On \((r^*g^*)^*\) Closed Sets in Topological Spaces

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Abstract: The aim of this paper is to introduce a new class of sets namely \((r^*g^*)^*\) closed sets in topological spaces. This class was obtained by generalizing closed sets via \(r^*g^*\) open sets which was introduced by N. Meenakumari and T. Indira[16]. This new class falls strictly between the class of closed sets and \(rg\) closed sets.

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1. Introduction

In 1970, Levine [12] introduced the concept of generalized closed set in the topological spaces. Many mathematicians started generalizing closed sets in recent years. In 1993, N. Palaniappan and K. Chandrasekhara Rao [22] introduced regular generalized closed (rg-closed) sets. M. K. R. S. Veerakumar introduced \(g^*\) closed sets [26], \(g\#\) closed sets [25] in topological spaces. The aim of this paper is to introduce a new class of sets namely \((r^*g^*)^*\) closed sets in topological spaces and study some basic properties.

2. Preliminaries

Definition 2.1: A subset \(A\) of a space \(X\) is called

1. A \(X\)-preopen set if \(\text{int}(A) \subseteq \text{int}(\text{cl}(A))\) and a \(\alpha\)-closed set if \(\text{cl}(\text{int}(A)) \subseteq A\).
2. A \(\alpha\)\(-\)open set if \(A \subseteq \text{cl}(\text{int}(A))\) and a \(\alpha\)-closed set if \(\text{int}(\text{cl}(A)) \subseteq A\).
3. A \((\beta\ast\) closed if \(\text{cl}(\text{int}(A)) \subseteq A\).
4. A \((\beta\ast\) preopen set if \(A \subseteq \text{cl}(\text{int}(A))\) and a \(\beta\ast\) \(-\)preclosed set if \(\text{int}(\text{cl}(A)) \subseteq A\).

Definition 2.2: A subset \(A\) of a space \(X\) is called

1. A regular closed if \(\text{cl}(\text{int}(A))=A\).
2. A generalized closed \((g\text{-closed})\) set if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open.
3. A regular generalized closed \((rg\text{-closed})\) set if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular open.
4. An \(\alpha\)\(-\)generalized closed \((\alpha g\text{-closed})\) set if \(\alpha\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open.
5. A semi generalized closed \((\text{briefly sg-closed})\) set if \(\text{scl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is semiopen in \(X\).
6. A generalized semi closed \((\text{briefly gs-closed})\) set if \(\text{scl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open.
7. A weekly generalized closed \((\text{briefly wg-closed})\) set if \(\text{cl}(\text{int}(A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open.
8. A generalized \(\pre\) regular \(\text{closed \text{(gpr-closed)}}\) set if \(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular open.
9. A \(g^*\) closed set if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g\)-open.
10. A regular weakly generalized semi \text{closed \text{(rwg-closed)}}\) set if \(\text{cl}(\text{int}(A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular open.
11. A \(g^{**}\) \text{closed set if } \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g^*\)-open.
12. A \(g^\#\) \text{closed set if } \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g\#\)-open.
13. A \(g^{**}\) \text{generalized semi-preclosed \text{closed \text{(gsp-closed)}}\) set if \(\text{spcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open.
14. A \(g^{**}\) \text{generalized semi-preclosed star \text{closed \text{(gsp*)}}\) closed set if \(\text{spcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g^{**}\) \text{open.}
15. A \(g^{**}\) \text{generalized \text{preclosed \text{closed \text{(gp)}}}\) set if \(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g^{**}\) \text{open.}
16. A \(g^{**}\) \text{generalized \text{semi-closed \text{(gs-closed)}}\) set if \(\text{sgcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g^{**}\) \text{open.}
17. A \(\text{regular}\) \(g^\ast\) \text{generalized closed \text{(rgg-closed)}}\) set if \(\text{gcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g^\ast\) \text{open.}
18. A \(\text{regular}\) \(g^{**}\) \text{generalized closed \text{(rgg*)}}\) set if \(\text{gcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g^{**}\) \text{open.}
19. A \(g^\#\) \text{generalized closed \text{(mgg-closed)}}\) set if \(\text{gcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g^\#\) \text{open.}
20. A \(g^{**}\) \text{generalized closed \text{(rgg**-closed)}}\) set if \(\text{rggcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g^{**}\) \text{open.}
3. Basic Properties of \((r^g^*)^\bullet\) -Closed Sets

**Definition 3.1:** A subset \(A\) of a topological space \((X, \tau)\) is called a \((r^g^*)^\bullet\)-closed set if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(r^g^*\)-open.

