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Coherent Signal's DOA Estimation and its Relationship with SNR and Number of Samples

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Abstract: ***A novel method for direction of arrival (DOA) estimation and its relationship with various signal to noise ratios (SNRs) and number of samples is proposed in this paper. For achieving this a full rank Toeplitz matrix is formed using each row of input covariance matrix. By exploiting the joint diagonalization structure of the full set of Toeplitz matrices, a new cost function is designed. This cost function does not require a priori information of the source number. Thus a new spatial spectrum can be estimated and the DOAs can be estimated from it subsequently.***

Keywords: Coherent signals, Direction of arrival (DOA), spatial spectrum, Covariance matrix

1. Introduction

Array signal processing is an important branch in the field of array signal processing. The field of array signal processing can be classified into two; self-adaptation array signal processing and spatial spectrum estimation. Spatial spectrum estimation is focused on investigating the system of spatial multiple sensor arrays, with the main purpose of estimating the signal's spatial parameters and the location of the signal source. The spatial spectrum expresses the signal distribution in the space from all directions to the receiver. Hence, if one can get the signal's spatial spectrum, then the direction of arrival (DOA) can be obtained. As thus, spatial spectrum estimation is also called as DOA estimation.

DOA estimation using sensor arrays has wide range of application prospect in radar, sonar, wireless communications, seismology measurement and biomedicine. There are many kinds of algorithms for DOA estimation like spectral estimation, Bartlett, Capon and subspace based algorithms. The subspace based direction of arrival (DOA) estimation methods like MUSIC and ESPIRIT are super resolution algorithms for uncorrelated and partially correlated signals. Under these circumstances the subspace based methods fail due to rank deficiency of input covariance matrix.

Another drawback of subspace based algorithm is that they need a prior knowledge about the source number. Some algorithms exists which can be used where the source number is unknown. But a major problem with those approaches is that they cannot be applicable to and the case of coherent signals. Although some algorithms have been modified to eliminate this issue, the probability of successfully detecting the number of sources is still low when the signal to noise ratio (SNR) and sample size are smaller than a certain threshold.

In this paper, a new method for direction of arrival (DOA) estimation of coherent signals is proposed which can overcome the limitations of existing algorithms. Along with this, relationship between DOA estimation with SNR and number of snapshots are devised in this paper.

In this paper, we use boldface uppercase letters to denote matrices, bold face lower case letters for column vectors and lower case letter for scalar quantities. Superscripts (.)T, (.)*,(.)H, and (.)-1 represent transpose, complex conjugate, conjugate transpose and inverse, respectively. The operator $\mathrm{E}\{a\}$ denotes expectation value of a, 0_m is the $m\times 1$ zero vector and I_m is the $m\times m$ identity matrix. The □ denotes the set of complex numbers and $\|\cdot\|$, $\|\cdot\|_F$ represent the Euclidean norm of a vector, Frobenius norm and trace of a matrix respectively.

2. Mathematical Model of DOA Estimation

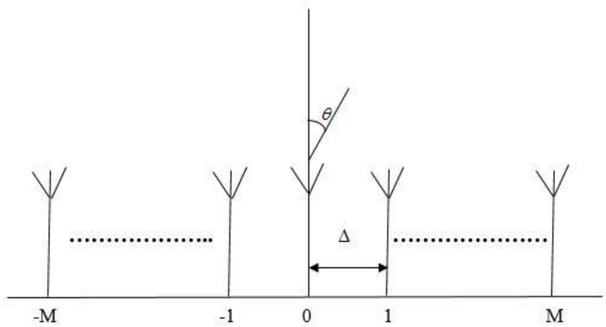

Figure 1: symmetric ULA model

Consider a Uniform Linear Array (ULA) with (2M+1) isotropic sensors as shown in the figure 1. Each test signal source has the same but unrelated polarization. Generally consider that the signal sources are narrow bands. There are narrow band source signals impinging on the array from different directions in the far field and the first K signals are mutually coherent while the others are uncorrelated and independent of first K signals. Consider the first signal as the reference, the kth coherent signal becomes

$$p_k(t)=\rho_k e^{j\delta\phi_k} p_1(t), k=2,.......,K \tag{1}$$

Where ρ_k is the amplitude fading factor and $\delta\phi_k$ is the phase change.

