Form Factors Calculations of $^{54,56}Fe$ Isotopes with Core Polarization Effects

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Abstract: ***The form factors for the inelastic electron scattering to 2^+,6^+ states in $^{54,56}Fe$ are studied in the framework of shell model. The calculation is performed in (0f7/2,1p3/2,0f5/2,1p1/2) model space as well as extended 2ħω model space. The predictions of charge form factors which includes core-polarization effects to first order were compared with the experimental values. It is noticed that the core polarization effects are essential in obtaining a good agreement between the calculated inelastic longitudinal F(q)'s and those of experimental data.***

Keywords: form factar, core polarization

1. Introduction

The core-polarization effect has been included in order to account for the contribution of configurations from outside of the model space in the transition. Three model spaces are adopted in this work which are fp shell model space for, , ^{54}Fe and nuclei, 1f7/2 subshell orbit as a model space for ^{56}Fe nuclei and 1f7/21d3/2 configuration as a model space for ^{54,56}Fe Core-polarization effects are taken into account through first order perturbation theory, which allows particle-hole excitation from shell core orbits 1s1/2, 1p3/2, 1p1/2, 1d5/2, 1d3/2 and 2S1/2.

The cp effects are calculated with the realistic effective interaction modified surface delta interaction (MSDI) [1] as a residual interaction. In this interaction, the Elliot fitting has been used to calculate the radial integral. In this approach one-derived two-body matrix elements in a particular single-particle basis directly from the scattering phase shift. The HO potential is used to calculate the radial part of the single-particle wave functions, with size parameter b fitted to get the root mean square radius(r m s) of each nuclei

Comparison between calculated and measured longitudinal electron scattering form factors has long been used as stringent tests of models for transition densities. Various microscopic and macroscopic theories have been used to study excitations in nuclei [2]. Calculations of form factors using the model space wave function alone is inadequate for reproducing the data of electron scattering [3]. Therefore, effects out of the model space, which are called core polarization effects, are necessary to be included in the calculations. The intermediate one-particle one-hole states are taken up to $2\hbar\omega$ excitation. These effects are found essential for obtaining a quantitative agreement with the experimental data[9]. The (MSDI) were used in this case as a residual interaction. The strength of the MSDI denoted by A_T ,B and C are set equal to A_0=A_1=B= 4.212 MeV and C=0. The single particle wave function were those of the harmonic oscillator potential (HO) with size parameter *b* chosen to reproduce the measured ground state root mean square charge radii of these nuclei. The one-body density matrix (OBDM) elements ($\chi^{\Lambda}\Gamma_f\Gamma_i(\alpha_f,\alpha_i)$) are calculated using the shell model code OXBASH[4]

2. Theory

The electron scattering form factor for a given multipolarity λ and momentum transfer q is expressed as[1],

$$\left|F_\lambda(q)\right|^2=\frac{1}{2j_i+1}\left(\frac{4\pi}{Z^2}\right)\left|\left\langle\Gamma_f \parallel \hat{T}_\lambda^\xi \parallel \Gamma_i\right\rangle\right|^2 \left| F_{f.s}F_{c.m}\right|^2 \quad (1)$$

Where $F_{f.s}=e^{-0.43q^2/4}$ is the finite nucleon-size correction and $F_{c.m}=q^2b^2/4A$ is the center of mass correction, A is the mass number and b is the harmonic oscillator size parameter.

The effect of the core polarization on the form factors is based on a microscopic theory, which combines shell-model wave functions and configuration with higher energy as particle-hole perturbation expansion. The reduced matrix element of the electron scattering operator $\hat{T}_\lambda^\xi$ is expressed as a sum of the fp-model space (p) contribution and the core-polarization (cp) contribution, as follows [5]

$$\left\langle\Gamma_f \parallel \hat{T}_\lambda^\xi \parallel \Gamma_i\right\rangle=\left\langle\Gamma_f \parallel \hat{T}_\lambda^\xi \parallel \Gamma_i\right\rangle_{ms}+\left\langle\Gamma_f \parallel \delta\hat{T}_\lambda^\xi \parallel \Gamma_i\right\rangle_{cp} \quad (2)$$