**Example 3.2:**
Let \(X = \{a, b, c\}\), \(\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}\)
Closed sets = \(\{\phi, X, \{b, c\}, \{a, c\}, \{c\}\}\)
\(r^g^*\)-open sets of \(X\) are \(\phi, X, \{a\}, \{b\}, \{a, b\}\)
\((r^g^*)^\bullet\) closed set are \(\{\phi, X, \{c\}, \{b, c\}, \{a, c\}\}\)

**Proposition 3.3**
Every closed set is \((r^g^*)^\bullet\)-closed.

**Proof:**
Let \(A\) be any closed set in \(X\)
To prove : \(A\) is \((r^g^*)^\bullet\)-closed set
Let \(A \subseteq U\) and \(U\) be any \(r^g^*\)-open set in \(X\)
\(A\) is closed \(\Rightarrow\) \(\text{cl}(A) = A \subseteq U\)
\(\Rightarrow\) \(\text{cl}(A) \subseteq U\)
\(\Rightarrow\) \(A\) is \((r^g^*)^\bullet\)-closed in \(X\).
Hence every closed set is \((r^g^*)^\bullet\)-closed.
The converse is not true as seen from the following example.

**Example 3.4**
Let \(X = \{a, b, c\}\), \(\tau = \{\phi, X, \{a\}, \{a, c\}\}\)
Here \(\{a, b\}\) is \((r^g^*)^\bullet\)closed but not \((r^g^*)^\bullet\)-closed.

**Proposition 3.5**
Every \(g^*\) closed set is \((r^g^* g^*\) \(\bullet\) closed set.

**Proof:**
Let \(A \subseteq U\) Where \(U\) is \(r^g^*\)open. Since every \(r^g^*\)open set is \(g^*\) open set we have \(\text{cl}(A) \subseteq U\). Therefore \(\text{cl}(A) \subseteq U\) where \(U\) is \(r^g^*\) open. Hence \(A\) is \((r^g^*)^\bullet\)closed.
The converse is not true as seen from the following example.

**Example 3.6**
Let \(X = \{a, b, c\}\), \(\tau = \{\phi, X, \{a\}, \{b, c\}\}\)
Here \(\{b\}\) is \((r^g^*)^\bullet\)closed but not \(g^*\)-closed.

**Proposition 3.7**
Every \((r^g^*)^\bullet\)-closed set is \(rg\)-closed

**Proof:**
Let \(A\) be any \((r^g^*)^\bullet\)-closed set in \(X\)
To prove : \(A\) is \(rg\)-closed set
Let \(A \subseteq U\) and \(U\) be a regular-open set in \(X\).
Since every regular open set is \(r^g^*\)open we have \(\text{cl}(A) \subseteq U\). Therefore \(\text{cl}(A) \subseteq U\) where \(U\) is \(r^g^*\) open. Hence \(A\) is \((r^g^*)^\bullet\)closed.
The converse is not true as seen from the following example.

**Example 3.8**
The converse need not be true as seen from the following example
Let \(X = \{a, b, c\}\) and \(\tau = \{X, \phi, \{a\}, \{a, b\}\}\)
Here \(\{a, b\}\) is \(rg\) closed but not \((r^g^*)^\bullet\) closed.

