Regular Interval Valued Fuzzy Soft Matrices

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Abstract: In this paper, we define regular interval valued fuzzy soft matrices as a generalization of regular fuzzy soft matrices and obtain the structure of Row space, column space of an Interval valued fuzzy soft matrices (IVFSM).

Keywords: Soft set, fuzzy soft set, Interval valued fuzzy soft matrix, Interval valued fuzzy soft set, Interval valued fuzzy soft Matrix, Regular Interval valued fuzzy soft Matrix.

1. Introduction

We deal with interval-valued fuzzy matrices (IVFM) that is, matrices whose entries are intervals and all the intervals are subintervals of the interval [0,1]. Recently the concept of IVFM a generalization of fuzzy matrix was introduced and developed by shymal and pal[5], by extending the max. min operations on fuzzy algebra F=0,1, for elements, a, b∈F, a+b=max{a,b} and a-b=min{a,b}. In [2], Meenakshi and Kaliraja have represented and IVFM as an interval matrix of its lower and upper limit fuzzy matrices. In [3], Meenakshi and Jenita have introduced the concept of K-regular fuzzy matrix and discussed about inverses associated with a K-regular fuzzy matrix as a generalization of results on regular fuzzy matrix developed in [1]. A matrix A∈Fnxn, the set of all nxn fuzzy matrices is said to be right(left) K-regular if there exists x(y)∈Fnxn such that A=x(y)=A(A=AYA=Ax) , x(y) is called a right(left) K-g inverses of A, Where K is a positive integer. Recently we have expended characterization of K-regular fuzzy matrix and discussed about inverses associated with a K-regular fuzzy matrix as a generalization of results on regular fuzzy matrix developed in [1]. A matrix A∈Fnxn, the set of all nxn fuzzy matrices is said to be right(left) K-regular if there exists x(y)∈Fnxn such that A=x(y)=A(A=AYA=Ax) , x(y) is called a right(left) K-g inverses of A, Where K is a positive integer. Recently we have expended characterization of K-regular fuzzy matrix and discussed about inverses associated with a K-regular fuzzy matrix as a generalization of results on regular fuzzy matrix developed in [1].

In this paper, we define regular interval valued fuzzy soft matrices as a generalization of regular interval valued fuzzy soft matrices and obtain the structure of Row space, column space of an Interval valued fuzzy soft matrices (IVFSM).

2. Preliminaries

Soft set 2.1 [9]
Suppose that U is an initial Universe set and E is a set of parameters, let P(U) denotes the power set of U.A pair (F,E) has called a soft set over U where F is a mapping given by F :E→P(U). Clearly a soft set is a mapping from parameters to P(U)and it is not a set, but a parameterized family of subsets of the Universe.

Fuzzy soft set 2.2 [10]
Let U be an initial Universe set and E be the set of parameters, let A⊆E. A pair (F,A) has called fuzzy soft set over U where F is a mapping given by F :A→1U, where 1U denotes the collection of all fuzzy subsets of U.

Fuzzy soft Matrices 2.3 [12]
For A = (aij), the set of all aij element of A is called a fuzzy soft set over U where F is a mapping given by F :A→1U, where 1U denotes the collection of all fuzzy subsets of U.

Interval valued fuzzy soft set 2.4 [11]
Let U be an initial Universe set and E be the set of parameters, let A⊆E. A pair (F,A) has called Interval valued fuzzy soft set over U where F is a mapping given by F :A→1U, where 1U denotes the collection of all Interval valued fuzzy soft matrices of U.

Definition 2.1.
An Interval-Valued Fuzzy Matrix(IVFM) of order mxn is defined as A=(a[i][j]), Where a[i][j]=[a[i][j],a[i][j]], the ijth element of A is an interval representing the membership value. All the elements of an IVFM are intervals and all the intervals are the subintervals of the interval [0,1].

For A = (a[i][j]) and B = (b[i][j]) of order mxn their sum denoted as A+B defined as, A+B = (a[i][j]+b[i][j]) = ([a[i][j]+b[i][j],(a[i][j]+b[i][j])]) ...(2.1)

In particular if a[i][j] = a[i][j] and b[i][j] = b[i][j] then (2.2) reduces to the standard max.min composition of fuzzy matrices [1].