with ξ selection the longitudinal (L), electric(E) and magnetic(M) transverse form factors, respectively. The Greek symbols were used to denote quantum numbers in coordinate space and isospace, i.e. $\Gamma_i\equiv J_iT_i$, $\Gamma_f\equiv J_fT_f$ and $\lambda\equiv JT$. The fp-shell model space element can be expressed as linear combination of the single-particle matrix element[6]

$$\left\langle\Gamma_f \parallel \hat{T}_\lambda^\xi \parallel \Gamma_i\right\rangle_{ms}=\sum_{\alpha_f\alpha_i}\chi^\lambda\Gamma_f\Gamma_i(\alpha_f,\alpha_i)\left\langle\alpha_f \parallel \hat{T}_\lambda^\xi \parallel \alpha_i\right\rangle \quad .....(3)$$

where $\chi^{\lambda}\Gamma_f\Gamma_i(\alpha_f,\alpha_i)$ are the structure factors (one body density matrix element), given by,

$$\chi^{\lambda}\Gamma_f\Gamma_i(\alpha_f,\alpha_i)=\frac{\left\langle \Gamma_f \left\| \left[a^{+}(\alpha_f)\otimes\tilde{a}(\alpha_i)\right]^{\lambda} \right\| \Gamma_i \right\rangle}{\sqrt{2\lambda+1}} \quad \text{.....(4)}$$

The α_f and α_i label are single-particle states for the fp-shell model space. Similarly, core-polarization matrix element as follow:

$$\left\langle \Gamma_f \left\| \delta\hat{T}_{\lambda}^{\xi} \right\| \Gamma_i \right\rangle_{cp}=\sum_{\alpha_f\alpha_i}\chi^{\lambda}\Gamma_f\Gamma_i(\alpha_f,\alpha_i)\left\langle \alpha_f \left\| \delta T_{\lambda}^{\xi} \right\| \alpha_i \right\rangle \quad \text{.....(5)}$$

Up to the first order perturbation theory, the single-particle matrix element for the higher-energy configuration is given by[7]

$$\left\langle \alpha_f \left\| \delta T_{\lambda}^{\xi} \right\| \alpha_i \right\rangle=\left\langle \alpha_f \left\| \hat{T}_{\lambda}^{\xi}\frac{Q}{E_i-H_0}V_{res} \right\| \alpha_i \right\rangle+\left\langle \alpha\Gamma_f \left\| V_{res}\frac{Q}{E_f-H_0}\hat{T}_{\lambda}^{\xi} \right\| \alpha\Gamma_i \right\rangle \quad \text{......(6)}$$

The operator Q is the projection operator on the space outside the model space. E_i and E_f are the energies of initial and final states. For the residual interaction V_{res} the MSDI and M3Y were adopted.

The two term in right hand side of Eq.(6) can be written as[8]

$$\sum_{\alpha_1\alpha_2\Gamma}\frac{(-1)^{\alpha_i+\alpha_2+\Gamma}}{e_{\alpha_i}-e_{\alpha_f}-e_{\alpha_1}+e_{\alpha_2}}(2\Gamma+1)\begin{Bmatrix}\alpha_f & \alpha_i & \lambda\\ \alpha_2 & \alpha_1 & \Gamma\end{Bmatrix}\sqrt{(1+\delta_{\alpha_1\alpha_f})(1+\delta_{\alpha_2\alpha_i})}\left\langle \alpha_2 \right\| T_{\lambda} \left\| \alpha_1 \right\rangle \times\left\langle \alpha_f\alpha_1 \right| V_{res} \left| \alpha_i\alpha_2 \right\rangle_{\Gamma} \quad \text{.....(7)}$$

Where α_1 runs over particle states and α_2 over hole state and e is the single-particle energy .

$$e_{nlj}=(2n+l-\frac{1}{2})\hbar\omega+\begin{cases}-\frac{1}{2}(l+1)\langle f(r)\rangle_{nl} & for\ j=l-\frac{1}{2}\\ \frac{1}{2}l\langle f(r)\rangle_{nl} & for\ j=l+\frac{1}{2}\end{cases} \quad \text{.....(8)}$$

With $\langle f(r)\rangle_{nl}\approx -20A^{-2/3}$ and $\hbar\omega=45A^{-1/3}-25A^{-2/3}$

The electric transition strength is given by,

$$B(C\lambda)=\frac{|(2\lambda+1)!!|^2}{4\pi}\frac{Z^2}{k^{\lambda}}|F_{\lambda}(k)|^2$$

Where k = E_x /ħc .

