

Cooperative Information Transmission in Wireless Communication Networks

Asmaa Helmy Zaid¹, Fayez W. Zaki², Sherif Kishk³

¹Mansoura University, Faculty of Engineering, Egypt

²Professor Electronics and Communications Engineering Dept. Mansoura University, Egypt

³Associate Professor Electronics and Communications Engineering Dept. Mansoura University, Egypt

Abstract: This paper indicates the comparison of outage probability for cooperative protocols such as: amplify-and-forward and decode-and-forward over Rayleigh and Nakagami-m fading channels.

Keywords: Cooperative protocols, Rayleigh fading, Nakagami-m fading.

1. Introduction

Cooperative communications schemes that occurs when a helper node assists the source and destination nodes. We have different relaying protocols:

- **Amplify and Forward (AF):** system amplifies the source signal which as advantage of simple processing of the received signal relay node.
- **Decode and Forward (DF):** the relay decodes and encodes the received signal then retransmits it again, decoding errors may occur in this method.

System performance was analyzed of AF and DF over Rayleigh fading [1]. Performance analysis for AF and DF over Nakagami-m fading indicated in [2],[3]. Comparison between different protocols over different channel fading in [4].

2. Relay Channels

The classical relay channel models a class of three-terminal communication channels originally examined. Discrete memoryless and additive white Gaussian noise relay channels are treated, and they determine channel capacity for the class of physically degraded relay channels. More generally, they develop lower bounds on capacity, i.e., achievable rates, via three structurally different random coding schemes:

- Facilitation, in which the relay does not actively help the source, but rather, facilitates the source transmission by inducing as little interference as possible.
- Cooperation, in which the relay fully decodes the source message and retransmits, jointly with the source, a bin index of the previous source message.

Observation, in which the relay encodes a quantized version of its received signal, using ideas from source coding with side information.

3. Channel Models

3.1 Rayleigh Fading

Under the orthogonality constraints, we can characterize our channel models using a time-division notation, frequency-division counterparts to this model are straightforward. Due to the symmetry of the channel allocations, we focus on the message of the source terminal, which potentially employs terminal R as a relay in transmitting to the destination terminal D , where $s, r \in \{1,2\}$ and $d \in \{3,4\}$. We utilize a baseband-equivalent, discrete-time channel model for the continuous-time channel, and we consider N consecutive uses of the channel, where N is large.

For direct transmission, our baseline for comparison, we model the channel as

$$y_n[n] = a_{s,d}X_s[n] + Z_d[n] \quad (1)$$

for, say $n = 1, \dots, N/2$, where $X_s[n]$ is the source transmitted signal, and $y_n[n]$ is the destination received signal. The other terminal transmits for $n = N/2 + 1, \dots, N$. Thus, in the baseline system, each terminal utilizes only half of the available degrees of freedom of the channel.

For cooperative diversity, we model the channel during the first half of the block as

$$y_r[n] = h_{s,r}X_s[n] + Z_r[n] \quad (2)$$

$$y_d[n] = h_{s,d}X_s[n] + Z_d[n] \quad (3)$$

for, say $n = 1, \dots, N/4$, where $X_s[n]$ is the source transmitted signal and $y_r[n]$ and $y_d[n]$ are the relay and destination received signals, respectively. For the second half of the block, we model the received signal as

$$y_d[n] = h_{r,d}X_r[n] + Z_d[n] \quad (4)$$

for $n = N/4 + 1, \dots, N/2$, where $X_r[n]$ is the relay transmitted signal and $y_d[n]$ is the destination received signal. A similar setup

is employed in the second half of the block, with the roles of the source and relay reversed. Note that, while again half the degrees of freedom are allocated to each source terminal for transmission to its destination, only a quarter of the degrees of freedom are available for communication to its relay.

In (1)–(4), $h_{i,j}$ captures the effects of path-loss, shadowing, and frequency nonselective fading, and $Z_j[n]$ captures the effects of receiver noise and other forms of interference in the system, where $i \in \{s, r\}$ and $j \in \{r, d\}$. We consider the scenario in which the fading coefficients are known to, i.e., accurately measured by, the appropriate receivers, but not fully known to, or not exploited by, the transmitters. Statistically, we model $h_{i,j}$ as zero-mean, independent, circularly symmetric complex Gaussian random variables with variances $\sigma_{i,j}^2$. Furthermore, we model $Z_j[n]$ as zero-mean mutually independent, circularly symmetric, complex Gaussian random sequences with variance N_0 .