In [2], representation of an IVFM as an interval matrix of its lower and upper limit fuzzy matrices in introduced and...
discussed the regularity of an IVFM in terms of its lower and upper limit fuzzy matrices.

**Definition 2.2**

For a pair of Fuzzy Matrices $E=(e_i)$ and $F=(f_j)$ in $F_{mn}$ such that $E \leq F$, it can be defined that the interval matrix is denoted as $[E,F]$, whose $ij^{th}$ entry is the interval with lower limit $e_{ij}$ and upper limit $f_{ij}$, that is $([e_{ij},f_{ij}])$. In particular for $E=F$, IVFM $[E,E]$ reduces to the fuzzy matrix $E \in F_{nn}$.

For $A=(a_{ij}) \in (IVFM)_{mn}$, let us define $A_L=(a_{ijL})$ and $A_U=(a_{ijU})$. Clearly $A_L$ and $A_U$ belongs to $F_{mn}$ such that $A_L \leq A_U$ and from Definition (2.2) A can be written as $A=[A_L,A_U]$.

For $A \in (IVFM)_{nn}$, $A^T,R(A),C(A)$ denotes the transpose of $A$, row space of $A$, column space of $A$ respectively.

**Interval valued fuzzy soft matrix 2.5[13]**

Let $U = \{c_1,c_2,c_3,...,c_m\}$ be the Universe set and $E$ be the set of parameters given by $E = \{e_1,e_2,e_3,...,e_n\}$. Let $A \subseteq E$ and $(F,A)$ be a interval valued fuzzy soft set over $U$, where $F$ is a mapping given by $F: A \rightarrow I^U$, where $I^U$ denotes the collection of all Interval valued fuzzy subsets of $U$. Then the Interval valued fuzzy soft set can expressed in matrix form as

\[
A = \begin{bmatrix}
[a_{11}] & [a_{12}] & \cdots & [a_{1m}]
\end{bmatrix}
\]

Where $a_{ij} = \left[ \begin{array}{c}
\mu_{IL}(c_i) \\
\mu_{IU}(c_i)
\end{array} \right]$ if $e_j \in A$

\[
\begin{bmatrix}
[0,0] \\
[0,0]
\end{bmatrix}
\]

if $e_j \notin A$

$[\mu_{IL}(c_i), \mu_{IU}(c_i)]$ represents the membership of $c_i$ in the Interval valued fuzzy set $F(c_i)$.

Note that if $\mu_{IL}(c_i)=\mu_{IL}(c_j)$ then the Interval valued fuzzy soft matrix (IVFSM) reduces to an FSM.

**Example 2.1**

Suppose that there are four houses under consideration, namely the universes $U = \{h_1,h_2,h_3,h_4\}$, and the parameter set $E = \{e_1,e_2,e_3,e_4\}$ where $e_1$ stands for “beautiful”, “large”, “cheap”, and “in green surroundings” respectively. Consider the mapping $F$ from parameter set $A = \{e_1,e_2\} \subseteq E$ to all interval valued fuzzy subsets of power set $U$. Consider an interval valued fuzzy soft set $(F,A)$ which describes the “attractiveness of houses” that is considering for purchase. Then interval valued fuzzy soft set $(F,A)$ is

\[(F,A) = \{(h_1, [0.6,0.8]), (h_2, [0.8,0.9]), (h_3, [0.6,0.7]), (h_4, [0.5,0.6])\}\]

We would represent this Interval valued fuzzy soft set in matrix form as

\[
\begin{bmatrix}
[0.6,0.8] & [0.7,0.8] & [0.0,0.0] & [0.0,0.0] \\
[0.8,0.0] & [0.6,0.7] & [0.0,0.0] & [0.0,0.0] \\
[0.6,0.7] & [0.5,0.7] & [0.0,0.0] & [0.0,0.0] \\
[0.5,0.0] & [0.8,0.9] & [0.0,0.0] & [0.0,0.0]
\end{bmatrix}
\]

**Definition 2.3**

A matrix $A \in (IVFM)_n$ is said to be right $k$-regular if there exist a matrix $X \in (IVFM)_n$ such that $A^k XA=A^k$, for some positive integer $k$. $X$ is called a right $k$-inverse of $A$. Let $A\{1^k\} = \{X/ A^k XA = A^k\}$.

