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Heating Two-Layer System with Pulsed Laser

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Abstract: ***Laser heating of two layer system is studied. The front layer is a homogeneous thin film having constant optical surface absorptance "A_f". This layer is of finite thickness. It is deposited on a thick substrate. The temperature profiles within the front layer and the substrate are obtained. Laplace integral transform technique is applied to solve the related heat diffusion equation of parabolic type. Computations for silicon-glass two layer system are given as an illustrative example.***

Keywords: Pulsed laser heating, Laplace integral transform, heating Silicon-Glass two layer systems, constant optical absorptance, laser pulse parameters effect.

1. Introduction

Laser-solid interaction is a serious problem that has aroused the interest of many investigators [1-28]. It has different industrial applications, especially in the fields of material processing, such as spot welding, scribing, drilling of holes, laser cutting and laser glazing [1-5].

Pulsed lasers have been successfully applied for annealing of radiation damage associated with ion implantation in "Si" during device fabrication [9].

The pulse shape is important in determining the resulting thermal response of the target. In the semiconductor industry laser is used for local diffusion and alloying to form p-n junctions. Different theoretical models are made to study the laser heating problem.

Laser pulses of moderate power are used in a variety of fields, for example, to alter thin films on low-conductivity substrates especially for information recording [2-6].

The laser source function is assumed either constant (CW) laser or pulsed laser.

In the present trial, laser thermal response of a two-layer system is studied. The two-layer system is composed of a thin film of finite thickness "d" is deposited on a thick substrate.

Both layers are in perfect thermal contact. The system is subjected to laser source. The pulse shape is published elsewhere [25]. Laplace integral transform is applied to solve the heating problem and to get the thermal fields within the film layer and within the substrate. These functions are obtained in terms of the laser pulse parameters.

This makes it possible to determine the thermal penetration depth in the target and to determine the time required to initiate melting in the front layer. This in turn is important in studying the dynamics of pulsed laser interaction.

Computations on Silicon-Glass two-layer system is made and is given as an illustrative example.

2. Mathematical Formulation of the Problem:

In setting up the problem, it is assumed that laser $q_o(t)$ is incident on the front surface of the system in a normal direction (Fig. 1). The received power is partly absorbed and partly reflected. The absorbed part is $(A_f\ q_0\ (t))$ where A_f is the absorptance of the thin film front surface. The thickness of the thin film is "d" microns. Two coincident axes x and z = (x-d), along the direction of the incident laser beam are considered.

The boundary z = 0 represents the interface between the two layers [19]. Thermal losses arising from any mode other than convection are neglected. The physical parameters together with the optical absorptance "A_f" are assumed to be temperature independent.

The heat flow is considered one-dimensional [1, 19], Laplace integral transform technique is used to obtain the required solutions.

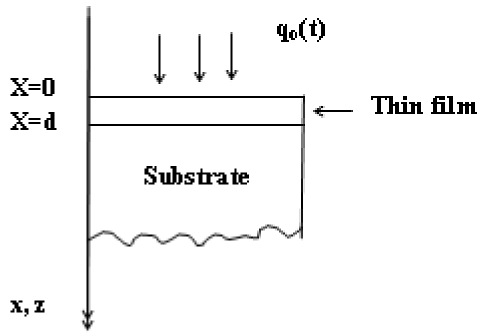

Figure 1: Schematic view of the considered model.

The heat diffusion equation in the film layer can be written in the form:

$$\frac{\partial \theta_f(x,t)}{\partial x} = \alpha_f \frac{\partial^2 \theta_f(x,t)}{\partial x^2}, \quad t_m > t > 0, \quad 0 \le x \le d \qquad (2\text{-}1)$$

Where, $\alpha = \lambda/\rho c_p$, is the thermal diffusivity m^2/s, t_m, is the time required to initiate melting, $\theta = (T-T_o)$ is the excess temperature relative to the ambient temperature T_o.