3.1.1 Outage probability of fixed relaying over Rayleigh fading

(a) Amplify-and-forward

The amplify-and-forward protocol produces an equivalent one-input, two-output complex Gaussian noise channel with different noise levels in the outputs. The maximum average mutual information between the input and the two outputs, achieved by i.i.d. complex Gaussian inputs, is given by

$$I_{AF} = \frac{1}{2} \log \left(1 + \gamma |h_{s,d}|^2 + f(\gamma |h_{s,r}|^2, \gamma |h_{r,d}|^2) \right) \quad (5)$$

as a function of the fading coefficients, where

$$f(x, y) := \frac{xy}{x+y+1} \quad (6)$$

where γ is the signal to noise ratio (SNR). The outage event for spectral efficiency is given by and is equivalent to the event

$$|h_{s,d}|^2 + \frac{1}{\gamma} f(\gamma |h_{s,r}|^2, \gamma |h_{r,d}|^2) < \frac{2^{2R}-1}{\gamma} \quad (7)$$

For Rayleigh fading, i.e., $|h_{s,d}|^2$ independent and exponentially distributed with parameters $\sigma_{i,j}^{-2}$, analytic calculation of the outage probability becomes involved, but we can approximate its high-SNR behavior as

$$P_{AF}^{out}(\gamma, R) := \Pr[I_{AF}, R] \sim \left(\frac{1}{2\sigma_{s,d}^2} \frac{\sigma_{s,r}^2 + \sigma_{r,d}^2}{\sigma_{s,r}^2 \sigma_{r,d}^2} \right) \left(\frac{2^{2R}-1}{\gamma} \right)^2 \quad (8)$$

(b) Decode-and-forward

To analyze decode-and-forward transmission, we examine a particular decoding structure at the relay. Specifically, we require the relay to fully decode the source message; examination of symbol-by-symbol decoding at the relay becomes involved because it depends upon the particular coding and modulation choices. The maximum average mutual information for repetition-coded decode-and-forward can be readily shown to be

$$I_{DF} = \frac{1}{2} \min \left\{ \log \left(1 + \gamma |h_{s,r}|^2 \right), \log \left(1 + \gamma |h_{s,d}|^2 + \gamma |h_{r,d}|^2 \right) \right\} \quad (9)$$

as a function of the fading random variables. The first term in (9) represents the maximum rate at which the relay can reliably decode the source message, while the second term in (9) represents the maximum rate at which the destination can reliably decode the source message given repeated transmissions from the source and destination. Requiring both the relay and destination to decode the entire codeword without error results in the minimum of the two mutual informations in (9). We note that such forms are typical of relay channels with full decoding at the relay.

The outage event for spectral efficiency R is given by $I_{DF} < R$ and is equivalent to the event

$$\min \left\{ |h_{s,r}|^2, |h_{s,d}|^2 + |h_{r,d}|^2 \right\} < \frac{2^{2R}-1}{\gamma} \quad (10)$$

For Rayleigh fading, the outage probability for repetition coded decode-and-forward can be computed according to

$$P_{AF}^{out}(\gamma, R) := \Pr[I_{AF}, R] = \Pr \left[|h_{s,r}|^2 < g(\gamma) \right] + \Pr \left[|h_{s,r}|^2 < g(\gamma) \right] \Pr \left[|h_{s,d}|^2 + |h_{r,d}|^2 < g(\gamma) \right] \quad (11)$$

where $g(\gamma) = (2^{2R} - 1)/\gamma$, we conclude that

$$P_{AF}^{out}(\gamma, R) \sim \frac{1}{\sigma_{s,r}^2} \frac{2^{2R}-1}{\gamma} \quad (12)$$

The fixed decode-and-forward does not offer diversity gains for large SNR, because requiring the relay to fully decode the source information limits the performance of decode-and-forward to that of direct transmission between the source and relay.