**Definition 2.4**

A matrix $A \in (IVFM)_n$ is said to be left $k$-regular if there exist a matrix $Y \in (IVFM)_n$ such that $AYA^k = A^k$, for some positive integer $k$. $Y$ is called a left $k$-inverse of $A$. Let $A\{1^k\} = \{Y/ A^k YA = A^k\}$. In particular for a fuzzy matrix $A$, $a_{il} = a_{jl}$ then definition (2.3) and definition (2.4) reduce to right left $k$-regular fuzzy matrices respectively found in [3].

In the sequel we shall make use of the following results on IVFM found in [2] and [4].

**Lemma 2.1**

For $A= [A_L,A_U] \in (IVFM)_{nn}$ and $B= [B_L,B_U] \in (IVFM)_{np}$ the following hold.


(ii) $AB= [A_L B_L, A_U B_U]$.

**Lemma 2.2**

For $A,B \in (IVFM)_m$

(i) $R(B) \subseteq R(A) \iff B=XA$ for some $X \in (IVFM)_n$

(ii) $C(B) \subseteq C(A) \iff B=XY$ for some $Y \in (IVFM)_n$.

**Theorem 2.1**

For $A,B \in (IVFM)_m$ with $R(A)=R(B)$ and $R(A^k)=R(B^k)$ then $A$ is right $k$-regular IVFM $\iff B$ is right $k$-regular IVFM.

**Theorem 2.2**

For $A,B \in (IVFM)_m$ with $C(A)=C(B)$ and $C((A^k)=C(B^k)$ then $A$ is left $k$-regular IVFM $\iff B$ is left $k$-regular IVFM.

**3. Regular Interval Valued Fuzzy Soft Matrices**

In this section, we define regular interval valued fuzzy soft matrices and obtain the structure of row spaces, column spaces of an interval valued fuzzy soft matrices (IVFSM).

**Definition 3.1**

Let $\hat{A}$ is called Regular interval valued fuzzy soft matrix, then there exists $\hat{X} \in (IVFSM)_{nn}$ such that $\hat{A} \times \hat{A} = \hat{A}$.

**Lemma 3.1:**

For $\hat{A} = [\hat{A}_L, \hat{A}_U] \in (IVFSM)_{nn}$ and $\hat{B} = [\hat{B}_L, \hat{B}_U] \in (IVFSM)_{np}$, the following hold.

(i) $\hat{A}^T = [\hat{A}_L^T, \hat{A}_U^T]$.

(ii) $\hat{A} \hat{B} = [\hat{A}_L \hat{B}_L, \hat{A}_U \hat{B}_U]$. 

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**References**


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Let us define the interval valued fuzzy soft matrix
$$Z = [\tilde{X}, \tilde{Y}]$$
Then by the definition of regular
$$\tilde{A} \tilde{Z} \tilde{A} = [\tilde{A}_L, \tilde{A}_U] [\tilde{X}, \tilde{Y}] [\tilde{A}_L, \tilde{A}_U] = [\tilde{A}_L, \tilde{A}_U] = \tilde{A}.$$ 
Thus $\tilde{A}$ is regular
Hence the theorem.

**Example : 3.1**
Let us consider $A = [\tilde{A}_L, \tilde{A}_U] \in \text{(IVFSM)}_{nm}$.

$$\text{When } \tilde{A}_L = \begin{bmatrix} 0.3 & 0.1 & 0.5 \\ 0.6 & 0.2 & 0.3 \\ -0.4 & 0.5 & 0.2 \end{bmatrix} \text{ and } \tilde{A}_U = \begin{bmatrix} 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 \end{bmatrix}$$

Then $\tilde{A}_L \leq \tilde{A}_U$.