For the substrate layer.

$$\frac{\partial \theta_\rho(z,t)}{\partial t} = \alpha_\rho \frac{\partial^2 \theta_\rho(z,t)}{\partial z^2}, \quad o \le z \le \infty, \ t < t_m \qquad (2\text{-}2)$$

λ, W/m.K is the thermal conductivity,
ρ, kg/m^3 is the density,
C_p, (J/kg.K) is the specific heat.

The system of equation 1 and 2 is subjected to the following initial and boundary conditions:

$$\begin{array}{lll} \text{i)} & \theta_f(x,o) = 0 & \\ \text{ii)} & \theta_\rho(z,o) = 0 & (2\text{-}3) \\ \text{iii)} & \theta_\rho(\infty,t) = 0 & \end{array}$$

The condition at the front surface x = 0

$$\lambda_f \left.\frac{\partial \theta_f(x,t)}{\partial x}\right|_{x=0} = A_f\ (T) q_o(t) - h_o\, \theta_f(o,t) \qquad (2\text{-}4)$$

Where "h_o" is the heat transfer coefficient at the front surface of the system.

The source laser function is suggested by the author elsewhere [25] in the form:

$$q(t) = q_{\max}\left(\frac{t_d - t}{t_d - t_o}\right)^m \left(\frac{t}{t_o}\right), \quad m = \left(\frac{t_d}{t_o} - 1\right) \qquad (2\text{-}5)$$

Where q_{max}, is the maximum power density W/m2 of the pulse.

t_d is the time duration of the pulse,
t_o is the time interval after which the power density reaches its maximum value q_{max}.

The condition of continuity of the interface x = d,

$$\theta_f\ (d,t) = \theta_\rho\,(z{=}o,t) \qquad (2\text{-}6)$$

$$-\lambda_f \left.\frac{\partial \theta_f(x,t)}{\partial x}\right|_{x=d} = -\lambda_\rho \left.\frac{\partial \theta_\rho(z,t)}{\partial z}\right|_{z=(x-d)=0} \qquad (2\text{-}7)$$

Together with the integrated heat balance equation:

$$\int_o^t A_f\, q_o(t)dt = \int_o^d \lambda_f\ C_{p_f}\, \theta_f(x,t)dx + \int_o^\infty \rho_\rho C_{p_\rho}\, \theta_\rho(z,t)dz + \int_o^t h_o\, \theta_f(o,t)dt \qquad (2\text{-}8)$$

Taking Laplace transform with respect to the time variable for both equations (1) and (2) one gets:

$$s\bar\theta_f(x,s) - \theta_f(x,o) = \alpha_f \frac{\partial^2 \theta_f(x,s)}{\partial x^2} = o \qquad (2\text{-}9)$$

and

$$s\bar\theta_\rho(z,s) - \theta_\rho(z,o) = \alpha_\rho \frac{\partial^2 \theta_\rho(z,s)}{\partial z^2} \qquad (10)$$

Where $\bar\theta$ is the Laplace transform of θ, s is the variable in Laplace domain.

One has also to transform the boundary and initial conditions to Laplace domain, this gives:

$$\begin{array}{ll} \text{a)} & \bar\theta_f(x,o) = 0 \\ \text{b)} & \bar\theta_\rho(z,o) = 0 \\ \text{c)} & \bar\theta_\rho(\infty,s) = 0 \\ \text{d)} & \bar\theta_\rho(\infty,o) = 0 \end{array} \qquad (11)$$

Equation (2-4) is transformed in the form:

$$-\lambda_f \left.\frac{\partial \bar\theta_f(x,s)}{\partial x}\right|_{x=0} = A_f G(s) - h_o \bar\theta_f(o,s) \qquad (12)$$