3. Results and discussion

The core polarization effect is calculated with the MSDI as residual interaction for ^{54,56}Fe. The parameters of MSDI are denoted by A_T , B and C. where T indicates the isospin (0,1). These parameters are taken to be A_0=A_1=B= 4.212MeV and C=0. The results including cp effects are shown by solid curve.

3.1 The Charge Form Factor for 0^+2 State

The charge form factor C2 is first property to be discussed where the charge distribution scale is presented with excitation energy E_x=1.913MeV. Fig.(1) shows the C2 electron scattering form factor as a function of momentum transfer (q). From fig.(1) the experimental data and theoretical curve at the first maximum are coincident but at the second the theoretical one quenched. We show three peaks in the theoretical data which due to core polarization effect.

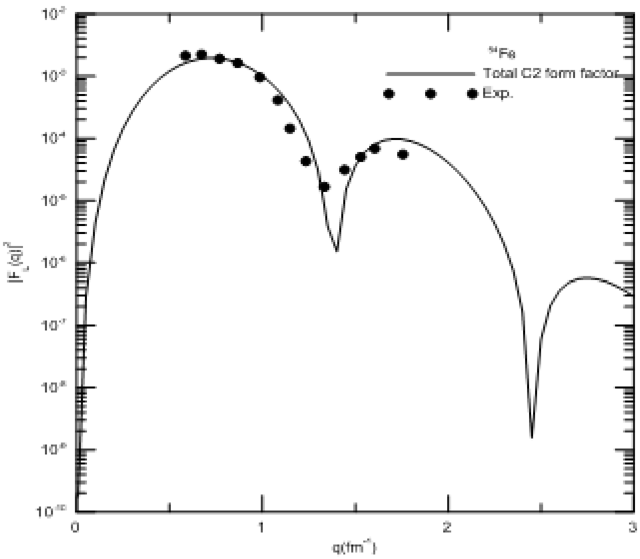

Figure 1: Inelastic longitudinal form factors for the transition to the 2^+ in the ^{54}Fe,the experimental data are taken from ref. [9]

3.2 The charge form factor for 0^+4 state

The quadrupole transition C2 charge form factor for the 1f2p-shell model calculation in ^{56}Fe with excitation energy 1.08MeV is illustrated in fig.(2) for the first state. An excellent agreement is obtain for the first maximum of form factors where the data are correctly reproduced up to q=1.2 fm^{-1} . For the second maximum the data are over estimate and the third maximum the curve are under estimate

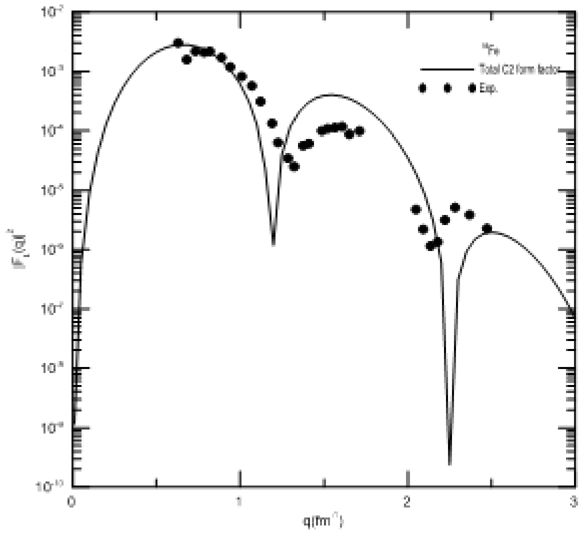

Figure 2: Inelastic longitudinal form factors for the transition to the 2^+ in the ^{56}Fe the experimental data are taken from ref. [9]

3.3 The Charge Form Factor for 0^+2 State

Fig.(3) displayed the total form factor of ^{54}Fe with excitation energy 2.2MeV. It is clear that the first maximum is excellent agreement with experimental data especially at 1.4 fm^{-1} of momentum transfer, we notice that the theoretical curve has two peaks this due to core polarization. The contribution of core polarization are give a good agreement with experimental data.