Then the signals received by the mth element can be expressed as

$$x_m(t)=\sum_{i=1}^{D} p_i(t)e^{-j2\pi m\sin(\theta_i)\frac{\Delta}{\lambda}}+n_m(t)$$

$$=p_1(t)\sum_{i=1}^{K} e^{-j2\pi m\sin(\theta_i)\frac{\Delta}{\lambda}}+\sum_{i=K+1}^{D}\beta_i p_i(t)e^{-j2\pi m\sin(\theta_i)\frac{\Delta}{\lambda}}+n_m(t) \quad (2)$$

Where $p_i(t)$ the complex envelope of the i[th] signal is, λ is the carrier wavelength, $\Delta=\frac{\lambda}{2}$ is the array element spacing. Assume that the noise $n_m(t)$ is a white Gaussian process with zero mean and covariance σ^2 at the m[th] element. The observation vector is

$$\mathbf{X}\ (\mathbf{t})=\left[x_{-M}(t),......,0,.......,x_M(t)\right]^T$$

$$=\mathbf{B}\ \mathbf{p}\ (\mathbf{t})+\mathbf{n}\ (\mathbf{t}) \quad (3)$$

Where $\mathbf{p}\ (\mathbf{t})=\left[p_1(t),..........,p_D(t)\right]$ is the source signal vector and **B= [b(θ₁)……. b(θ_D)]** is the array manifold where

$$\mathbf{b}\ (\theta_D)=\left[e^{j2\pi M\sin(\theta_D)\frac{\Delta}{\lambda}},..........,1,..........,e^{-j2\pi M\sin(\theta_D)\frac{\Delta}{\lambda}}\right] \quad (4)$$

Be the d[th] steering vector.

3. Proposed Algorithm

Here, we develop a new algorithm for DOA estimation of coherent signals without knowing the source number.
The input covariance matrix of x (t) can be obtained as,

$$\square=E\left\{x(t)x^H(t)\right\} \quad (5)$$

The (m,n) entry of $\square$ can be expressed as

$$r(m,n)=\sum_{i=1}^{D}N_{m,i}e^{j2\pi n\sin(\theta_i)\frac{\Delta}{\lambda}}+\sigma^2\delta_{m,n}\quad m,n=-M,......,0,......,M \quad (6)$$

Where

$$N_{m,i}=\begin{cases}D_{1,1}\beta_i^*\sum_{k=1}^{K}\beta_k e^{-j2\pi m\sin(\theta_k)\frac{\Delta}{\lambda}},i=1,.........,K\\ D_{i,i}e^{-j2\pi m\sin(\theta_i)\frac{\Delta}{\lambda}},i=K+1,................,D\end{cases} \quad (7)$$

$$D_{k,i}=E\left\{p_k(t)p_i^*(t)\right\}k,i=K+1,.............,D \quad (8)$$

$$\delta_{m,n}=\begin{cases}1,m=n\\0,m\neq n\end{cases} \quad (9)$$

Using the m[th] row of the input covariance matrix R, we can form the Toeplitz matrix R_m

$$R_m=\begin{bmatrix}r(m,0) & r(m,1)...............r(m,M)\\ r(m,-1) & r(m,0)...............r(m,-M)\\ : & :\\ : & :\\ r(m,-M) & r(m,-M+1).......r(m,0)\end{bmatrix}$$

$$R_m=\bar{B}N_m\bar{B}^H+\sigma^2 I_{M+1,m}\in\square^{(M+1)\times(M+1)} \quad (10)$$

Where $I_{M+1,m}$ is the $(M+1)\times(M+1)$ matrix with one on its m[th] diagonal and zero else where
B = [b(θ₁)……. b(θ_D)] denotes a new steering matrix with the d[th] steering vector being

$$\mathbf{b}(\theta_D)=\left[1,e^{-j2\pi\sin(\theta_D)\frac{\Delta}{\lambda}},..........,e^{-j2\pi M\sin(\theta_D)\frac{\Delta}{\lambda}}\right]$$

And $N_m=diag\left\{N_{m,1},........,N_{m,D}\right\}$ denotes pseudo signal covariance matrix.