Where $\bar G(s)$ is the Laplace transform of the source function $q_o(t)$,

Equation (6) is transformed in the form:

$$\bar\theta_f\,(d,s) = \bar\theta_\rho\,(o,s) \qquad (13)$$

Equation (7) is transformed as:

$$-\lambda_f \left.\frac{\partial \bar\theta_f(x,s)}{\partial x}\right|_{x=d} = -\lambda_\rho \left.\frac{\partial \bar\theta_\rho(z,s)}{\partial z}\right|_{z=0} \qquad (14)$$

The heat balance equation (2-8) is transformed in the form:

$$\frac{A_f G(s)}{s} = \int_0^d \rho_f C_{p_f} \bar\theta_f(x,s)dx + \int_0^\infty \rho_s C_{p_s} \bar\theta_s(z,s)dz + \frac{h_o \bar\theta_f(0,s)}{s} \qquad (15)$$

Considering equation (11), one can write equations (9) and (10) in the form:

$$s\bar\theta_f(x,s) - \alpha_f \frac{\partial^2 \bar\theta_f(x,s)}{\partial x^2} = 0 \qquad (16)$$

and

$$s\bar\theta_\rho(z,s) - \alpha_\rho \frac{\partial^2 \bar\theta_\rho(z,s)}{\partial z^2} = 0 \qquad (17)$$

The solution of both equations (16) and (17) can be written in the form:

$$\bar{\theta}_f(x,s) = C_1 \exp\left(-\sqrt{\frac{s}{\alpha_f}}\, x\right) + C_2 \exp\left(-\sqrt{\frac{s}{\alpha_f}}\, x\right) \tag{18}$$

and

$$\bar{\theta}_\rho(z,s) = C_3 \exp\left(-\sqrt{\frac{s}{\alpha_\rho}}\, z\right) + C_4 \exp\left(+\sqrt{\frac{s}{\alpha_\rho}}\, z\right) \tag{19}$$

To get finite solution at $z = \infty$, one must put $C_4 = 0$.

Thus, the solution can be rewritten in the form:

$$\bar{\theta}_\rho(z,s) = C_3 \exp\left(-\sqrt{\frac{s}{\alpha_\rho}}\, z\right) \tag{20}$$

Substituting solutions (18) and (20) into equations (12), (13), (14) and (15) one gets the following system of equations:

$$\left(h_o - \lambda_f \sqrt{\frac{s}{\alpha_f}}\right) C_1 + \left(h_o + \lambda_f \sqrt{\frac{s}{\alpha_f}}\right) C_2 = A_f G(s) \tag{21}$$

$$\exp\left(\sqrt{\frac{s}{\alpha_f}}\, d\right) C_1 + \exp\left(-\sqrt{\frac{s}{\alpha_f}}\, d\right) C_2 - C_3 = 0 \tag{22}$$

$$-\lambda_f \sqrt{\frac{s}{\alpha_f}} \exp\left(\sqrt{\frac{s}{\alpha_f}}\, d\right) C_1 + \lambda_f \sqrt{\frac{s}{\alpha_f}} \exp\left(\sqrt{\frac{s}{\alpha_f}}\, d\right) C_2 - \lambda p \sqrt{\frac{s}{\alpha_p}}\, C_3 = 0 \tag{23}$$

$$\left[\rho_f C_{p_f} \sqrt{\frac{\alpha_f}{s}} \exp\left(\sqrt{\frac{s}{\alpha_f}}\, d\right) - \rho_f C_{p_f} \sqrt{\frac{\alpha_f}{s}} + \frac{h_o}{s}\right] C_1 + \left[-\rho_f C_{p_f} \sqrt{\frac{\alpha_f}{s}} \exp\left(-\sqrt{\frac{s}{\alpha_f}}\, d\right) + \rho_f C_{p_f} \sqrt{\frac{\alpha_f}{s}} + \frac{h_o}{s}\right] C_2 + \rho_\rho C_{p_\rho} \sqrt{\frac{\alpha_\rho}{s}}\, C_3 = \frac{A_f G(s)}{s} \tag{24}$$