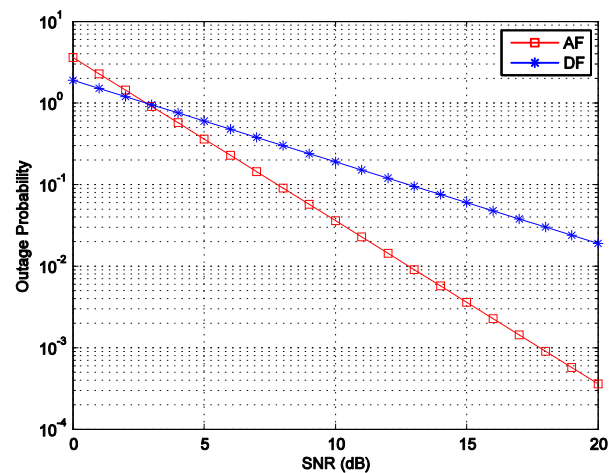


Figure 1: Outage probability of fixed relaying over Rayleigh fading, where SNR (dB) is normalized SNR $\sigma_{i,j}^2 = 1$, $R = 0.5$ b/s/Hz

3.2 Nakagami- m Fading

Let's consider the specific cooperative relay-based wireless system with $K + 2$ terminals: one source S , K relays R_k with $k = 1, \dots, K$, and one destination D . The source S

broadcasts the signal to K relays in the first-hop transmission (broadcasting phase). During the second-hop transmission (relaying phase), selection diversity is applied, i.e., only the best relay is selected for forwarding the message to the destination. We also assume that channels in the two hops are quasistatic i.n.i.d. Nakagami- m fading. Specifically, we denote h_{SR_k} and h_{RD_k} as the independent channel gains for the source to relay R_k ($S \rightarrow R_k$) link and the k -th relay-to-destination ($R_k \rightarrow D$) link, respectively. These channel gains are modeled as Nakagami- m random variables. Then, the effective power channel gains $|h_{SR_k}|^2$ and $|h_{RD_k}|^2$ follow the gamma distribution with different fading parameters $1/\Omega_{SR_k}$, $1/\Omega_{RD_k}$, and fading severity parameters m_{1k} , m_{2k} , respectively. Moreover, the instantaneous SNR for $S \rightarrow R_k$ and $R_k \rightarrow D$ are given by $\gamma_{SR_k} = \gamma_0 |h_{SR_k}|^2$ and $\gamma_{RD_k} = \gamma_0 |h_{RD_k}|^2$ where γ_0 is the average SNR. Due to the imperfect detection at the relay, incorrectly decoded signals may be forwarded to the destination. Hence, for any modulation scheme the dual-hop $S \rightarrow R_k \rightarrow D$ channel can be modeled as an equivalent single hop whose output SNR γ_{eq_k} can be tightly approximated in the high SNR regime as follows

$$\gamma_{eq_k} = \min\{\gamma_{SR_k}, \gamma_{RD_k}\} \quad (13)$$

For the selection combining scheme, the signal with largest equivalent received SNR is selected. Then the instantaneous SNR at the output of the relay selection combiner is given by

$$\gamma_{sc_k} = \max_{k=1, \dots, K} \gamma_{eq_k} \quad (14)$$

3.2.1 Outage probability of fixed relaying over Nakagami- m fading

(a) Amplify-and-forward

The mutual information of the opportunistic AF relaying scheme is given by

$$I_{AF} = \frac{1}{2} \log_2 \left(1 + \gamma_s \max_{k \in [1, K]} \frac{X_k Y_k}{\varpi_k + Y_k} \right) \quad (15)$$

where $\gamma_s = E_s/\sigma_n^2$, $\varpi_k = (\lambda_k \gamma_s + 1)/\gamma_R$ and $\gamma_R = E_R/\sigma_n^2$ with E_s , E_R and σ_n^2 being the transmit energy at the source node, the transmit energy at the relay node, and the power of additive white Gaussian noise (AWGN), respectively, and ϖ_k corresponds to the power constraint factor. The outage probability is mathematically defined as the probability that I_{AF} is less than a target rate R , denoted as $P_{AF}^{out} = \Pr(I_{AF} < R)$. Equivalently, we write

