

On Migration Aspects and Population Growth

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Abstract: Increase in birth rate, decrease in death rate are responsible mainly for the growth of population however, migration also plays an important role on it. Various push and pull factors based on opportunities available in the study area affects migration process and migration selectivity. Different population growth models of exponential, geometric, logistic, generalized logistic models and relationship with probability distributions are considered in this chapter. Fitting logistic curve taking into account of the contributions of migration on population growth are applied to the population in Jiribam Sub Division in Manipur for drawing analytical results.

Keywords: Exponential, logistic, generalized logistic, population growth models

1. Introduction

Logistic distribution has attracted the attention of many researchers due to the application of this distribution in various fields. Balakrishnan (1992) [1] gave a systematic study of the logistic distribution. Tsokos and Dicroce (1992) [2] discussed the applications of logistic distribution in population growth, medical diagnosis and public health. Shah (1992) [3] discussed the analysis of bio-availability data when successive samples are from logistic distribution.

2. Population Growth Models

A mathematical model for natural population growth that occurs as a result of constant birth and death rates is given by the exponential differential equation:

$$\frac{dp}{dt} = r p \quad \text{with the solution } p(t) = p_0 e^{rt} \quad [1]$$

where P_0 is the population initially (i.e. at $t = 0$)

$$r = \beta - \delta, \quad \beta = \beta(t), \quad \delta = \delta(t) P_0$$

where, r is a specific parameter.

Here β and δ are constants and (2) represents the natural growth model also called exponential growth model. For human population (or population of a nation) it is observed that birth rate decreases as the population itself increases.

If β , the birth rate is a linear decreasing function of population size p , then $\beta = \beta_0 - \beta_1 p$ where β_0 and β_1 are positive constants.

If $\delta = \delta_0$ remains constants we can give a modified form of exponential model as

$$\left(\frac{dp}{dt}\right) = (\beta_0 - \beta_1 p - \delta_0) p$$

$$\left(\frac{dp}{dt}\right) = (a - b p^2) \quad \text{where } a = \beta_0 - \delta_0 \quad \text{and } b = \beta_1 \quad [2]$$

$$\left(\frac{dp}{dt}\right) = K P (M - P) \quad \text{where } K = b, \quad M = \frac{a}{b} \quad \text{for } 0 < M < P \quad [3]$$

a, b are specific parameters for logistic model. The solution of the logistic model is given by

$$P(t) = \frac{MP_0}{P_0 + (M - P_0)e^{-KMt}} \quad [4]$$

3. Logistic Population Growth Model

Let N_t and N_{t+1} be the population in time t and $(t + 1)$ respectively where, $(t + i)$ refers to $(t + im)$ if census is taken in intervals of m years.

$N_{t+1} = N_t F(N_t) = f(N_t)$ where, $f(N_t)$ is in general a non-linear function of N_t .

$$\text{If } F(N_t) = r > 0, \text{ then } N_{t+1} = rN_t \Rightarrow N_t = r^t N_0 \quad [5]$$

In this case, population grows or decays geometrically accordingly as $r > 1$ or $r < 1$ respectively.

Verhulst {1938} model can be represented as;

$$N_{t+1} = rN_t \left(1 - \frac{N_t}{k}\right), \quad r > 0, k > 0 \quad [6]$$

This continuous logistic growth model is valid for $N_t < k$

Another frequently used model is

$$N_{t+1} = N_t \exp \left[r \left(1 - \frac{N_t}{k}\right) \right] \quad r > 0, k > 0 \quad [7]$$

This model is not reasonable for most population not for taking time but, even so, has used with some justification for the early stages of growth of certain factor. A slight modification to bring in crowding effect could be

$$N_{t+1} = rN_s \quad \text{or,} \quad N_s = N_t^{1-b} \quad [8]$$

Where N_s is the population that surviving to breed. There must be restriction on b of course, so $N_s \leq N_t$ otherwise these surviving to breed would be more than the population of which they form a part.

Malthus (1798) developed the population model as:

$$\frac{dN(t)}{dt} = rN(t) \Rightarrow N(t) = N(0)e^{rt}, \quad 0 \leq t < \infty \quad [9]$$

Where $N(t)$ is the population at the time t with respect to the whole population, $N(0)$ is the initial size of the population and r is the constant of proportionality.