Solving this system of equations for C_1, C_2 and C_3 one gets:

$$C_1 = A_f G(s) \frac{\lambda_f \sqrt{\frac{s}{\alpha_f}} \exp\left(-\sqrt{\frac{s}{\alpha_f}}\, d\right) - \lambda_s \sqrt{\frac{s}{\alpha_s}} \exp\left(-\sqrt{\frac{s}{\alpha_f}}\, d\right)}{\left(\lambda_s \sqrt{\frac{s}{\alpha_s}} h_o + \lambda_f^2 \frac{s}{\alpha_f}\right) \sinh\left(\sqrt{\frac{s}{\alpha_f}}\, d\right) + \left(\lambda_p \lambda_f \sqrt{\frac{s}{\alpha_f}} \sqrt{\frac{s}{\alpha_s}} + h_o \lambda_f \sqrt{\frac{s}{\alpha_f}}\right) Cosh\left(\sqrt{\frac{s}{\alpha_f}}\, d\right)} \tag{25}$$

$$C_2 = A_f G(s) \frac{\lambda_f \sqrt{\frac{s}{\alpha_f}} \exp\left(-\sqrt{\frac{s}{\alpha_f}}\, d\right) + \lambda p \sqrt{\frac{s}{\alpha_s}} \exp\left(\sqrt{\frac{s}{\alpha_f}}\, d\right)}{\left(\lambda_s \sqrt{\frac{s}{\alpha_s}} h_o + \lambda_f^2 \frac{s}{\alpha_f}\right) \sinh\left(\sqrt{\frac{s}{\alpha_f}}\, d\right) + \left(\lambda_p \lambda_f \sqrt{\frac{s}{\alpha_f}} \sqrt{\frac{s}{\alpha_p}} + h_o \lambda_f \sqrt{\frac{s}{\alpha_f}}\right) Cosh\left(\sqrt{\frac{s}{\alpha_f}}\, d\right)} \tag{26}$$

$$C_3 = A_f G(s) \frac{2\lambda_f \sqrt{\frac{s}{\alpha_f}}}{\left(\lambda_p \sqrt{\frac{s}{\alpha_p}} h_o + \lambda_f^2 \frac{s}{\alpha_f}\right) \sinh\left(\sqrt{\frac{s}{\alpha_f}}\, d\right) + \left(\lambda p \lambda_f \sqrt{\frac{s}{\alpha_f}} \sqrt{\frac{s}{\alpha_p}} + h_o \lambda_f \sqrt{\frac{s}{\alpha_f}}\right) Cosh\left(\sqrt{\frac{s}{\alpha_f}}\, d\right)} \tag{27}$$

Substituting for C_1, C_2 and C_3 in the solutions expressed by equations (18) and (20), and rearranging the obtained terms one can finally write the solutions in the form:

$$\bar{\theta}_f(x,s)=A_f G(s)\left[\left\{\exp\left(-\sqrt{\frac{s}{\alpha_f}}x\right)+\exp\left(-\sqrt{\frac{s}{\alpha_f}}(2d-x)\right)\right\}+\frac{\lambda_\rho}{\lambda_f}\sqrt{\frac{\alpha_f}{\alpha_\rho}}\right.$$
$$\left.\left[\exp\left(-\sqrt{\frac{s}{\alpha_f}}x\right)-\exp\left(-\sqrt{\frac{s}{\alpha_f}}(2d-x)\right)\right]\right]\div\left[\left\{\frac{\lambda_\rho h_o}{\lambda_f}\sqrt{\frac{\alpha_f}{\alpha_\rho}}+\lambda_\rho\sqrt{\frac{s}{\alpha_\rho}}+\lambda_f\sqrt{\frac{s}{\alpha_f}}+h_o\right\}\right.$$
$$\left.\left\{1-\gamma\ \exp\left(-2\sqrt{\frac{s}{\alpha_f}}d\right)\right\}\right] \tag{28}$$