**Proposition 3.9**
Every \((r^g^*)^\bullet\)-closed set is \(gpr\)-closed.

**Proof:**
Let \(A \subseteq U\) and \(U\) be an regular open set in \(X\). Since every regular open set is \(r^g^*\)open we have \(\text{cl}(A) \subseteq U\). But \(\text{cl}(A) \subseteq \text{cl}(A) \subseteq U\)
\(\Rightarrow\) \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular open in \(X\) which implies \(A\) is \(gpr\)-closed.
Hence every \((r^g^*)^\bullet\) closed set is \(gpr\)-closed.

**Example 3.10**
The converse need not be true as seen from the following example.
Let \(X = \{a, b, c\}\) and \(\tau = \{X, \phi, \{a, b\}\}\)
Here \(\{a\}\) is \(gpr\) closed but not \((r^g^*)^\bullet\) closed.

**Proposition 3.11**
Every \((r^g^*)^\bullet\)-closed set is \(rg\)-closed.

**Proof:**
Let \(A \subseteq U\) and \(U\) be an regular open set in \(X\). Since every regular open set is \(r^g^*\)open we have \(\text{cl}(A) \subseteq U\).
But \(\text{cl}(\text{int}(A)) \subseteq \text{cl}(A) \subseteq U\)
\(\Rightarrow\) \(\text{cl}(\text{int}(A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular open in \(X\) which implies \(A\) is \(rg\)-closed.
Hence every \((r^g^*)^\bullet\) closed set is \(rg\)-closed.

**Example 3.12**
The converse need not be true as seen from the following example.
Let \(X = \{a, b, c\}\) and \(\tau = \{X, \phi, \{a\}\}\)
Here \(\{a\}\) is \(rg\)-closed but not \((r^g^*)^\bullet\) closed.

**Proposition 3.13**
Every \(g^**\) closed set is \((r^g^*)^\bullet\)-closed set.

**Proof:**
Let \(A \subseteq U\) and \(U\) be a regular-open set in \(X\). Since every regular-open set is \(r^g^*\)open we have \(\text{cl}(A) \subseteq U\).
Hence \(\text{cl}(A) \subseteq U\) where \(U\) is \(r^g^*\) open. Hence \(A\) is \((r^g^*)^\bullet\)closed.
The converse is not true as seen from the following example.

**Example 3.14**
The converse need not be true as seen from the following example.
Let \(X = \{a, b, c\}\) and \(\tau = \{X, \phi, \{a\}, \{a, c\}\}\)
Here \(\{a\}\) is \((r^g^*)^\bullet\) closed but not \(g^**\) closed.

**Proposition 3.15**
Every \(g^#\) closed set is \((r^g^*)^\bullet\)-closed set.

**Proof:**
Let \(A \subseteq U\) and \(U\) be an \(r^g^*\)open set in \(X\). Since every \(r^g^*\)open set is \(g^*\)open we have \(\text{cl}(A) \subseteq U\).
Hence \(\text{cl}(A) \subseteq U\) where \(U\) is \(r^g^*\)open. Hence \(A\) is \((r^g^*)^\bullet\)closed.

**Example 3.16**
The converse need not be true as seen from the following example.

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Let $X=\{a,b,c\}$, $\tau = \{X, \emptyset, \{a\}, \{b,c\}\}$

Here $\{b\}$ is $(r^g)^*$ closed but not $g^#$ closed.

**Proposition 3.17:**

Every $(gsp)^*$ closed set is $(r^g)^*$-closed.

**Proof:**

Proof follows from the definition of $(gsp)^*$ closed set and fact that $r^g$ open implies $gsp$ open.

**Example 3.18**

The converse need not be true as seen from the following example

Let $X=\{a,b,c\}$, $\tau = \{X, \emptyset, \{b\}, \{a,c\}\}$

Here $\{a,c\}$ is not $(gsp)^*$ closed but $(r^g)^*$ closed.

**Proposition 3.19:**

Every $gp^*$ closed set is $(r^g)^*$-closed.

**Proof:**

Proof follows from the definition of $(gp)^*$ closed set and fact that $r^g$ open implies $gp$ open.