Verhulst (1838) modified limitation of Malthus's observation (1798) as:

$$\frac{dN(t)}{dt} = rN(t) \left\{ 1 - \frac{N(t)}{\pi} \right\} \Rightarrow \frac{N(t)}{\pi - N(t)} = e^{rt+c}$$

If $\frac{N(t)}{\pi} = f(t) \Rightarrow \frac{N(0)}{\pi} = f(0)$

$$f(t) = \frac{f(0)e^{rt}}{\{1 - f(0) + f(0)e^{rt}\}} \quad [10]$$

Fellar (1940) and Leslie (1948) proposed that population growth rate follows logistic pattern i.e. if the initial size of distribution from which we start is stable and stationary.

Moran (1961) expressed to deterministic model as:

$$\frac{dN}{dt} = N(B - D) \text{ where, } B \text{ and } D \text{ are the instantaneous}$$

birth and death rate at time t.

Since B and D are the function of N,

Hence $\frac{dN}{dt} = N\{B(N) - D(N)\}$

If B(N) and D(N) are the linear function of N.

$$\{B(N) - D(N)\} = \alpha - \beta N$$

Using (1) $\frac{dN}{dt} = N(\alpha - \beta N)$, where α and β are the positive constant.

$$\frac{dN}{\alpha - \beta N} = dt \Rightarrow \frac{1}{\alpha} \left\{ \frac{1}{N} - \frac{1}{N - \frac{\alpha}{\beta}} \right\} dN = dt \quad [11]$$

Integrating both sides, $\log \frac{N}{N - \frac{\alpha}{\beta}} = \alpha t + \alpha \log c$

$$\Rightarrow N = \frac{\frac{\alpha}{\beta}}{(1 + c'e^{-\alpha t})} \text{ where, } c' = \frac{-1}{c^\alpha} \quad [12]$$

According the initial value of $t_0 = 0$ gives N_0 , and

$$N_0 = \frac{\alpha}{\beta} (1 + c') \text{ also } N_0 \geq N \Rightarrow c' < 0$$

When $N_0 < \frac{\alpha}{\beta}$ gives as S-shaped curve of growth.

Let $y(t)$ be the population at the time t with respect to the whole.

Hence $y(t) = y(0) e^{rt}$

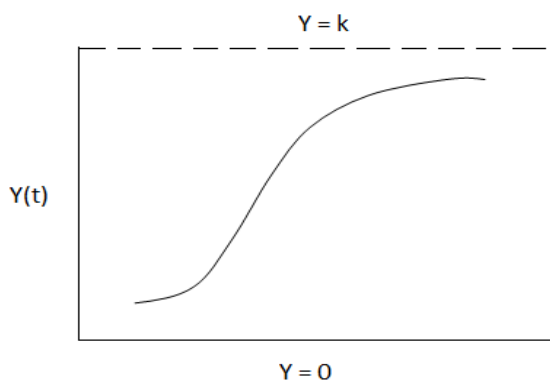


Figure 1: Logistic curve

Using two asymptotes $y = 0$ and $y = k$

Asymptotes for the logistic curve

$$\frac{d^2y}{dt^2} = 0, \quad y = \frac{k}{2}$$

And $\frac{dy(t)}{dt} = \frac{y(k-y)}{k} \phi'(t)$ where, $\phi'(t)$ = time trend of the population

Integrating both sides, $\int \left(\frac{1}{y} + \frac{1}{k-y} \right) dy(t) = \phi(t)$

It gives $y(t) = \frac{k}{1 + be^{-at}} \quad [13]$

Where, k is point of inflexion and a, b are real parameters .

4. Survival Functions and Logistic Distributions

Gupta, Jayakumar, Mathew (2004) [4] studied the characteristics of logistic, generalized logistic and mixed generalized logistic distributions. Arnold and Robertson (1989) [5] studied first order autoregressive logistic process. Arnold (1993) constructed a logistic process using geometric minimization. Tan (1992) [6] discussed logistic stochastic growth models and their applications. Accordingly the relationship between the distributions and the survival functions can be expressed:

(a) We consider the logistic distribution

Suppose, X be a random variable and $X \stackrel{d}{=} L(c, \alpha)$, then

$$e^{\alpha x} \bar{F}_X(x) = \int_{-\infty}^x G_Y^2(t) \alpha e^{\alpha t} dt$$

$$\Rightarrow \frac{e^{\alpha x}}{1 + ce^{\alpha x}} = \alpha \int_{-\infty}^x G_Y^2(t) e^{\alpha t} dt$$