$$P_{AF}^{out} = \Pr \left(\max_{k \in [1, K]} \frac{X_k Y_k}{\varpi_k + Y_k} < \eta_0 \right) \quad (16)$$

where $\eta_0 = (2^{2R} - 1)/\gamma_s$, due to the independent channel assumption, it is given by

$$P_{AF}^{out} = \prod_{k=1}^K P_{AF_k}^{out} \quad (17)$$

with

$$P_{AF_k}^{out} = \int_0^\infty \int_0^\infty \frac{\eta_0(\varpi_k + y)}{y} f_{X_k}(x) f_{Y_k}(y) dx dy \\ = \frac{1}{\Gamma(m_k)} \int_0^\infty \Gamma_{inc} \left(m_k, \eta_0 \theta_k \left(1 + \frac{\varpi_k}{y} \right) \right) f_{Y_k}(y) dy \quad (18)$$

where $\Gamma_{inc}(a, x)$ is the Pearson's incomplete gamma function expressed as $\Gamma_{inc}(a, x) = \int_0^x e^{-t} t^{a-1} dt$.

Subsequently, we assume that all coefficients $\{m_k\}$ are natural numbers. This does not represent a strong limitation for the following reasons. First, the channel may sometimes merely be characterized or measured to an accuracy corresponding to whole integer arithmetic. Second, if channels are known more accurately, the upper and lower bounds of natural numbers can be employed to offer bounds for the outage probabilities, and a linear approximation between these results may be used to obtain an accurate approximations. Under this assumption, we can get

$$\Gamma_{inc}(m, x) = \Gamma(m) \left(1 - e^{-x} \sum_{n=0}^{m-1} \frac{x^n}{n!} \right) \quad (19)$$

Then,

$$P_{AF_k}^{out} = 1 - e^{-\eta_0 \theta_k} \int_0^\infty e^{-\frac{\eta_0 \varpi_k \theta_k}{y}} \sum_{n=0}^{m_k-1} \frac{1}{n!} (\eta_0 \theta_k)^n \left(1 + \frac{\varpi_k}{y} \right)^n f_{Y_k}(y) dy \quad (20)$$

In (20), rewriting $(1 + \varpi_k/y)^n$ by virtue of the power series, which is

$$(a, x)^n = \sum_{j=0}^n C_n^j x^j a^{n-j} \quad (21)$$

and

$$C_n^k = \frac{n!}{k!(n-k)!} \quad (22)$$

It finally yields

$$P_{AF_k}^{out} = 1 - \frac{2e^{-\eta_0 \theta_k} (\bar{\theta}_k)^{\bar{m}_k}}{\Gamma(\bar{m}_k)} \sum_{n=0}^{m_k-1} \frac{1}{n!} (\eta_0 \theta_k)^n \\ \cdot \sum_{j=0}^n C_n^j \varpi_k^j \left(\frac{\eta_0 \theta_k \varpi_k}{\bar{\theta}_k} \right)^{\frac{\bar{m}_k - j}{2}} K_{\bar{m}_k - j} \left(2\sqrt{\eta_0 \bar{\theta}_k \theta_k \varpi_k} \right) \quad (23)$$

where $K_\nu(z)$ denotes the Bessel function of the imaginary argument. The function $K_\nu(z)$ is not available directly in popular symbolic software such as MATLAB. Whereas its zero and first orders ($K_0(z)$ and $K_1(z)$) have a concise and closed-form expression, the higher orders can be calculated via the formula $zK_{\nu+1}(z) = zK_{\nu-1}(z) + 2\nu K_\nu(z)$. We did not constrain \bar{m}_k to be a natural number during the derivation of (23). As is well known, the Nakagami fading channel with parameter $m_k = 1$ will become the Rayleigh fading channel.