and

$$\bar{\theta}_s(z,s)=A_f G(s)\left\{4\exp\left(-\left(\frac{z}{\sqrt{\alpha_p}}+\frac{d}{\sqrt{\alpha_f}}\right)\sqrt{s}\right)\right\}\div\left\{\left(\frac{\lambda_p h_o}{\lambda_f}\sqrt{\frac{\alpha_f}{\alpha_p}}+\lambda_p\sqrt{\frac{s}{\alpha_p}}+\lambda_f\sqrt{\frac{s}{\alpha_f}}+h_o\right)\right..$$
$$\left.\left[1-\gamma\exp\left(-2\sqrt{\frac{s}{\alpha_f}}d\right)\right]\right\} \tag{29}$$

Where

$$\gamma=\frac{\left(\frac{\lambda_p h_o}{\lambda_f}\sqrt{\frac{\alpha_f}{\alpha_\rho}}+\lambda_f\sqrt{\frac{s}{\alpha_f}}\right)-\left(\lambda_\rho\sqrt{\frac{s}{\alpha_\rho}}+h_o\right)}{\left(\frac{\lambda_\rho h_o}{\lambda_f}\sqrt{\frac{\alpha_f}{\alpha_\rho}}+\lambda_f\sqrt{\frac{s}{\alpha_f}}\right)+\left(\lambda_\rho\sqrt{\frac{s}{\alpha_\rho}}+h_o\right)} \tag{30}$$

Equation (30) indicates that:

$$0\leq\gamma\leq 1 \tag{31}$$

Moreover, discussing the order of magnitudes of the different terms in the expression (eq. 30) for γ, one can conclude that :

$$\gamma\cong 1 \tag{32}$$

$$Since\ \frac{1}{1-a}=\sum_{n=o}^{\infty}a^n,\ \ |a|<1\qquad [29] \tag{33}$$

One can write

$$\frac{1}{1-\gamma\ \exp\left(-2\sqrt{\frac{s}{\alpha_f}}d\right)}=\sum_{n=o}^{\infty}\gamma^n\exp\left(-2n\sqrt{\frac{s}{\alpha_f}}d\right) \tag{34}$$

Substituting equations (34) into equations (28) and (29) one can write the solutions (28) and (29) in the following form:

$$\bar{\theta}_f(x,s)=\frac{A_f G(s)\sqrt{\alpha_f\alpha_\rho}}{\lambda_f\sqrt{\alpha_\rho}+\lambda_\rho\sqrt{\alpha_f}}\left\{\left[\exp\left(-\sqrt{\frac{s}{\alpha_f}}x\right)+\exp\left(-\sqrt{\frac{s}{\alpha_f}}(2d-x)\right)\right]+\frac{\lambda_\rho}{\lambda_f}\sqrt{\frac{\alpha_f}{\alpha_\rho}}\right.$$
$$\left.\left[\exp\left(-\sqrt{\frac{s}{\alpha_f}}x\right)-\exp\left(-\sqrt{\frac{s}{\alpha_f}}(2d-x)\right)\right]\right\}\div\left\{\left(\sqrt{s}+\frac{h_o\sqrt{\alpha_f}}{\lambda_f}\right)\sum_{n=0}^{\infty}\gamma^n\exp\left(-2n\sqrt{\frac{s}{\alpha_f}}d\right)\right. \tag{35}$$

$$\bar{\theta}_\rho(z,s)=\frac{A_f G(s)\sqrt{\alpha_f\alpha_p}}{\lambda_f\sqrt{\alpha_\rho}+\lambda_\rho\sqrt{\alpha_f}}\left\{4\exp\left(-\left(\sqrt{\frac{z}{\alpha_\rho}}+\frac{d}{\sqrt{\alpha_f}}\right)\sqrt{s}\right)\right\}\div \left\{\left(\sqrt{s}+\frac{h_o\sqrt{\alpha_f}}{\lambda_f}\right)\sum_{n=0}^{\infty}\gamma^n\exp\left(-2n\sqrt{\frac{s}{\alpha_f}}d\right)\right\} \quad (36)$$