Differentiating both sides with respect to x and simplifying, we get

$$\bar{G}_Y(x) = \frac{1}{1 + ce^{\alpha x}} \quad [14]$$

Then X represents, $X = \min\{Y - \ln V, Z - \ln V\}$, where V, Y and Z are independent random variables such that V has the distribution function

$$F(v) = v^\alpha, \alpha > 0, 0 < v < 1 \text{ and } Y \stackrel{d}{=} Z, \text{ then}$$

$X \stackrel{d}{=} L(c, \alpha)$ if and only if $X \stackrel{d}{=} Y$.

It means that a random variable X on $(-\infty, \infty)$ has a logistic distribution and write

$X \stackrel{d}{=} L(c, \alpha)$ if its survival function is of the form

$$\bar{F}_X(x) = P(X > x) = \frac{1}{1 + ce^{\alpha x}}, \quad c > 0, \alpha > 0 \quad [15]$$

(b) If we consider the generalized logistic distribution Again suppose X is a random variable and if

$$X \stackrel{d}{=} GL(c, \alpha, \gamma), \text{ then}$$

$$\begin{aligned} \frac{1}{\overline{G}_Z(x)} &= \int_0^{\overline{F}_X\{x + \ln v\}} \frac{\alpha \gamma v^{\alpha-\gamma-1}}{\overline{F}_X(x)} dv \\ &= \int_0^1 \left(\frac{1 + ce^{\alpha x}}{1 + ce^{\alpha(x+\ln v)}} \right)^{\gamma+1} \alpha \gamma v^{\alpha-\gamma-1} dv \\ &= \int_0^1 \left(\frac{1 + ce^{\alpha x}}{1 + cv^{\alpha} e^{\alpha x}} \right)^{\gamma+1} \alpha \gamma v^{\gamma-1} dv \end{aligned} \quad [16]$$

If $v^{\alpha} = w$, then

$$\frac{1}{\overline{G}_Z(x)} = \int_0^1 \left(\frac{1 + ce^{\alpha x}}{1 + cw e^{\alpha x}} \right)^{\gamma+1} \gamma w^{\gamma-1} dw \quad [17]$$

Taking $cwx^{\alpha} = u$, then

$$\frac{1}{\overline{G}_Z(x)} = \frac{(1 + ce^{\alpha x})^{\gamma+1}}{(ce^{\alpha x})^{\gamma}} \int_0^{ce^{\alpha x}} \frac{\gamma u^{\gamma-1}}{(1+u)^{\gamma+1}} du \quad [18]$$

Let $t = \frac{u}{1+u}$, and on simplification, it gives

$$\overline{G}_Y(x) = \frac{1}{1 + ce^{\alpha x}} \quad [19]$$

Then X represents, $x = \min(Y - \ln V, Z)$, where Y, Z and V are independent random variables such that V has distribution function

$$F(v) = v^{\alpha\gamma} \quad \alpha, \gamma > 0, 0 < v < 1 \text{ and } X \stackrel{d}{=} Y, \text{ then}$$

$$X \stackrel{d}{=} GL(c, \alpha, \gamma) \text{ if and only if } Z \stackrel{d}{=} L(c, \alpha).$$

It means that a random variable X on $(-\infty, \infty)$ has a generalized logistic distribution and write as;

$$X \stackrel{d}{=} GL(c, \alpha, \gamma)$$

if its survival function is of the form

$$\overline{F}_X(x) = P(X > x) = \frac{1}{(1 + ce^{\alpha x})^{\gamma-1}}, \quad \alpha, c, \gamma > 0 \quad [20]$$

5. Migration and Population Growth

Suppose the census is taken in intervals of 't' years and p_a^{z+it} indicates the population with characteristics 'a' in the census year (z + it) where $i = 0, 1, 2 \dots q$, and (z + qt) indicates the latest census year. The Population in the census year (z + it) characterized by 'a' can be represented as

$$\begin{aligned} p^{(z+it)} &= \sum_a^f p_a^{(z+it)} + \sum_a^m p_a^{(z+it)} \\ &= {}^f p^{(z+it)} + {}^m p^{(z+it)} \end{aligned}$$

where, f and m indicate females and males respectively.

