) = \emptyset$ and $q\alpha cl^*(X) = X$
(c) If $A \subset B$, then $q\alpha cl^*(A) \subset q\alpha cl^*(B)$
(d) $q\alpha cl^*(A) \cup q\alpha cl^*(B) \subset q\alpha cl^*(A\cup B)$
(e) If $\mathcal{I} = \emptyset$, then $q\alpha cl^*(A) = q\alpha cl(A)$

Proof: Follows from Definition 2.2.

Definition 2.3. A subset A of an ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is said to be:

(a) $q\alpha\mathcal{I}$- open if $A \subset q\alpha Int(A^*_{q\alpha})$.
(b) $q\alpha\mathcal{I}$- closed if its complement is $q\alpha\mathcal{I}$- open.

The family of all $q\alpha\mathcal{I}$- open (respectively $q\alpha\mathcal{I}$- closed) sets of an ideal bitopological space $(X, \tau_1,\tau_2,\mathcal{I})$ is denoted by $QA\mathcal{I}O(X)$ (respectively $QA\mathcal{I}C(X)$) .
The family of all $q\alpha\mathcal{I}$- open sets of $(X,\tau_1,\tau_2,\mathcal{I})$ containing a point x is denoted by $QA\mathcal{I}O(X, x)$.

Remark 2.1. Every $q\mathcal{I}$- open set is $q\alpha\mathcal{I}$- open but the converse is not true. For,

Example 2.1. Let X = {a,b,c,d} and τ_1 = { X, ∅, {b},{a,c}, {a,b,c}}, τ_2 = {X, ∅, {d} {a,b},{a,b,d}} be topologies on X and $\mathcal{I}$ = {∅,{a}} be an ideal on X. Then the set A = {c,d} is $q\alpha\mathcal{I}$- open but not $q\mathcal{I}$- open in $(X,\tau_1,\tau_2,\mathcal{I})$.

Remark 2.2. The concepts of $q\alpha\mathcal{I}$- open sets and quasi α- open sets are independent. For, in an ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ of Example 2.1, the set {b,c} is $q\alpha\mathcal{I}$- open but not quasi α- open and the set {a,b,d} is quasi α- open but not $q\alpha\mathcal{I}$- open.

Remark 2.3. For an ideal bitopological space $(X,\tau_1, \tau_2,\mathcal{I})$ we have the following:

(a) X need not be a $q\alpha\mathcal{I}$- open set.
(b) If $\mathcal{I} = \mathcal{P}(X)$, then only the empty set is $q\alpha\mathcal{I}$- open.
(c) If $\mathcal{I} = \emptyset$, $q\alpha\mathcal{I}$- openness and quasi α- openness are equivalent.

Theorem 2.5. If A is $q\alpha\mathcal{I}$- open, then $A^*_{q\alpha} = (q\alpha Int(A^*_{q\alpha}))^*_{q\alpha}$

Proof: Since A is qα$\mathcal{I}$- open, A ⊂ qαInt($A^*_{q\alpha}$). Therefore, $A^*_{q\alpha}$ ⊂ (qαInt($A^*_{q\alpha}$))$^*_{q\alpha}$ Also we have qαInt($A^*_{q\alpha}$) ⊂ $A^*_{q\alpha}$(qαInt($A^*_{q\alpha}$))* ⊂ ($A^*_{q\alpha}$)$^*_{q\alpha}$ ⊂ ($A^*_{q\alpha}$). Hence, $A^*_{q\alpha}$ = (qαInt($A^*_{q\alpha}$))$^*_{q\alpha}$

Theorem 2.6. Any union of a family of qα$\mathcal{I}$- open sets in an ideal bitopological space (X,τ_1,τ_2,$\mathcal{I}$) is qα$\mathcal{I}$- open in X.

Proof: Let {U_∂: ∂∈Δ} be a family of qα$\mathcal{I}$- open sets of an ideal bitopological space (X,τ_1, τ_2 ,$\mathcal{I}$). Then U_∂ ⊂ qαInt((U_∂)$^*_{q\alpha}$) ∀ ∂ ∈ Δ. It follows that $\cup_{\partial \in \Delta}$ U_∂ ⊂ $\cup_{\partial \in \Delta}$ (qαInt((U_∂)$^*_{q\alpha}$))) ⊂ qαInt($\cup_{\partial \in}$ (U_∂) ⊂ qαInt($\cup_{\partial \in \Delta}$($U_\partial$)$^*_{q\alpha}$). Hence $\cup_{\partial \in \Delta}$ U_∂ is qα$\mathcal{I}$- open set in X.