For γ = 1, then solutions (35) and (36) can be rewritten in the form:

$$\bar{\theta}_f(x,s)=\frac{A_f G(s)\sqrt{\alpha_f\alpha_p}}{\lambda_f\sqrt{\alpha_\rho}+\lambda_\rho\sqrt{\alpha_f}}\left\{\sum_{n=0}^{\infty}\left[\exp\left(-\sqrt{\frac{s}{\alpha_f}}(2nd+x)\right)+\exp\left(\sqrt{\frac{s}{\alpha_f}}((2n+2)d-x)\right)+\frac{\lambda_p}{\lambda_f}\sqrt{\frac{\alpha_f}{\alpha_\rho}}\left(\exp\left(-\sqrt{\frac{s}{\alpha_f}}((2nd+x))-\exp\sqrt{\frac{s}{\alpha_f}}((2n+2)d-x)\right)\right)\right]\right\}\div\left\{\left(\sqrt{s}+\frac{h_o\sqrt{\alpha_f}}{\lambda_f}\right)\right\} \quad (37)$$

$$\bar{\theta}_s(z,s)=\frac{A_f G(s)\sqrt{\alpha_f\alpha_\rho}}{\lambda_f\sqrt{\alpha_\rho}+\lambda_\rho\sqrt{\alpha_f}}\frac{\sum_{n=0}^{\infty}4\exp\left(-\left(\frac{2nd+d}{\sqrt{\alpha_f}}+\frac{z}{\sqrt{\alpha_\rho}}\right)\sqrt{s}\right)}{\sqrt{s}+\frac{h_o\sqrt{\alpha_f}}{\lambda_f}} \quad (38)$$

To get the inverse Laplace transform one has to apply the convolution theorem [29].

$$L^{-1}\{f_1(s)\ f_2(s)\}=\int_0^t F_1(u)\ F_2(t-u)du \quad (39)$$

Using the standard tables [29-32] one gets the inverse Laplace transform of equations (37) and (38) in the form:

$$\theta_f(x,t)=\frac{A_f q_m(s)\sqrt{\alpha_f\alpha_\rho}}{\left(\lambda_f\sqrt{\alpha_\rho}+\lambda_\rho\sqrt{\alpha_f}\right)(t_d-t_0)^m\ t_0}\left[\int_0^t\{(t_d-u)^m u\sum_{n=0}^{\infty}\left(\frac{\exp\left(-\frac{(2nd+x)^2}{4\alpha_f t}\right)}{\sqrt{\pi(t-u)}}-\frac{h_o\sqrt{\alpha_f}}{\lambda_f}\exp\left(\frac{h_o(2nd+x)}{\lambda_f}\right)\exp\left(\frac{h_o{}^2\alpha_f}{\lambda_f^2}(t-u)\right)erfc\left(\frac{h_o\sqrt{\alpha_f(t-u)}}{\lambda_f}+\frac{2nd+x}{\sqrt{4\alpha_f(t-u)}}\right)+\frac{\exp\left(-\frac{((2n+2)d-x)^2}{4\alpha_f(t-u)}\right)}{\sqrt{\pi(t-u)}}-\left(\frac{h_o\sqrt{\alpha_f}}{\lambda_f}\right)\exp\left(\frac{h_o((2n+2)d-x)}{\lambda_f}\right)\exp\left(\frac{h_o{}^2\alpha_f}{\lambda_f^2}(t-u)\right)erfc\left(\frac{h_o\sqrt{\alpha_f(t-u)}}{\lambda_f^2}+\frac{(2n+2)d-x}{\sqrt{4\alpha_f(t-u)}}\right)\right)+\frac{\lambda_\rho}{\lambda_f}\sqrt{\frac{\alpha_f}{\alpha_\rho}}\sum_{n=0}^{\infty}\left(\frac{\exp\left(-\frac{(2nd+x)^2}{4\alpha_f(t-u)}\right)}{\sqrt{\pi(t-u)}}-\frac{h_o\sqrt{\alpha_f}}{\lambda_f}\exp\left(\frac{h_o(2nd+x)}{\lambda_f}\right)\exp\left(\frac{h_o{}^2\alpha_f}{\lambda_f^2}t\right)erfc\left(\frac{h_o\sqrt{\alpha_f(t-u)}}{\lambda_f}+\frac{2nd+x}{\sqrt{4\alpha_f(t-u)}}\right)-\frac{\exp\left(\frac{((2n+2)d-x)^2}{4\alpha_f(t-u)}\right)}{\sqrt{\pi(t-u)}}+\frac{h_o\sqrt{\alpha_f}}{\lambda_f}\exp\left(\frac{h_o((2n+2)d-x)}{\lambda_f}\right)\exp\left(\frac{h_o{}^2\alpha_f}{\lambda_f^2}(t-u)\right)erfc\left(\frac{h_o\sqrt{\alpha_f(t-u)}}{\lambda_f^2}+\frac{(2n+2)d-x}{\sqrt{4\alpha_f(t-u)}}\right)\right)\}du\right] \quad (40)$$