The population in calendar year z in the age group (x, x+n), ${}_n p_x^z$ will become ${}_n p_{x+1}^{z+1}$, the population in calendar year (z + 1) in the age group (x + 1, x + n + 1) and so on. If migration is free, because of mortality factors it will satisfy ${}_n p_x^z \geq {}_n p_{x+1}^{z+1} \geq {}_n p_{x+2}^{z+2} \geq \dots$ and the population with the characteristics 'a' must satisfy:

$${}_n p_{a;x} \geq {}_n p_{a;x+1}^{z+1} \geq {}_n p_{a;x+2}^{z+2} \geq \dots \quad [21]$$

Any violation in this relation indicates in migration in the area of study.

Considering that

- Female population varies proportionally with the permanent or semi permanent settlers
- Male are of migratory nature to look for better net opportunities in advance whereas females generally follow their male members after ascertaining the opportunities available, subject to tolerable physical and cultural distances in the place of destination
- Migration due marriage is more or less remains constant and
- Census records are available in definite year intervals, (t years say).

- Suppose the sex ratio, S_a^z and growth rate between census years z and z+1, $g_a^{(z,z+1)}$ are known or taken from some standard population.

Expected male population,

$${}^m \hat{p}_a^z = {}^m p_a^z X \frac{100}{S_a^z} = {}^m p_a^z s_a^z \quad [22]$$

Opportunity looking male migrants ${}^m p_a^z - {}^m \hat{p}_a^z$

$${}^m M_{OL(a)}^z = {}^m p_a^z - {}^m \hat{p}_a^z \quad [23]$$

In-migration, out-migration of male members looking for better net opportunities in the area of study experienced accordingly ${}^m M_{OL(a)}^z > 0$ and < 0 .

Suppose $G_a^{(z+i, z+i+1)}$ be the growth rate between the census years (z + it) and {z + (i+1) t}, specified by the characteristic 'a' assumed to be known or taken from some standard population.

$$\text{Here, } G_a^{\{z+i, z+(i+1)\}} = \frac{P_a^{z+i+1} - P_a^{z+i}}{P_a^{z+i}} K,$$

where K is generally taken as 100 [24]

Expected population at census year z+1 is represented as;

$$\hat{P}^{z+1} \leq \sum_a \{ {}^f p_a^z + {}^f P_a^z S_a^z \} + \sum_a g_a^{(z,z+1)} \{ {}^f p_a^z + {}^f P_a^z S_a^z \}$$

where, $g_a^{(z,z+1)} = \frac{100}{G_a^{(z,z+1)}} \quad [25]$

No increase in male population of ${}^m M_{OL}^{(z)}$ can be expected due to mortality factors

$$\hat{p}^{z+1} \leq \sum_a \{ 1 + g_a^{(z,z+1)} \} {}^f p_a^z + \sum_a \{ 1 + g_a^{(z,z+1)} \} {}^m \hat{p}_a^z + \sum_a {}^m M_{OL(a)}^{(x)} \quad [26]$$

The growth of population are the main contributions from the married females, contributions from prior male migrants are negligible and supposed to be declined due to death factors.

The population in census year (z+1) can be expressed as:

$$\begin{aligned} p^{z+1} &= \sum_a ({}^f p_a^{z+1} + {}^m p_a^z) + \sum_a (B_a^z - D_a^z) + a \sum_a (M_{L(a)} - M_{O(a)}) \quad [27] \\ \therefore p^z &= \sum_a g_a^{(z,z+1)} \{ {}^f p_a^z + {}^m p_a^z \} \\ &= \sum_a \{ B_a^z - D_a^z \} + \sum_a \{ M_{L(a)} - M_{O(a)} \} \end{aligned} \quad [28]$$

Hence, opportunity seeking male migrants $M_{OL}^{(z,z+1)}$ available in the study area is represented as;

$$M_{OL}^{(z,z+1)} = P^{z+1} - \hat{P}^{z+1} \\ = \sum_a g_a^{(z,z+1)} \{ m p_a^{z-m} \hat{p}_a^z \} = \sum_a g_a^{(z,z+1)} M_{OL(a)}^z$$

Net migration is positive or negative accordingly as $M_{OL(a)}^{(z,z+1)} > 0$ or < 0

6. Cumulative Expected Migrants

Let $*p^z$ be the initial population in the calendar year z, from which cumulative expected migrants are to be determined for the census years (z+t), (z+2t), ... (z+mt) or (z+1), (z+2), ... (z+m) where, t is the year interval of census periods.