Definition 2.4. Let A be a subset of an ideal bitopological space (X,τ_1,τ_2,$\mathcal{I}$) and x ∈ X. Then:

(a) x is called a qα$\mathcal{I}$- interior point of A if ∃ V∈ Q$\mathcal{A}\mathcal{I}$O(X) such that x ∈ V ⊂ A.

(b) Set of all qα$\mathcal{I}$- interior points of A denoted by qα$\mathcal{I}$Int(A)is called the qα$\mathcal{I}$- interior of A.

The following theorem summarizes the properties of qα$\mathcal{I}$- interior of subsets in ideal bitopological spaces.

Theorem 2.7. Let A , B be subsets of an ideal bitopological space (X,τ_1,τ_2,$\mathcal{I}$). Then:

(a) qα$\mathcal{I}$Int(A) = ∪{T: T ⊂ A and A ∈ Q$\mathcal{A}\mathcal{I}$O(X) }

(b) qα$\mathcal{I}$Int(A) is the largest qα$\mathcal{I}$- open subset of X contained in A.

(c) A is qα$\mathcal{I}$- open if and only if A = qα$\mathcal{I}$Int(A)

(d) qα$\mathcal{I}$Int(qα$\mathcal{I}$Int(A)) = qα$\mathcal{I}$Int(A)

(e) If A ⊂ B, then qα$\mathcal{I}$Int(A) ⊂ qα$\mathcal{I}$Int(B)

(f) qα$\mathcal{I}$ Int(A) ∪ qα$\mathcal{I}$Int(B) ⊂ qα$\mathcal{I}$Int(A ∪ B)

(g) qα$\mathcal{I}$Int(A ∩ B) ⊂ qα$\mathcal{I}$Int(A) ∩ qα$\mathcal{I}$Int(B)

Proof: **(a)** Let x ∈ ∪ {T: T ⊂ A and A ∈ Q$\mathcal{A}\mathcal{I}$O(X) }. Then, there exists T ∈ Q$\mathcal{A}\mathcal{I}$O(X, x) such that x ∈ T ⊂ A and hence x ∈ qα$\mathcal{I}$Int(A). This shows that ∪{T: T ⊂ A and A ∈ Q$\mathcal{A}\mathcal{I}$O(X) } ⊂ qα$\mathcal{I}$Int(A) . For the reverse inclusion, let x ∈ qα$\mathcal{I}$ Int(A) , then there exists T ∈ Q$\mathcal{A}\mathcal{I}$O(X, x)}, such that x ∈ T ⊂ A and we obtain x ∈ ∪{T: T ⊂ A and A ∈ Q$\mathcal{A}\mathcal{I}$O(X) }. This shows that qα$\mathcal{I}$Int(A) ⊂ {∪{T: T ⊂ A and A ∈ Q$\mathcal{A}\mathcal{I}$O(X)}}. Therefore ∪{T: T ⊂ A and A ∈ Q$\mathcal{A}\mathcal{I}$O(X) }= qα$\mathcal{I}$Int(A).

The proof of properties **(b)** - **(e)** are obvious.

(f) Clearly qα$\mathcal{I}$Int(A) ⊂ qα$\mathcal{I}$Int(A ∪ B) and qα$\mathcal{I}$Int(B) ⊂ qα$\mathcal{I}$Int(A ∪ B). Thus qα$\mathcal{I}$Int(A) ∪ qα$\mathcal{I}$Int(B) ⊂ qα$\mathcal{I}$Int(A ∪ B)

(g) Since A ∩ B ⊂ A and A ∩ B ⊂ B, by (e) we have qα$\mathcal{I}$Int(A ∩ B) ⊂ qα$\mathcal{I}$Int(A) and qα$\mathcal{I}$ Int(A ∩ B) ⊂ qα$\mathcal{I}$Int(A). Then qα$\mathcal{I}$ Int(A ∩ B) ⊂ qα$\mathcal{I}$Int(A) ∩ qα$\mathcal{I}$Int(B)

Definition 2.5. Let A be a subset of an ideal bitopological space (X,τ_1,τ_2,$\mathcal{I}$) and x ∈ X. Then:

(a) X is called a qα$\mathcal{I}$- cluster point of A, if V ∩ A ≠ ∅. for every V ∈ Q$\mathcal{A}\mathcal{I}$O(X, x)

(b) The set of all qα$\mathcal{I}$- cluster points of A denoted by qα$\mathcal{I}$Cl(A) is called the qα$\mathcal{I}$- closure of A .

The following theorem summarizes the properties of qα$\mathcal{I}$- closure of subsets in an ideal bitopological spaces.