For this special case, (23) can be written as

$$P_{AF_k}^{out} = 1 - 2e^{-\eta_0 \theta_k} (\eta_0 \bar{\theta}_k \theta_k \varpi_k)^{\frac{\bar{m}_k}{2}} K_{\bar{m}_k} \left(2\sqrt{\eta_0 \bar{\theta}_k \theta_k \varpi_k} \right) \quad (24)$$

where $\theta_k = 1/\lambda_k$. If we further assume that $\bar{m}_k = 1$, that is, the links of relay-destination under Rayleigh fading, we have

$$P_{AF\ k}^{out} = 1 - 2e^{-\frac{\eta_0}{\lambda_k}} \sqrt{\frac{\eta_0 \varpi_k}{\lambda_k \lambda_k}} K_1 \left(2 \sqrt{\frac{\eta_0 \varpi_k}{\lambda_k \lambda_k}} \right) \quad (25)$$

In summary, we first derive $P_{AF\ k}^{out}$ based on the PDFs of X_k and Y_k . Then, a closed form expression for the outage probability P_{AF}^{out} can be obtained via a product of $P_{AF\ k}^{out} \forall k$.

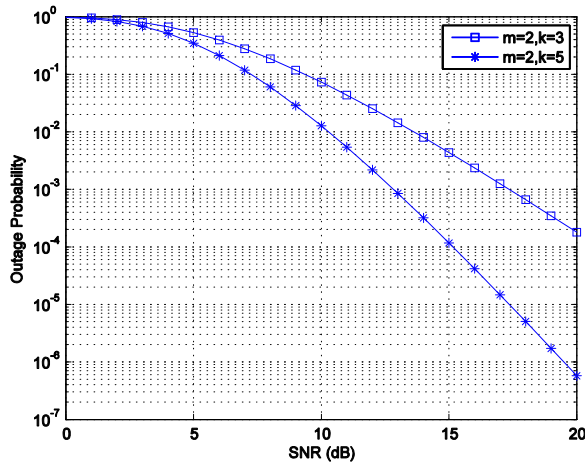


Figure 2: Outage probability of AF protocol over Nakagami- m fading, where SNR (dB) is average SNR

(b) Decode-and-forward

γ_{SR_k} and γ_{RD_k} are independent gamma distributed random variables, the cumulative distribution function (CDF) of γ_{eq_k} can be written as

$$F_{\gamma_{eq_k}}(\gamma) = 1 - \frac{\Gamma(m_{1k}, \alpha_k \gamma) \Gamma(m_{2k}, \beta_k \gamma)}{\Gamma(m_{1k}) \Gamma(m_{2k})} \quad (26)$$

where $\Gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt$, $\alpha_k = \frac{m_{1k}}{\Omega_{SR_k} \gamma_0}$ and $\beta_k = \frac{m_{2k}}{\Omega_{RD_k} \gamma_0}$. Assuming m_{1k} and m_{2k} are integers and using the fact that $\Gamma(n, x) = (n-1)! e^{-x} \sum_{i=0}^{n-1} \frac{x^i}{i!}$, we have

$$F_{\gamma_{eq_k}}(\gamma) = 1 - e^{-(\alpha_k - \beta_k)\gamma} \sum_{i=0}^{m_{1k}-1} \sum_{j=0}^{m_{2k}-1} \frac{\alpha_k^i \beta_k^j \gamma^{i+j}}{i! j!} \quad (27)$$

The outage probability P_{DF}^{out} is defined as the probability that the instantaneous SNR falls below a given threshold γ_{th} . It is easy to obtain P_{DF}^{out} as follows

$$P_{DF}^{out} = \prod_{k=1}^K \left[1 - e^{-(\alpha_k - \beta_k)\gamma_{th}} \sum_{i=0}^{m_{1k}-1} \sum_{j=0}^{m_{2k}-1} \frac{\alpha_k^i \beta_k^j \gamma_{th}^{i+j}}{i! j!} \right] \quad (28)$$

For symmetric case, $\Omega_{SR_k} = \Omega_{RD_k} = 3$, $m_{1k} = m_{2k} = 2$.