**Example 3.20**

The converse need not be true as seen from the following example

Let $X=\{a,b,c\}$, $\tau = \{X, \emptyset, \{c\}, \{a,c\}\}$

Here $\{a,c\}$ is not $(gp)^*$ closed but $(r^g)^*$ closed.

**Proposition 3.21:**

Every $(r^g)^*$ closed set is $r^g$-closed.

**Proof:**

Let $A$ and $U$ be an regular open set in $X$. Since every regular open set is $r^g$ open we have $cl(A) \subseteq U$

But $gcl(A) \subseteq cl(A) \subseteq U$

whenever $A$ and $U$ is regular open in X

Which implis $A$ is $r^g$ closed.

Hence every $(r^g)^*$ closed set is $r^g$ –closed.

**Example 3.22**

The converse need not be true as seen from the following example

Let $X=\{a,b,c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a,c\}\}$

Here $\{a,b\}$ is $(r^g)^*$ closed but not $(r^g)^*$ closed.

**Proposition 3.23:**

Every $(r^g)^*$ closed set is rgb-closed.

**Proof:**

Let $A$ and $U$ be an regular open set in $X$. Since every regular open set is $r^g$ open we have $cl(A) \subseteq U$

But $bcl(A) \subseteq cl(A) \subseteq U$

whenever $A$ and $U$ is regular open in X

Which implies $A$ is $rgb$ closed.

**Example 3.24**

The converse need not be true as seen from the following example

Let $X=\{a,b,c\}$, $\tau = \{X, \emptyset, \{c\}\}$

Here $\{b\}$ is $rgb$ closed but not $(r^g)^*$ closed. Thus we have the following diagram.

Here $A \Rightarrow B$ represents $A$ implies $B$. But not conversely.

**Remark 3.25**

$(r^g)^*$ closed sets and semi closed sets are independent of each other as seen from the following examples.

**Example 3.26**

Let $X=\{a,b,c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}$. Here $\{b\}$ is semi closed but not $(r^g)^*$ closed.

**Remark 3.27**

$(r^g)^*$ closed sets and semi pre closed sets are independent of each other as seen from the following examples.
Example 3.28
Let \( X=\{a,b,c\}\), \( \tau =\{X,\emptyset ,\{a\},\{b\},\{a,b\}\}. \) Here \{a\} is semi preclosed but not \((r^g)^*\)-closed.

Example 3.29: Let \( X=\{a,b,c\}\), \( \tau =\{X,\emptyset ,\{a\},\{a,c\}\}. \) Here \{b\} is \((r^g)^*\)-closed but not semi preclosed.

Remark 3.30
The following example shows that \((r^g)^*\) closedness is independent from sg closedness, gs closedness, pre closedness and wg closedness.

Example 3.31
Let \( X=\{a,b,c\}\), \( \tau =\{X,\emptyset ,\{a\},\{a,c\}\}\). Here

(i) \{c\} is sg closed but not \((r^g)^*\)-closed. \{a\} is \((r^g)^*\)-closed but not sg closed.
(ii) \{c\} is gs closed but not \((r^g)^*\)-closed. \{a\} is \((r^g)^*\)-closed but not gs closed.
(iii) \{c\} is pre closed but not \((r^g)^*\)-closed. \{a\} is \((r^g)^*\)-closed but not pre closed.
(iv) \{c\} is wg closed but not \((r^g)^*\)-closed. \{a\} is \((r^g)^*\)-closed but not wg closed.

Remark 3.32
\((r^g)^*\) closed sets and \( \alpha \) -closed sets are independent of each other as seen from the following examples.

Example 3.33
Let \( X=\{a,b,c\}\), \( \tau =\{X,\emptyset ,\{a\},\{b\}\}\). Here \{c\} is \((r^g)^*\)-closed but not \( \alpha \) -closed.

Let \( X=\{a,b,c\}\), \( \tau =\{X,\emptyset ,\{b\}\}\). Here \{a\} is \( \alpha \) -closed but not \((r^g)^*\)-closed.