$$\theta_\rho(z,t)=\frac{4A_f q_m\sqrt{\alpha_f\alpha_\rho}}{\left(\lambda_f\sqrt{\alpha_\rho}+\lambda_\rho\sqrt{\alpha_f}\right)\left(t_d-t_0\right)m\,t_0}\left[\int_0^t\left\{(t_d-u)^m u\sum_{n=0}^{\infty}\left(\frac{\exp\left(-\left(\frac{(2nd+d)^2}{4\alpha_f(t-u)}+\frac{z^2}{4\alpha_\rho(t-u)}\right)\right)}{\sqrt{\pi(t-u)}}\right.\right.\right.$$

$$-\frac{h_o\sqrt{\alpha_f}}{\lambda_f}\exp\left(\frac{\left(\frac{h_o(2nd+d)^2}{4\alpha_f(t-u)}\right)+\frac{z^2}{4\alpha_\rho(t-u)}}{\lambda_f}\right)-\exp\left(\frac{h_o{}^2\alpha_f}{\lambda_f^2}(t-u)\right)$$

$$\left.\left.\left.erfc\left(\frac{h_o\sqrt{\alpha_f t}}{\lambda_f}+\frac{2nd+d}{\sqrt{4\alpha_f(t-u)}}+\frac{z}{4\alpha_\rho(t-u)}\right)\right)\right\}du\right] \tag{41}$$

Equations (40) and (41) show the temperature profiles in the thin film layer and in the substrate of the chosen two layer target.

3. Computations

As illustrative example the temperature fields are commuted for the following two-layer system

i. Silicon – glass

The physical properties of the considered materials are given in Table 1 [32-34]

Table 1: The physical and optical properties of the considered materials [32-34]

Material	ρ (kgm^{-3})	λ (Wm^{-1}K^{-1})	α (m^{2}s^{-1})	C_p (Jkg^{-1}K^{-1})	A_1	A 2 K^{-1}	Tm,, K
Silicon	2.328x10^3	1.5x10^2	9.2x10^{-5}	700	0.678	3.12x10^{-5}	1688
Glass	2707	0.75	0.035x10^{-5}	0.8x10^3	-	-	-

The laser pulse is characterized by the following set of parameters:

$$q_{max}=1.25\ x10^9\ W/m^2,$$

$$t_d=40\,\mu s,\ to=10\,\mu s,\ and\ m=3$$

The heat transfer of the front surface

$$h_o=1000\,W/m^2K$$

The thickness of the thin film = d = 10 microns.