In the absence of noise R_m can be written as

$$Rm=\bar{B}N_m\bar{B}^H=\sum_{i=1}^{D}N_{m,i}\bar{b}(\theta_i)\bar{b}^H(\theta_i) \quad (11)$$

Since the –m[th] and m[th] rows of $\square$ are conjugate symmetric, there is no need to adopt all the $(2M+1)$ rows to form Toeplitz matrices. Without loss of generality, we employ the first $(M+1)$ rows of $\square$ and thus there are only $(M+1)$ Toeplitz matrices containing different static information. Since N_m has full rank, we utilize these $(M+1)$ Toeplitz matrices to identify the range space of array manifold matrix **B** estimate the DOA parameters.

$$\bar{b}^H(\theta_i)a_d=\begin{cases}\bar{b}^H(\theta_i)a_d,i=d\\0,i\neq d\end{cases} \quad (12)$$

Since

$$a_d\perp range\left\{\bar{b}(\theta_1),....,\bar{b}(\theta_{d-1}),\bar{b}(\theta_{d+1}),...........\bar{b}(\theta_d)\right\} \quad (13)$$

Substituting (12) into (11) yields

$$R_m a_d=\sum_{i=1}^{D}N_{m,i}\bar{b}(\theta_i)\bar{b}^H(\theta_i)a_d=v_m\bar{b}(\theta_d) \quad (14)$$

From (14) we decide that if θ is one of the true DOAs there always exists a scalar v_m that $R_m a$ and $\bar{b}(\theta)$ parallel. That is,

$$R_m a=v_m\bar{b}(\theta),-M\leq m\leq 0 \quad (15)$$

This leads to the optimization problem as follows,

$$\min_{\theta} J(\theta,v,a)=\sum_{m=-M}^{0}\left\|R_m b-v_m\bar{b}(\theta)\right\|^2 \quad (16)$$

Such that $\|v\|=1$

Where $\bar{b}(\theta)$ is the steering vector with parameter to be optimized, $a\in\square^{M+1}$, and $v=\left[v_{-M},...........,v_0\right]^T\in\square^{M+1}$.

Since **a** and **v** are unknown parameters. It is difficult to optimize (16) by searching for DOAs directly. To dodge this problem we try to simplify (16).
By expanding (16),

$$J(\theta,v,a)=a^H\left(\sum_{m=-M}^{0}R_m{}^H R_m\right)a-a^H\left(\sum_{m=-M}^{0}v_m R_m{}^H\bar{b}(\theta)\right)-\left(\sum_{m=-M}^{0}v_m{}^*\bar{b}^H(\theta)R_m\right)a+\bar{b}^H(\theta)\bar{b}(\theta)\sum_{m=-M}^{0}|g_m|^2 \quad (17)$$

$$\text{Let } Q=\sum_{m=-M}^{0}R_m{}^H R_m\in\square^{(M+1)\times(M+1)} \quad (18)$$

$$V(\theta)=\left[R_{-M}{}^H\bar{b}(\theta),...............,R_0{}^H\bar{b}(\theta)\right]\in\square^{(M+1)\times(M+1)} \quad (19)$$

Since $\sum_{m=-M}^{0}v_m=\|v\|^2=1$ and (16) can be rewritten as

$$J(\theta,v,a)=a^H Qa-a^H V(\theta)v-v^H V^H(\theta)a+M+1 \quad (20)$$

Keeping θ and v constant, differentiate (20) with respect to **a** and then setting the resultant expression to zero to obtain

$$\frac{\partial(\theta,v,a)}{\partial a}=2\left(Qa-V(\theta)v\right)\square\,0_{M+1} \quad (21)$$

Which leads to

$$a_{opt} = Q^{-1}V(\theta)v \quad (22)$$

Substituting (22) back into (16),

$$\min_{\theta} J(\theta, v) = M + 1 - v^H V^H(\theta) Q^{-1} V(\theta) v \quad (23)$$

Minimizing $-v^H V^H(\theta) Q^{-1} V(\theta) v$ is equal to maximizing its negative version.