Here $\tilde{A}_L$ is regular being idempotent and $\tilde{A}_L$ is not regular. There is no $X \in \text{(IVFSM)}_{nm}$ such that, $\tilde{A}_L \times \tilde{X}_L \tilde{A}_L = \tilde{A}_L$. $A$ is not regular for, if $A$ is regular, then for some $X \in \text{(IVFSM)}_{nm}$, $\tilde{A} \times X \tilde{A} = \tilde{A} \Rightarrow \tilde{A}_L \times \tilde{X}_L \tilde{A}_L = \tilde{A}_L$ is regular which is not possible.

**Theorem : 3.1**
Let $A = [\tilde{A}_L, \tilde{A}_U]$ be an $(\text{IVFSM})_{mn}$. Then

(i) $\mathcal{R}(A) = \{ \mathcal{R}(\tilde{A}_L), \mathcal{R}(\tilde{A}_U) \}$ $\in \text{(IVFSM)}_{mn}$

(ii) $C(A) = \{ C(\tilde{A}_L), C(\tilde{A}_U) \}$ $\in \text{(IVFSM)}_{mn}$

Proof:
(i) Since $A \in \text{(IVFSM)}_{mn}$, any vector $x \in \mathcal{R}(A)$ is of the form $x = y \tilde{A}$ for some $y \in \text{(IVFSM)}_{mn}$ such that $x$ is an interval valued vector with $n$ components.

Let is compute $x \in \mathcal{R}(A)$ as follows. $x$ is a linear combination of the low of $A \Rightarrow x = \sum_{j=1}^{n} a_j \tilde{A}_j$ when $\tilde{A}_j$ is the $j$th low of $A$.

Equating the $j_{th}$ component on both sides yield $x = \sum_{j=1}^{n} a_j \tilde{A}_j$

\begin{equation}
\begin{bmatrix} a_j \\ \vdots \\ a_j \end{bmatrix} = \begin{bmatrix} \text{im} \mathcal{R}(A), \text{im} \mathcal{R}(A) \end{bmatrix}
\end{equation}

$x_j$ is the $j$th component of $x_j \in \mathcal{R}(\tilde{A}_L)$ and $x_{jL}$ is the $j$th component of $x_j \in \text{R}(\tilde{A}_U)$. Hence $x = \begin{bmatrix} x_L \times x_U \end{bmatrix}$.

Therefore $\mathcal{R}(A) = \{ \mathcal{R}(\tilde{A}_L), \mathcal{R}(\tilde{A}_U) \}$

(ii) For $A = [\tilde{A}_L, \tilde{A}_U]$, the transpose of $A$ is $\tilde{A}^T = ([\tilde{A}^T_L], [\tilde{A}^T_U])$.

By using (i) we get

$C(\tilde{A}) = \mathcal{R}(\tilde{A}) = \mathcal{R}(\tilde{A}^T_L), \mathcal{R}(\tilde{A}^T_U)$

$= \{ C((\tilde{A}_L), C(\tilde{A}_U)) \}$

Hence the theorem.
Theorem : 3.2
For \( \tilde{A}, \tilde{B} \in (IVFSM)_{mn} \),
(i) \( \mathcal{R}(\tilde{B}) \subseteq \mathcal{C}(\tilde{A}) \) if and only if \( \tilde{B} = \tilde{X} \tilde{A} \) for some \( \tilde{X} \in (IVFSM)_{mn} \).
(ii) \( \mathcal{C}(\tilde{B}) \subseteq \mathcal{C}(\tilde{A}) \) if and only if \( \tilde{B} = \tilde{Y} \tilde{A} \) for some \( \tilde{Y} \in (IVFSM)_{mn} \).