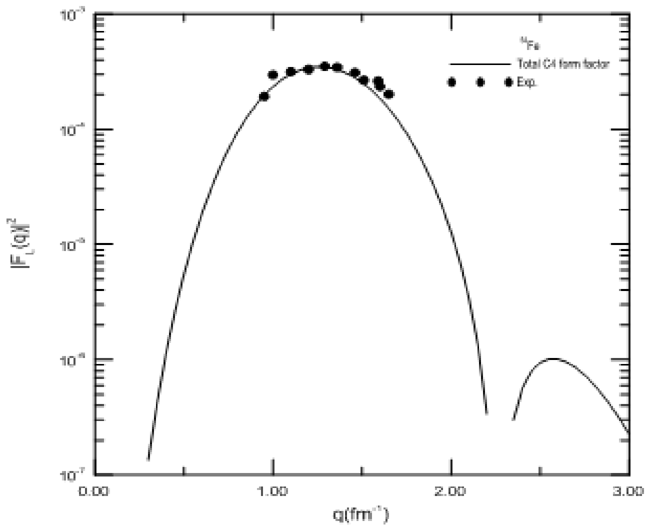

Figure 3: Inelastic longitudinal form factors for the transition to the 4^+ in the ^{54}Fe, the experimental data are taken from ref. [9]

The form factor for C4 transition in ^{54}Fe with an excitation energy E_x=2.018MeV is displayed in Fig. (4) where the total contribution are due to core polarization. The data are well explained for the first maximum and also up to q=1.6fm^{-1} . Higher q value are over estimated. The second peak is over estimated.

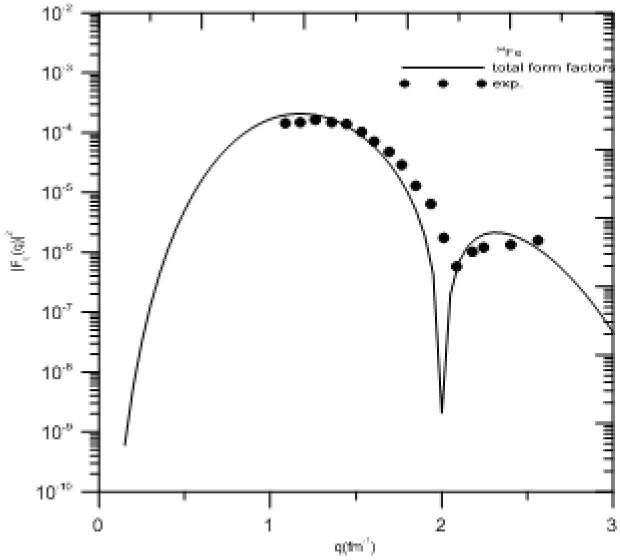

Figure 4: Inelastic longitudinal form factors for the transition to the 4^+ in the ^{56}Fe ,the experimental data are taken from ref. [9]

3.4 The Charge Form Factor for$2\ ^+4$ State

From Fig.(5) it is clear that the core model space and total form factor have well distribution in this type of transition, the first maximum of the core has a positive contribution with respect of model space, but the second maximum has negative with respect of model space. This behavior comes from the fact that the core contribution is weak in the second maximum.

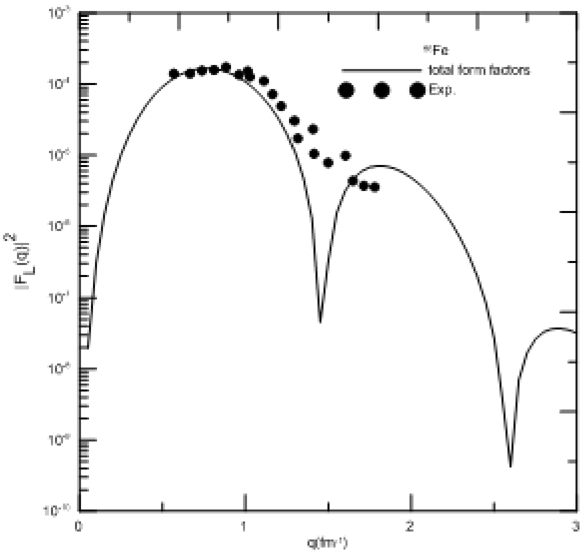

Figure 5: Inelastic longitudinal form factors for the transition to the 4^+ in the ^{56}Fe, the experimental data are taken from ref. [9]

4. Conclusions

1) Core polarization has an enhancement property that makes the use of the core necessary and beneficial
2) Radial wave functions of some single particle states of the inclusion and exclusion from the model space will make the form factor shift along the q-values

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