Let $\sum_{i=1}^{M+1} \lambda_i z_i z_i^H$ be the Eigen value decomposition of $V^H(\theta) Q^{-1} V(\theta)$ with $\lambda_1 \geq .... \geq \lambda_{M+1}$ being the Eigen values and $\{z_i\}_{i=1}^{M+1}$ being the corresponding eigenvectors. As a result we have

$$\max_{\theta}\{v^H V^H(\theta) Q^{-1} V(\theta) v\} = \max_{\theta}\left\{\sum_{i=1}^{M+1} \lambda_i v^H z_i z_i^H v\right\} = \max_{\theta}\left\{\sum_{i=1}^{M+1} \lambda_i \left|v^H z_i\right|^2\right\} = \lambda_1 \quad (24)$$

Where the last equation holds if and only if **v** is the Eigen vector of $V^H(\theta) Q^{-1} V(\theta)$ corresponding to its maximum Eigen value that is $v = z_1$ and λ_1 is the maximum Eigen value. Therefore (23) can be further simplified as

$$\min_{\theta} J(\theta) \quad (25)$$

$$J(\theta) = M + 1 - \max eig\{V^H(\theta) Q^{-1} V(\theta)\} \quad (26)$$

Where $\max eig(.)$ denotes the maximum Eigen value of a matrix. Thus we can have the pseudo output power spectrum as

$$D(\theta) = \frac{1}{M + 1 - \max eig\{V^H(\theta) Q^{-1} V(\theta)\}} \quad (27)$$

The DOAs are selected as the angles corresponding to the highest local maxima of $D(\theta)$

4. Simulation Results

4.1 Spatial spectrum of proposed algorithm and MUSIC algorithm

The figure 2 & 3 displays how two coherent signals are recognized by MUSIC algorithm and proposed algorithm. Here two signals with equal powers arrive at a 5 element ULA from angles and the number of snapshots is N=400. The SNR is set to be 10dB. The simulation results are shown in the figure 2 & 3. As can be seen from the figures for coherent signals, classic MUSIC algorithm has lost effectiveness while proposed algorithm can be effectively applied to remove the signal correlation feature, which can distinguish the coherent signals and estimate the angle of arrival more accurately. This verifies the fact that when given a 5 element ULA, the proposed method can resolve both the coherent signals where the classic MUSIC algorithm fails.

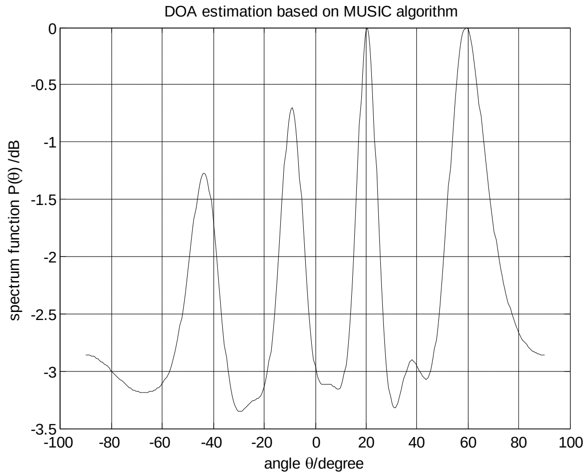

Figure 2: Spatial spectrum of DOA estimation of coherent signals using MUSIC algorithm