Thus we have the following diagram:

![Diagram]

Theorem 3.34
The union of two \((r^g)^*\)-closed sets is \((r^g)^*\)-closed set.

Example 3.35
Let \( A \subseteq U \) and \( B \subseteq U \) be any \( r^g^*\)-open sets containing \( A \) and \( B \) such that \( \text{cl}(A) \subseteq U \) and \( \text{cl}(B) \subseteq U \). Whenever \( A \subseteq U \) and \( B \subseteq U \) is \( r^g^*\)-open and \( \text{cl}(B) \subseteq U \), whenever \( B \subseteq U \) and \( U \) is \( r^g^*\)-open. Let \( A \cup B \subseteq U \). Then \( A \subseteq U \) and \( B \subseteq U \).

Remark 3.36
The intersection of two \((r^g)^*\) closed sets need not be \((r^g)^*\)-closed. Let \( X=\{a,b,c\}\), \( \tau =\{\emptyset ,X,\{a\}\}\). \((r^g)^*\) closed sets are \( \emptyset ,X,\{b\},\{c\},\{a,b\},\{b,c\},\{a,c\}\). The intersection of \( \{a,b\} \) and \( \{a,c\} \) is not a \((r^g)^*\) closed set.

Proposition 3.37:
If \( A \) is both \((r^g)^*\)-open and \((r^g)^*\)-closed, then \( A \) is closed.

Proof:
Let \( A \) be \( r^g^*\)-open set in \( X \) and also \((r^g)^*\)-closed set in \( X \). Since \( A \) is \( r^g^*\)-open, \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( (r^g)^*\)-open set in \( X \).

Since \( A \) is \( r^g^*\)-open. Take \( U =A \Rightarrow \text{cl}(A) \subseteq U \Rightarrow A \Rightarrow \text{cl}(A) \subseteq A \).

But \( A \subseteq \text{cl}(A) \Rightarrow \text{cl}(A) = A \). Hence \( A \) is closed.

Proposition 3.38:
If \( A \) is mildly \( g \) closed and open then \( A \) is \((r^g)^*\)-closed.

Proof:
Let \( A \) be \( r^g^*\)-open set in \( X \) and also \((r^g)^*\)-closed set in \( X \). Since \( A \) is \( r^g^*\)-open, \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( r^g^*\)-open set in \( X \).

Proposition 3.39:
If \( A \) is \((r^g)^*\)-closed set of \((X,\tau)\), then \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \((r^g)^*\)-open set in \( X \).

Proof:
Given that \( A \) is \((r^g)^*\)-closed set of \((X,\tau)\) such that \( A \subseteq B \subseteq \text{cl}(A) \), then \( B \) is also \((r^g)^*\)-closed set of \((X,\tau)\).

Example 3.33
Let \( X=\{a,b,c\}\), \( \tau =\{X,\emptyset ,\{a\}\}\). Here \( \{c\} \) is \((r^g)^*\)-closed but not \( \alpha \)-closed.

Example 3.34
Let \( X=\{a,b,c\}\), \( \tau =\{X,\emptyset ,\{a\}\}\). Here \( \{a\} \) is \( \alpha \)-closed but not \((r^g)^*\)-closed.

Thus we have the following diagram:

![Diagram]

A \( \nless \) B represents A and B are independent of each other.

Theorem 3.34
The union of two \((r^g)^*\)-closed sets is \((r^g)^*\)-closed set.

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To prove : B is \((r^g)^*\)-closed set.

Let \( B \subseteq U \). Now \( \text{cl}(B) \subseteq \text{cl}(A) \subseteq U \)

\( \Rightarrow \text{cl}(B) \subseteq U \) whenever \( B \subseteq U \) and \( U \) is \((r^g)^*\)-open.

\( \Rightarrow B \) is \((r^g)^*\)-closed.

4. \((r^g)^*\) Open Sets

Definition: 4.1 A set \( A \subseteq X \) is \((r^g)^*\) open set if its complement is \((r^g)^*\) closed.

