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On Ternary Quadratic Diophantine Equation $8x^2 + 8y^2 - 15xy = 40z^2$

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Abstract: ***Five different methods of the non-zero integral solutions of the ternary quadratic homogeneous Diophantine equation $8x^2 + 8y^2 - 15xy = 40z^2$ are obtained. Some interesting relations among the special numbers and the solutions are exposed.***

Keywords: Ternary Quadratic, homogenous cone, integer solutions, special numbers.

2010 Mathematics Subject Classification: 11D09

Notations used

$t_{m,n}$ - Polygonal number of rank n with sides m. $Ct_{m,n}$ - Centered Polygonal number of rank n with sides m s_n - Star number p_n - Pronic number G_n - Gnomonic number

1. Introduction

The number theory is queen of Mathematics. In particular, the Diophantine equations have a blend of attracted interesting problems. For an extensive review of variety of problems, one may refer to [3-12]. In this work, we are observed another interesting five different methods of the non-zero integral solutions the ternary quadratic homogeneous Diophantine equation $8x^2 + 8y^2 - 15xy = 40z^2$. Further, some elegant properties among the special numbers and the solutions are observed.

2. Method of Analysis

The ternary quadratic homogeneous Diophantine equation to be solved is

$$8x^2+8y^2-15xy=40z^2 \quad (1)$$

The substitution of linear transformations

$$x = u + v \text{ and } y = u - v \ (u \neq v \neq 0) \quad (2)$$

in (1) leads to

$$u^2 + 31v^2 = 40z^2 \quad (3)$$

The above equation is solved through different methods and different patterns of integer solution to (1) are obtained.

2.1 Pattern: I

Write 40 as

$$40 = (3 + i\sqrt{31})(3 - i\sqrt{31}) \quad (4)$$

$$\text{Assume } z = a^2 + 31b^2, \text{ where } a, b > 0 \quad (5)$$

Using (4) and (5) in (3) and applying the method of factorization, define

$$(u + i\sqrt{31}\,v) = (3 + i\sqrt{31})(a + i\sqrt{31}\,b)^2 \quad (6)$$

Equating the real and imaginary parts, we have

$u = u(a, b) = 3a^2 - 93b^2 - 62ab$

$v = v(a, b) = a^2 - 31b^2 + 6ab$

Substituting the above u and v in equation (2) the value of x and y are given by

$x = x(a, b) = 4a^2 - 124b^2 - 56ab$

$$y = y(a, b) = 2a^2 - 62b^2 - 68ab \quad (7)$$

Thus (5) and (7) represents non-zero distinct integral solutions of (1) in two parameters.

Properties:-

1. $x(a, 1) - 4t_{4,a} + G_{28a} \equiv 0 \pmod{123}$
2. $z(a, a+1) - 32t_{4,a} - G_{31a} \equiv 0 \pmod{32}$
3. $y(1, b) - 62t_{4,b} + G_{34b} \equiv 0 \pmod{3}$
4. $x(a, 1) + z(a, 1) - 5t_{4,a} + G_{28a} \equiv 0 \pmod{92}$
5. $y(2, b) - x(2, b) + 186t_{4,b} + G_{124b} \equiv 0 \pmod{7}$

2.2 Pattern: II

Consider (3) as $u^2 - 9z^2 = 31(z^2 - v^2)$ (8)

Write (8) in the form of ratio as

$$\frac{u+3z}{z-v} = 31\frac{(z+v)}{u-3z} = \frac{\alpha}{\beta}, \beta \neq 0$$

This is equivalent to the following two equations

$-\alpha u + 31\beta v + z(31\beta + 3\alpha) = 0$

$\beta u + \alpha v + z(3\beta - \alpha) = 0$

On employing the method of cross multiplication, we get.

$u = -3\alpha^2 + 93\beta^2 - 62\alpha\beta$

$v = -\alpha^2 + 31\beta^2 + 6\alpha\beta$ (9)

$z = -\alpha^2 - 31\beta^2$ (10)

Substituting the values of u and v from (9) in (2) the non-zero distinct integer values of x, y are given by

$x = x(\alpha,\beta) = -4\alpha^2 + 124\beta^2 - 56\alpha\beta$

$y = y(\alpha,\beta) = -2\alpha^2 + 62\beta^2 - 68\alpha\beta$ (11)

Thus (10) and (11) represent the non- zero distinct integer solution of equation (1) in two parameters.

Properties:-

1. $z(\beta+3, \beta+3) + 32\, t_{4,\beta} + G_{96\beta} \equiv 0 \pmod{287}$
2. $y(3, 2\beta) - 248\, t_{4,\beta} + G_{204\beta} \equiv 0 \pmod{17}$
3. $x(\alpha, 1) + 4\, t_{4,\alpha} + G_{23\alpha} \equiv 0 \pmod{125}$
4. $x(2, \alpha) + y(2, \alpha) - 186 t_{4,\alpha} + 124 G_{\alpha} + 23 = 1.$
5. $x(3,2\beta) - y(2, 3\beta) + t_{150\beta} + 151 p_{\beta} + 151 t_{4,\beta} + 28 = 0$

Note:-

(8) also be expressed in the form of ratio in three different wags as follows.

(1) $\frac{(u+3z)}{31(z+v)} = \frac{z-v}{u-3z} = \frac{\alpha}{\beta}, \beta \neq 0.$

(2) $\frac{(u+3z)}{z+v} = \frac{31(z-v)}{(u-3z)} = \frac{\alpha}{\beta}, \beta \neq 0.$

(3) $\frac{(u+3z)}{31(z-v)} = \frac{z+v}{u-3z} = \frac{\alpha}{\beta}, \beta \neq 0.$

Repeating the analysis as above, we get different pattern of solution to (1).

2.3 Pattern: III

Rewrite (3) as $31v^2 = 40z^2 - u^2$ (12)
Write 31 as,
$31 = (2\sqrt{10} + 3)(2\sqrt{10} - 3)$ (13)
Let $v = 40a^2 - b^2$ (14)

Using (13) and (14) in (12) and employing the method of factorization, we write
$(2\sqrt{10}\, z + u) = (2\sqrt{10} + 3)(2\sqrt{10}\, a + b)^2$

Equating the rational and irrational parts, we have
$z = z(a, b) = 40a^2 + b^2 + 6ab$ (15) $u = u(a, b) = 120a^2 + 3b^2 + 80ab$ (16) Substituting (14) & (16) in (2), the values of x and y are
$x = x(a, b) = 160a^2 + 2b^2 + 80ab$
$y = y(a, b) = 80a^2 + 4b^2 + 80ab$
Thus (17) and (15) represent the integer solution to (1)

Properties:

1. $x(a, 1) - 160 t_{4,a} - G_{40a} \equiv 0 \pmod{3}$
2. $z(3a, 1) - 360\, t_{4,a} - G_{9a} \equiv 0 \pmod{2}$
3. $x(a, 2) - z(a, 2) - 160\, t_{4,a} - G_{80a} \equiv 0 \pmod{9}$
4. $y(a, 3) - 80\, t_{4,a} - G_{120a} \equiv 0 \pmod{37}$
5. $y(a, a) - 164\, t_{4,a} = 0$

3. Conclusion

In this paper, we have presented different patterns of integer solutions to the ternary quadratic equation $8x^2+8y^2 - 15xy = 40z^2$ representing the cone. As this Diophantine equation is rich in variety, one may attempt to find integer solutions to other choices of equations along with suitable properties.

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