$$\text{Then } *p^z = *f p^z + *m p^z \quad [29]$$

The opportunity looking male migrants can be determined using sex-ratio (s^z) of the census year z.

$$*m \hat{p}^z = *f p^z \cdot \frac{S^z}{100} = *f p^z \cdot s^2 \quad [30]$$

$$\therefore *m M_{OL} = *m p^z - *m \hat{p}^z \quad [31]$$

Neglecting $*m p_{OL}$ factor which does not contribute to the natural increase through births, we have $*\hat{p}^z = *f p^z + *m \hat{p}^z$ [32]

Considering the growth rate, $g^{(z,z+1)}$ between inter-censal period z and z+1, the cumulative expected migrants can be estimated as follows:

$$*\hat{p}^{z+1} = *\hat{p}^z + g^{(z,z+1)} * \hat{p}^z \\ = \{ *f p^z + g^{(z,z+1)} * f p^z \} + \{ *m \hat{p}^z + g^{(z,z+1)} * m \hat{p}^z \} \\ = *f p^{z+1} + *m \hat{p}^{z+1} \quad [33]$$

Hence cumulative expected migrants during census year z, z+1 will be represented by

$$M_c^{(z,z+1)} = p^{z+1} - *\hat{p}^{z+1}$$

where, p^{z+1} is the actual population in calendar year (z+1)

$$\text{Similarly } M_c^{(z+1,z+2)} = p^{z+2} - *\hat{p}^{z+2}$$

$$\text{and hence } M_c^{(z+n-1,z+n)} = p^{z+n} - *\hat{p}^{z+n-1} \quad [34]$$

7. Analysis

Cumulative migration trend can be studied by drawing the curves of actual population p^{z+j} and $*p^{z+j}$. The difference will show the cumulative migration between successive census years which shows in-migration and out-migration depending upon the net opportunities available in the study area from time to time.

For fitting the logistic curve, we consider the Method of three points. Considering 3-equidistant points say $t = 0, t = n, t = 2n$ with corresponding population y_0, y_n and y_{2n} , the curve is allowed to pass through 3- equidistant points $(0, y_0)$,

$(n, y_n), (2n, y_{2n})$. Then the parameters can be estimated by the following relations.

$$\hat{a} = \frac{1}{n} [\log_e d_1 - \log_e d_2] \\ \frac{1}{k} = \frac{1}{y_0} - \frac{d_1^2}{d_1 - d_2} \\ \hat{b} = \frac{k}{y_0} - 1$$

$$\text{where, } d_1 = \frac{1}{y_0} - \frac{1}{y_n} \text{ and } d_2 = \frac{1}{y_n} - \frac{1}{y_{2n}} \quad [35]$$

Comparing the logistic growth and cumulative expected migrants, analysis on migrants available in the study area and population growth can be made.

8. Applications

8.1 The growth of population in North Eastern Region, Manipur and Jiribam sub-division of Manipur from 1951-2001 as per census records are as follows:

Table 1: Population in Manipur, NER and Jiribam SD

Census Year	Population		
	Jiribam sub-division	Manipur	North Eastern Region
1951	7,541	5,77,635	1,02,61,257
1961	18,126	7,80,037	1,45,00,134
1971	23,368	10,72,753	1,95,82,296
1981	26,367	14,11,375	2,65,79,000
1991	28,077	18,37,149	3,15,47,314
2001	37,828	23,88,634	3,84,95,089

Exponential growth model of [21]

a) North-Eastern Region is $P(t) = P_0 e^{0.0293t}$

b) Manipur population is $P(t) = P_0 e^{0.0287t}$

c) Jiribam population is $P(t) = P_0 e^{0.0412t}$

Exponential errors created with the fitting of exponential curves are shown in Table-2.