Theorem 2.8. Let A and B be subsets of an ideal bitopological space (X,τ_1,τ_2,$\mathcal{I}$), Then:

(a) qα$\mathcal{I}$cl(A) = ∩{F: A⊂ F and F ∈ Q$\mathcal{A}\mathcal{I}$C(X)}

(b) qα$\mathcal{I}$cl(A) is the smallest qα$\mathcal{I}$- closed subset of X containing A.

(c) A is qα$\mathcal{I}$- closed if and only if A = qα$\mathcal{I}$cl(A).

(d) qα$\mathcal{I}$cl(qα$\mathcal{I}$Int(A)) = qα$\mathcal{I}$cl(A)

(e) If A ⊂ B, then qα$\mathcal{I}$cl(A) ⊂ qα$\mathcal{I}$cl(A)

(f) qα$\mathcal{I}$cl(A) ∪ qα$\mathcal{I}$cl(B) = qα$\mathcal{I}$cl(A ∪ B)

(g) qα$\mathcal{I}$cl(A ∩ B) ⊂ qα$\mathcal{I}$cl(A) ∩ qα$\mathcal{I}$cl(B)

Proof: (a) Suppose x ∉ qα$\mathcal{I}$cl(A). Then, there exists F ∈ Q$\mathcal{A}\mathcal{I}$O(X) such that F ∩ A = ∅. Since X - F is qα$\mathcal{I}$- closed set containing A and x ∉ X-F, we obtain x ∉ ∩{F: A ⊂ F and F ∈ Q$\mathcal{A}\mathcal{I}$C(X)}. For the reverse, there exists F ∈ Q$\mathcal{A}\mathcal{I}$O(X) such that A ⊂ F and x ∉ F. Since X-F is qα$\mathcal{I}$- closed set containing x, we get (X-F) ∩ A = ∅ This shows that x ∉ qα$\mathcal{I}$cl(A). Therefore qα$\mathcal{I}$cl(A) = ∩ {F: A ⊂ F and F∈ Q$\mathcal{A}\mathcal{I}\mathcal{C}$(X)}.

Statements **(b)** - **(g)** have obvious proofs.

Theorem 2.9. Let (X,τ_1,τ_2,$\mathcal{I}$) be an ideal bitopological space and A ⊂ X. Then the following properties hold:

(a) qα$\mathcal{I}$cl(X-A) = X-qα$\mathcal{I}$Int(A)

(b) qα$\mathcal{I}$Int(X-A) = X-qα$\mathcal{I}$cl(A)

Proof: **(a)** Let W be a subset of X. W ⊂ A if and only if (X-A) ⊂ (X-W), W is qα$\mathcal{I}$- open if and only if (X-W) is qα$\mathcal{I}$- closed. Thus, qα$\mathcal{I}$cl(X-A) = ∩{(X-W): W ⊂ A and W ∈ Q$\mathcal{A}\mathcal{I}$O(X)} = X - ∪{W ⊂ A and W ∈ Q$\mathcal{A}\mathcal{I}$O(X)} = (X - qα$\mathcal{I}$Int(A)).

(b) Follows from (a).

Definition 2.6. A subset B_x of an ideal bitopological space (X,τ_1,τ_2,$\mathcal{I}$) is said to be a qα$\mathcal{I}$-neighbourhood of a point x∈X if there exists a qα$\mathcal{I}$- open set U of X such that x ∈ U ⊂ B_x.

Theorem 2.10. A subset of an ideal bitopological space (X,τ_1,τ_2,$\mathcal{I}$) is qα$\mathcal{I}$- open if and only if it is a qα$\mathcal{I}$- neighbourhood of each of its points.

Proof: Necessary: Let G be a qα$\mathcal{I}$- open set of X. Then by definition, it is clear that G is a qα$\mathcal{I}$- neighbourhood of each of its points, since ∀ x ∈ G, x ∈ G ⊂ G and G is qα$\mathcal{I}$- open.

Sufficient: Suppose G is a qα$\mathcal{I}$- neighbourhood of each of its points. Then for each x ∈ G there exists S_x ∈ Q$\mathcal{A}\mathcal{I}$O(X) such that S_x ⊂ G. Therefore G = ∪{ S_x : x ∈ G}. Since each

S_x is q$\alpha\mathcal{I}$- open and arbitrary union of q$\alpha\mathcal{I}$- open sets is q$\alpha\mathcal{I}$- open, G is q$\alpha\mathcal{I}$- open in $(X,\tau_1,\tau_2,\mathcal{I})$.