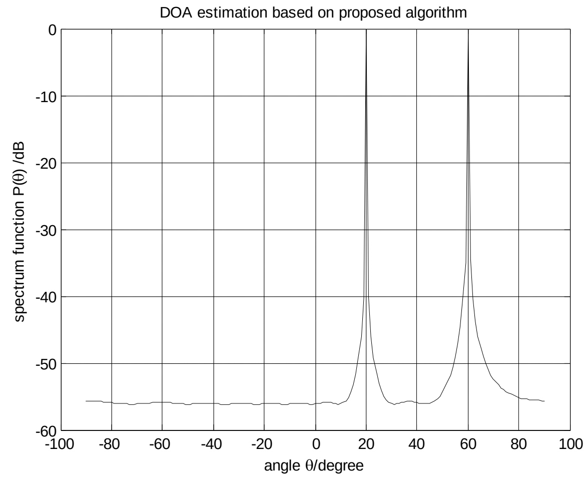

Figure 3: Spatial spectrum of DOA estimation of coherent signals using proposedalgorithm

4.2 The Relationship between DOA estimation &SNR

The figure 4 shows how two signals are recognized by the proposed algorithm under different SNRs. There are two independent narrow band signals, the incident angle is $20^0$and 60^0 respectively. The number of snapshots is 400, the SNR is -20dB, 0dB and 20db.The simulation results are shown in the figure 4.

As can be seen from the figure the dashed line shows the SNRs is -20dB, the solid line shows the SNR is 0dB & dash-dotted line shows the SNR is 20dB, with the other conditions remaining unchanged, with the increase in SNR, the beam width of DOA estimation spectrum becomes narrow, the direction of signal becomes clearer and the accuracy of proposed algorithm is also increased.

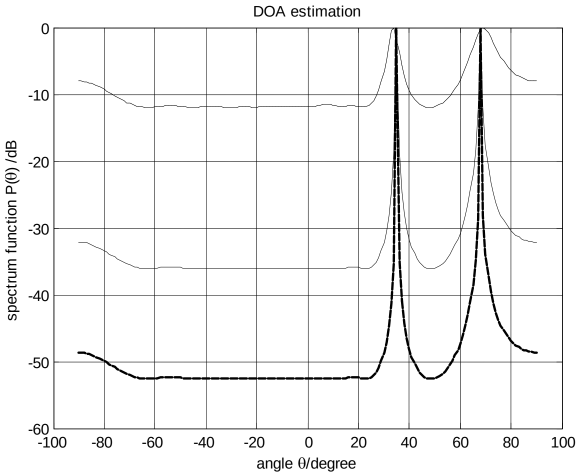

Figure 4: Relationship between DOA estimation and SNR

4.2 The Relationship between DOA estimation & number of snapshots

The figure 5 shows how two coherent signals are recognized by the proposed algorithm when different snapshots are used. There are two independent narrow band signals, the incident angle is 20^0 and 60^0 respectively, the SNR is 20dB, the number of snapshots is 5, 50 & 200.

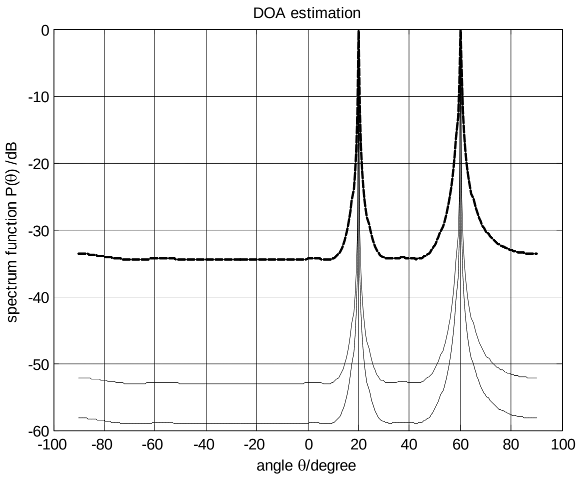

Figure 4: Relationship between DOA estimation and number of snapshots

As can be seen from the figure, the dashed line shows the number of snapshots are 5, the solid line shows the number of snapshots are 50 and the dash dotted line shows the number of snapshots are 200. With the other conditions remaining unchanged and with the increase in the number of snapshots the beam width of DOA estimation spectrum becomes narrow, the direction of array element becomes good and accuracy of proposed algorithm is increased.

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