Proof:
Let \( \tilde{A} = [\tilde{A}_L, \tilde{A}_U] \) and \( \tilde{B} = [\tilde{B}_L, \tilde{B}_U] \).
Since \( \tilde{B} = \tilde{X} \tilde{A} \) for some \( \tilde{X} \in (IVFSM) \),
Put \( \tilde{X} = [\tilde{X}_L, \tilde{X}_U] \). Then by lemma 3.1
\[
\begin{align*}
\text{(iii) } & \tilde{B}_L = \tilde{X}_L \tilde{A}_L \quad \text{and } \tilde{B}_U = \tilde{X}_U \tilde{A}_U, \\
& \text{Hence lemma } 2.4 \mathcal{R}(\tilde{B}_L) \subseteq \mathcal{R}(\tilde{A}_L) \text{ and } \mathcal{R}(\tilde{B}_U) \subseteq \mathcal{R}(\tilde{A}_U). \\
& \text{By theorem 3.1 (i)} \mathcal{R}(\tilde{B}) = [\mathcal{R}(\tilde{B}_L), \mathcal{R}(\tilde{B}_U)] \subseteq [\mathcal{R}(\tilde{A}_L), \mathcal{R}(\tilde{A}_U)] = \mathcal{R}(\tilde{A}) \text{ (by theorem 3.1, (ii))}. \\
& \text{Conversely } \mathcal{R}(\tilde{B}) \subseteq \mathcal{R}(\tilde{A}_L) \text{ and } \mathcal{R}(\tilde{B}_U) \subseteq \mathcal{R}(\tilde{A}_U) \text{ (by theorem 3.1, (iii))}. \\
& \Rightarrow \tilde{B}_L = \tilde{Y}_L \tilde{A}_L \text{ and } \tilde{B}_U = \tilde{Y}_U \tilde{A}_U \text{ (by lemma 2.3).} \\
& \text{Then } \tilde{B} = [\tilde{B}_L, \tilde{B}_U] = [\tilde{Y}_L \tilde{A}_L, \tilde{Y}_U \tilde{A}_U] \text{ (by lemma 3.1)}. \\
& \Rightarrow \tilde{X} = [\tilde{X}_L, \tilde{X}_U] \text{ where } \tilde{X} = [\tilde{Y}_L, \tilde{Y}_U] \in (IVFSM)_{mn}. \\
& \tilde{B} = \tilde{X} \tilde{A}. \\
\end{align*}
\]

(i) This can be proved along the same lines as that of (i) that hence omitted.

Theorem : 3.3
For \( \tilde{A} \in (IVFSM)_{mn} \), \( \tilde{B} \in (IVFSM)_{mn} \), the following hold.
\( \mathcal{R}(\tilde{A} \tilde{B}) \subseteq \mathcal{C}(\tilde{A}) \mathcal{R}(\tilde{B}) \subseteq \mathcal{C}(\tilde{B}). \)

Proof:
Let \( \tilde{A} = [\tilde{A}_L, \tilde{A}_U] \) and \( \tilde{B} = [\tilde{B}_L, \tilde{B}_U] \).
\( \tilde{A}^T = [\tilde{A}_L^T, \tilde{A}_U^T] \) and \( \tilde{B}^T = [\tilde{B}_L^T, \tilde{B}_U^T] \).
Then by lemma 3.1
\[ \tilde{A} \tilde{B} = [\tilde{A}_L \tilde{B}_L, \tilde{A}_U \tilde{B}_U]. \]
By theorem 3.1
\[ \mathcal{R}(\tilde{A}) = \{[\tilde{A}_L \tilde{B}_L, \tilde{A}_U \tilde{B}_U] \}. \]
\[ = \mathcal{R}(\tilde{A}_L, \tilde{B}_L) \subseteq \mathcal{R}(\tilde{A}_L) \mathcal{R}(\tilde{B}_L) = \mathcal{R}(\tilde{A}_L). \]
Hence \( \mathcal{R}(\tilde{A} \tilde{B}) \subseteq \mathcal{R}(\tilde{A}) \mathcal{R}(\tilde{B}) \).
\[ \mathcal{C}(\tilde{A} \tilde{B}) \supseteq \mathcal{C}(\tilde{A}) \mathcal{C}(\tilde{B}) = \mathcal{C}(\tilde{B}). \]
Hence the theorem.

References