3. q$\alpha\mathcal{I}$- Continuous Mappings

Definition 3.1. A mapping f: $(X,\tau_1,\tau_2,\mathcal{I}) \rightarrow (Y,\sigma_1,\sigma_2)$ is called a q$\alpha\mathcal{I}$- continuous if $f^{-1}(V)$ is a q$\alpha\mathcal{I}$- open set in X for every quasi open set V of Y .

Remark 3.1. Every q$\mathcal{I}$- continuous mapping is q$\alpha\mathcal{I}$- continuous but the converse is not true. For,

Example 3.1. Let X = {a,b,c,d} and τ_1 = {X, ∅, {b}, {a,c}, {a,b,c}}, τ_2 = {X, ∅, {d}, {a,b}, {a,b,d}} be topologies on X and $\mathcal{I}$ = {∅, {a}} be an ideal on X. σ_1= {X, ∅, {b}, {c,d}, {b,c,d}}, and σ_2 = {X, ∅} be topologies on X. Then the identity mapping f: $(X,\tau_1,\tau_2,\mathcal{I}) \rightarrow (X,\sigma_1,\sigma_2)$ is q$\alpha\mathcal{I}$- continuous but not q$\mathcal{I}$- continuous.

Theorem 3.1. Let f: $(X,\tau_1, \tau_2, \mathcal{I}) \rightarrow (Y,\sigma_1,\sigma_2)$ be a mapping. Then the following statements are equivalent:

(a) f is q$\alpha\mathcal{I}$- continuous.

(b) $f^{-1}(V)$ is q$\alpha\mathcal{I}$- closed in X for every quasi closed set V of Y.

(c) for each x ∈ X and every quasi open set V of Y containing f(x), ∃ W ∈ Q$\mathcal{AI}$O(X, x) such that f(W) ⊂ V.

(d) for each x ∈ X and every quasi open set V of Y containing f(x), $f^{-1}(V)^*_{q\alpha}$ is a q$\alpha\mathcal{I}$- neighbourhood of x.

Proof: (a) ⇔ (b). Obvious.

(a) ⇒ (c). Let x ∈ X and V be a quasi open set of Y containing f(x). Since f is q$\alpha\mathcal{I}$ continuous, $f^{-1}(V)$ is a q$\alpha\mathcal{I}$ open set. Putting W = $f^{-1}(V)$, we get f(W) ⊂ V.

(c) ⇒ (a). Let A be a quasi open set in Y. If $f^{-1}(A)$ = ∅, then $f^{-1}(A)$ is clearly a q$\alpha\mathcal{I}$- open set. Assume that $f^{-1}(A) \neq$ ∅ and x ∈ $f^{-1}(A)$, then f(x) ∈ A ⇒ ∃ a q$\alpha\mathcal{I}$- open set W containing x such that f(W) ⊂ A. Thus W ⊂ $f^{-1}(A)$. Since W is q$\alpha\mathcal{I}$- open, x ∈ W ⊂ qαInt($W^*_{q\alpha}$) ⊂ qαInt($f^{-1}(A)^*_{q\alpha}$) and so $f^{-1}(A)$ ⊂ qαInt($f^{-1}(A)^*_{q\alpha}$). Hence $f^{-1}(A)$ is a q$\alpha\mathcal{I}$- open set and therefore f: $(X,\tau_1, \tau_2, \mathcal{I}) \rightarrow (Y,\sigma_1,\sigma_2)$ is q$\alpha\mathcal{I}$- continuous.

(c) ⇒ (d). Let x ∈ X and V be a quasi open set of Y containing f(x) then ∃ a q$\alpha\mathcal{I}$- open set W containing x such that f(W) ⊂ V. It follows that W ⊂ $f^{-1}(f(W)^*_{q\alpha}))$ ⊂ $f^{-1}(V)$. Since W is a q$\alpha\mathcal{I}$- open set, x ∈ W ⊂ qαInt(W^*) ⊂ qαInt($f^{-1}(V)^*_{q\alpha}$)⊂ $f^{-1}(V)^*$. Hence $f^{-1}(V)^*_{q\alpha}$) is a q$\alpha\mathcal{I}$- neighbourhood of x.

(d) ⇒ (c). Obvious.

Definition 3.2. A mapping f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,\mathcal{I})$ is said to be :

(a) q$\alpha\mathcal{I}$- open if f(U) is a q$\alpha\mathcal{I}$- open set of Y for every quasi open set U of X.

(b) q$\alpha\mathcal{I}$- closed if f(U) is a q$\alpha\mathcal{I}$- closed set of Y for every quasi closed set U of X.