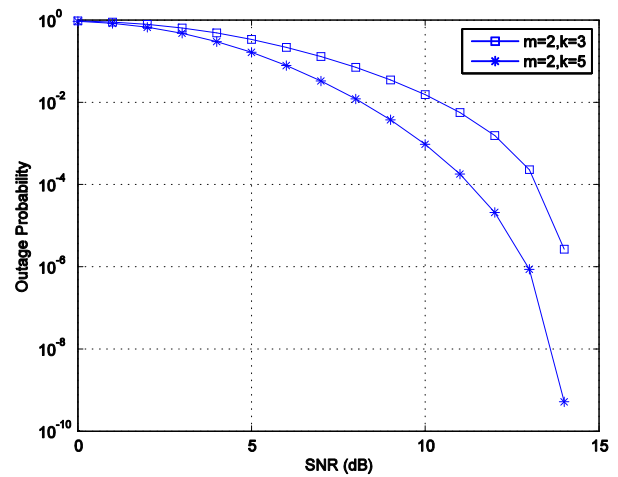


Figure 3: Outage probability of DF protocol over Nakagami- m fading, $\gamma_{th} = 3$

4. Conclusions

In this paper, we have derived an expression for outage probability of Rayleigh and Nakagami- m fading channel models for different fixed strategy protocols such as Amplify-and-Forward and Decode-and-Forward protocols. It can be concluded that outage probability in Nakagami- m fading model is less than outage probability in Rayleigh fading model.

References

- [1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.
- [2] Qinghai Yang, Yingji Zhong, Kyung Sup Kwak, and Fenglin Fu, "Outage probability of opportunistic amplify-and-forward relaying in Nakagami- m fading channels", *ETRI Journal*, Volume 30, Number 4, August 2008.
- [3] T. Q. Duong, Vo Nguyen Quoc Bao and H. J Zepernick, "On the performance of selection decode-and-forward relay networks over Nakagami- m fading channels," *IEEE Communications Letters*, vol. 13, no. 3, pp. 172-174, Mar. 2009.
- [4] Behrouz Maham, "Transmission Strategies and Performance Analysis of Resource-Constrained Wireless relay Networks," Ph.D. thesis, Faculty of Mathematics and Natural Sciences, University of Oslo, 2010.

Author Profiles



Asmaa Helmy Zaid is born in Egypt 1989. She received the B.Sc. degree in Comm. Engineering from Mansoura University, Egypt in 2011. Her research interest is in the area of Wireless Telecommunication Systems. She is a research assistant at Faculty of Engineering, Mansoura University.



Fayez W. Zaki is born in Egypt 1957. He received B.Sc. in Communication Eng. from Menofia University Egypt 1969, M.Sc. Communication Eng. from Helwan University Egypt 1975, and Ph. D. from Liverpool University 1982. He is a professor at Faculty of Engineering, Mansoura University. He worked as a RA at Mansoura University, Egypt from 1969, Lecture assistant from 1975, lecturer from 1982, Associate Prof. from 1988, and Prof.

from 1994. Head of Electronics and Communication Engineering Department Faculty of Engineering, Mansoura University from 2002 till 2005. His experience covers watermarking, 3D object recognition, Optical signal processing, Wireless networks, Biomedical image processing. Prof. Zaki supervised several MSc and PhD theses. He has published several papers in refereed journals and international conferences. He is now a member of the professorship promotion committee in Egypt.



Sherif S. Kishk is born in Egypt 1969. He received the B.Sc. degree in Electronics and Communications Engineering in 1992, Mansoura University, Egypt. and the M.Sc. degree in Electrical Communications 1995 from Mansoura University, Egypt. He received another M.Sc. degree in Electrical and Computer Eng. in 2001 from the University of Connecticut, CT, USA. He received the Ph.D. degree in Electrical and Computer Eng. in 2004 from the University of Connecticut, CT, USA..He was a visiting scholar at the university of Minnesota SPINCOM lab in the period from Sep. 2104 to March 2015. Currently, he is an associate professor at Faculty of Engineering, Mansoura University. He spent 3 years as a project manager at Investment Program, at Ministry of State for Administrative Development. He has worked also as a consultant at BJ information technology (Connecticut USA), where he has established tracking and signal processing systems. He spent five years working in research projects under National Science Foundation NSF, DARPA, CT DOT, etc... His experience covers watermarking, 3D object recognition, Optical signal processing, Wireless networks, Biomedical image processing. Prof. Kishk has been a reviewer for applied optics, Optics Express, Optical Communications, He has published several papers in refereed journals and international conferences.