The function $\theta_f\ (0,t)\,and\ \theta_f\ (x,t=34\mu s)$, are computed for the system Silicon-glass. The obtained results are illustrated graphically in figures (1) and (2) respectively.

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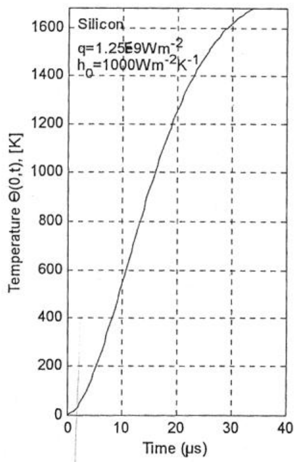

Fig. 2 : The variation of the temperature of the front surface as a function of the exposure times for the two-layer system (silicon-glass)

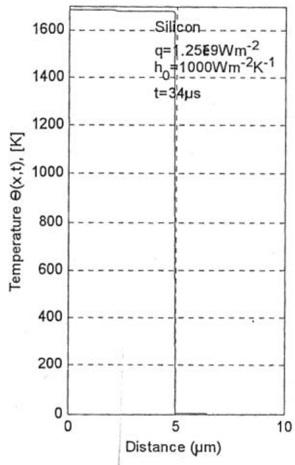

Fig. 3: The variation of the temperature of the thin film of silicon with the distance at t = 34μs. For silicon-glass two layer system.

4. Results

The obtained results reveal that:

1) The critical time required to initiate melting in silicon for the considered operation condition is $t_m = 34$ µs.
2) The thermal penetration depth in the front Silicon layer after exposure interval of time t = 34 µs for the two layer Silicon – glass system in 5 µm for the considered operating conditions.
3) The obtained mathematical expressions for the thermal profiles in the thin film front layer (eq. 40) and that for the substrate layer (eq. 41) reveal clearly that :
   a) They depend linearly on the maximum value q_{max} of the laser source power density.
   b) The dependence on the cooling conditions (h_a, W/m^2K) is not linear.
   c) Such profiles depend on the value of the absorption coefficient at the front surface (A_f) of the irradiated system such dependence is linear for A_f constant.
   d) These profiles depend principally on the general shape of the heating laser source pulse, through the parameter "m"
   e) The dependence on the pulse duration td and the time "to" required to reach q_{max} is revealed through the factor $(t_d - t_0)^m$. Small values of such factor give higher values for the temperature of the heated target (see equations (40) and (41).

5. Conclusions

1) The thermal response of the laser irradiated target is a function of the pulse shape and the operating conditions.
2) The dependence of the temperature field on the maximum power density q_{max} of the laser pulse is linear.
3) The dependence of the temperature field within the heated target on the absorption coefficient of constant value is linear.
4) The dependence of the temperature field within the irradiated target on the cooling conditions is not linear.

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NOMECLATURE

A_f Optical surface absorptance of the thin film front layer, dimensionless.
C_p Specific heat [J kg^{-1} K^{-1}].
d Thickness of the thin film [m].
h_o Heat transfer coefficient at the front surface [Wm^{-2}K^{-1}].
m A fitting parameter defined in the text, dimensionless.
q_o Laser irradiance [Wm^{-2}].
S Laplace transform variable that stands for the time variable t.
T The temperature in degrees Kelvins
T_m The melting temperature [K].
t Time variable [s]
t_m Time interval required to initiate damage [s].
t_o The time interval required to reach q_{max}.
x,z Spatial variables [m].

Greek Symbols:

α Thermal diffusivity [m^{2}S^{-1}]
λ Thermal conductivity [Wm^{-1}K^{-1}]
ρ Density of the material [kg m^{-3}]
θ Excess temperature relative to the ambient temperature ($T-T_0$). [K].

Subscripts:
f Related to the film layer.
p related to the substrate layer