Theorem: 4.2 A subset \( A \subseteq X \) is \((r^g)^*\) open iff there exists an \((r^g)^*\) closed set \( F \) such that \( X \subseteq \text{int}(A) \).

Proof: Let \( A \) be \((r^g)^*\) closed and \( F \subseteq A \) then \( \text{X-A} \subseteq \text{r} \wedge \text{i} \text{n} \text{t} \text{a} \text{s} \text{u} \text{c} \text{t} \text{e} \text{d} \text{a} \text{s} \text{t} \text{e} \text{m} \) \( \subseteq F \).

Since \( X \subseteq \text{int}(A) \), \( X \subseteq \text{int}(A) \).

Hence \( A \) is \((r^g)^*\) open.

Theorem 4.3: Int \( A \subseteq B \subseteq A \) and \( A \subseteq \text{int} \subseteq \text{cl} \subseteq \text{r} \wedge \text{g} \text{g} \text{e} \text{n} \text{e} \text{r} \text{i} \text{a} \text{t} \text{i} \text{d} \text{d} \text{e} \text{m} \text{s} \text{s} \text{i} \text{s} \text{u} \text{b} \text{t} \text{e} \text{m} \)

Proof: \( B \subseteq X \), \( A \subseteq X \).

Int \( A \subseteq B \) implies \( \text{X-B} \subseteq \text{X-int} \).

Then \( X \subseteq \text{A} \subseteq \text{X} \subseteq \text{X-int} \).

Since \( X \subseteq \text{A} \), \( \text{A} \subseteq \text{X} \).

Hence \( A \) is \((r^g)^*\) open.

Theorem 4.4: If \( A \subseteq B \subseteq A \) and \( A \subseteq \text{int} \subseteq \text{cl} \subseteq \text{r} \wedge \text{g} \text{g} \text{e} \text{n} \text{e} \text{r} \text{i} \text{a} \text{t} \text{i} \text{d} \text{d} \text{e} \text{m} \text{s} \text{s} \text{i} \text{s} \text{u} \text{b} \text{t} \text{e} \text{m} \)

Proof: \( B \subseteq X \), \( A \subseteq X \).

Int \( B \subseteq \text{A} \subseteq \text{X} \).

By theorem 3.39 \( \text{A-\text{B}} \subseteq \text{r} \wedge \text{g} \text{g} \text{e} \text{n} \text{e} \text{r} \text{i} \text{a} \text{t} \text{i} \text{d} \text{d} \text{e} \text{m} \text{s} \text{s} \text{i} \text{s} \text{u} \text{b} \text{t} \text{e} \text{m} \).

Hence \( A \) is \((r^g)^*\) open.

Theorem 4.5: If \( A \subseteq B \subseteq A \) and \( A \subseteq \text{int} \subseteq \text{cl} \subseteq \text{r} \wedge \text{g} \text{g} \text{e} \text{n} \text{e} \text{r} \text{i} \text{a} \text{t} \text{i} \text{d} \text{d} \text{e} \text{m} \text{s} \text{s} \text{i} \text{s} \text{u} \text{b} \text{t} \text{e} \text{m} \)

Proof: \( B \subseteq X \), \( A \subseteq X \).

Int \( B \subseteq \text{A} \subseteq \text{X} \).

By theorem 3.39 \( \text{A-\text{B}} \subseteq \text{r} \wedge \text{g} \text{g} \text{e} \text{n} \text{e} \text{r} \text{i} \text{a} \text{t} \text{i} \text{d} \text{d} \text{e} \text{m} \text{s} \text{s} \text{i} \text{s} \text{u} \text{b} \text{t} \text{e} \text{m} \).

Hence \( A \) is \((r^g)^*\) open.

5. Conclusion

In this paper we have introduced \((r^g)^*\) closed sets and \((r^g)^*\) open sets and studied some properties. This class of sets can be used to discuss the notion of Continuity, Compactness and connectedness and also can be extended to other topological spaces like Fuzzy & Bitopological Spaces.