Table 2: Exponential Errors

Year	Exponential model			Exponential Error		
	NE R	Manipur	Jiribam SD	NE R	Manipur	Jiribam SD
1951	10.2610	5.7764	0.7541	0	0	0
1961	13.7489	7.6933	1.1391	0.7511	0.1071	0.6735
1971	18.4225	10.2464	1.7207	1.1597	0.4811	0.4811
1981	24.6846	13.6468	2.5991	1.8944	0.5627	0.5627
1991	33.0755	18.1756	3.9261	-1.5282	0.1958	0.1958
2001	44.3185	24.2073	5.9306	-5.8235	-2.5394	-2.5394
2011	59.3833	32.2407	8.9584	-	-	-

8.2 Applying logistic model in the population of NER, Manipur and Jiribam SD by using 3-point method, it shows very significantly different values of parameters as follows:

NE Region	Manipur	Jiribam SD of Manipur
$\hat{a} = 0.34169550$	$\hat{a} = 0.361177805$	$\hat{a} = 1.263504931$
$\hat{b} = 5.11155032$	$\hat{b} = 8.780774216$	$\hat{b} = 2.703499836$
$\hat{k} = 88667.47957$	$\hat{k} = 5649717.514$	$\hat{k} = 27931.86276$

8.3 Finding Cumulative Expected Migrants

Prior male migrants and settled migrants in the sub division considering a) population of Manipur as standard b) population of valley as standard are as follows:

Table 3: Prior male migrants (estimated) in Jiribam

Census Year	In migrants		Out migrants		Net migrants	
	(a)	(b)	(a)	(b)	(a)	(b)
1961	+ 709	+ 726	- 66	- 61	+ 643	+ 665
1971	+ 582	+ 614	- 179	- 154	+ 403	+ 460
1981	+ 801	+ 991	- 223	- 155	+ 598	+ 836
1991	+ 772	+ 958	- 124	- 88	+ 648	+ 870
2001	+ 448	+ 630	- 440	- 274	+ 8	+ 356

It is observed that immigrants exceeds out-migration in all the census years indicating positive net-migrants ranges from 356 to 870.

Table 4: Expected migrants (estimated) in Jiribam

Census Year	In migrants		Out migrants		Net migrants	
	(a)	(b)	(a)	(b)	(a)	(b)
1971	+ 3109	+ 3212	- 1482	- 1259	+ 1627	+ 1953
1981	+ 2538	+ 3105	- 1777	- 1373	+ 761	+ 1732
1991	+ 4852	+ 4105	- 5287	- 5385	- 435	- 1280
2001	+ 4224	+ 2054	- 7895	- 5401	- 3671	- 3347

8.4. Analytical Results

The population of North Eastern Region and population of Manipur are found increasing in a more or less homogeneous structure but the population of Jiribam as observed from the table, experienced sudden rise from 1951 to 1971 and smooth rise from 1961 to 1981 indicating signs of in-migration during 1951 to 1981. It experienced heavy in-migration during 1961 to 1981 and light in-migration during 1961 to 1981 and out-migration during 1981-2001.

In migrants in Manipur are shown with using asterisk marks in Table-5. It shows the in-migration of persons in the age group 10-19, 20-29 occurring during 1961-71 and 1971-81 as it violates (3).

Table 5: Indicators of in-migrants in Manipur

Age	p ¹⁹⁶¹	p ¹⁹⁷¹	p ¹⁹⁸¹	p ¹⁹⁹¹	p ²⁰⁰¹
0 - 9	2,37,296	≤ * 2,46,884	≤ * 2,61,863	≥ 2,44,129	≥ 2,15,434
10 - 19	1,53,485	≤ * 1,71,487	≥ 1,64,835	≥ 1,61,967	≥ 1,34,247
20 - 29	1,25,589	≥ 1,23,985	≥ 1,17,123	≥ 1,07,362	≥ 84,030
30 - 39	2,59,669	≥ 91,976	≥ 81,686	≥ 69,612	≥ 65,973
40 - 49	62,660	≥ 60,038	≥ 49,853	≥ 41,312	-
50 - 59	44,001	≥ 39,564	≥ 33,622	-	-
60 - 69	44,224	≥ 25,764	-	-	-

*indicating in-migrants in the study area

The above observation is found justifiable because, as per Census of India publication (Table D1) the out-migrant Manipuris enumerated in other States / Union Territories accounts for 54,036 persons (28,058 males and 25,978 females) while the in-migrant outsiders enumerated in

Manipur and immigrant foreigners enumerated at Manipur accounts for 18,529 persons (10,758 males and 7,773 females) and 2,148 persons (1,208 males and 940 females) respectively. Out-migrant Manipuris, enumerated in other States / Union Territories exceed in-migrant outsiders enumerated in Manipur and immigrant foreigners enumerated at Manipur together as per Census of India, D1 table, 2001.

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