Theorem 3.2. Let f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,\mathcal{I})$ be a mapping. Then the following statements are equivalent:

(a) f is q$\alpha\mathcal{I}$- open

(b) f(qInt(U)) ⊂ q$\alpha\mathcal{I}$Int(f(U) for each subset U of X.

(c) qInt($f^{-1}(V)$) ⊂ f^{-1}(q$\alpha\mathcal{I}$Int(V)) for each subset V of Y.

Proof: (a) ⇒ (b). Let U be any subset of X. Then qInt(U) is a quasi open set of X. Then f(qInt(U)) is a q$\alpha\mathcal{I}$- open set of Y. Since f(qInt(U)) ⊂ f(U), f(qInt(U)) = q$\alpha\mathcal{I}$Int(f(qInt(U)) ⊂ q$\alpha\mathcal{I}$ Int(f(U).

(b) ⇒ (c). Let V be any subset of Y. Obviously $f^{-1}(V)$ is a subset of X. Therefore by (b), f(qInt($f^{-1}(V)$)) ⊂ q$\alpha\mathcal{I}$Int(f($f^{-1}(V)$)) ⊂ q$\alpha\mathcal{I}$Int(V)). Hence, qInt($f^{-1}(V)$) ⊂ f^{-1}(f(qInt($f^{-1}(V)$))) ⊂ f^{-1}(q$\alpha\mathcal{I}$Int(V)).

(c) ⇒ (a). Let V be any quasi open set of X. Then qInt(V) = V and f(V) is a subset of Y. So V = qInt(V) ⊂ qInt(f^{-1}(f(V))) ⊂ f^{-1}(q$\alpha\mathcal{I}$Int(f(V))). Then f(V) ⊂ f(f^{-1}(q$\alpha\mathcal{I}$Int(f(V)))) ⊂ q$\alpha\mathcal{I}$Int(f(V) and q$\alpha\mathcal{I}$Int(f(V) ⊂ f(V). Hence, f(V) is a q$\alpha\mathcal{I}$- open set of Y and f is q$\alpha\mathcal{I}$- open.

Theorem 3.3. Let f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,\mathcal{I})$ be a q$\alpha\mathcal{I}$- open mapping. If V is a subset of Y and U is a quasi closed subset of X containing $f^{-1}(V)$, then there exists a q$\alpha\mathcal{I}$- closed set F of Y containing V such that $f^{-1}(F)$ ⊂ U.

Proof: Let V be any subset of Y and U a quasi closed subset of X containing $f^{-1}(V)$, and let F = (Y - (f(X-V))). Then f(X-V) ⊂ f (f^{-1}(X-V)) ⊂ (X-V) and X-U is a quasi open set of X. Since f is q$\alpha\mathcal{I}$- open, f(X-U) is a q$\alpha\mathcal{I}$- open set of Y. Hence F is a quasi closed subset of Y and $f^{-1}(F)$ = f^{-1}(Y-(f(X-U)) ⊂ U.

Theorem 3.4. A mapping f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,\mathcal{I})$ is q$\alpha\mathcal{I}$- closed if and only if q$\alpha\mathcal{I}$cl(f(V) ⊂ f(qcl(V) for each subset V of X.

Proof: <u>Necessary:</u> Let f be a q$\alpha\mathcal{I}$- closed mapping and V be any subset of X. Then f(V) ⊂ f(qcl(V) and f(qcl(V) is a q$\alpha\mathcal{I}$- closed set of Y. Thus q$\alpha\mathcal{I}$cl(f(V)) ⊂ q$\alpha\mathcal{I}$cl(f(qcl(V)) = f(qcl(V).

<u>Sufficient:</u> Let V be a quasi closed set of X. Then by hypothesis f(V) ⊂ q$\alpha\mathcal{I}$cl(f(V)) ⊂ f(qαcl(V) = f(V) . And so, f(V) is a q$\alpha\mathcal{I}$- closed subset of Y. Hence, f is q$\alpha\mathcal{I}$- closed.

Theorem 3.5. A mapping f:$(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,\mathcal{I})$ q$\alpha\mathcal{I}$- closed if and only if f^{-1}(q$\alpha\mathcal{I}$cl(V)) ⊂ qcl($f^{-1}(V)$) for each subset V of Y.

Proof: Obvious.

Theorem 3.6. Let f:$(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,\mathcal{I})$ be a q$\alpha\mathcal{I}$- closed mapping. If V is a subset of Y and U is a quasi open subset of X containing $f^{-1}(V)$, then there exists a q$\alpha\mathcal{I}$- open set F of Y containing V such that $f^{-1}(F)$ ⊂ U.

Proof: Obvious.

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