

Generation of Gravitational Waves

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Abstract: Gravitational waves are the consequence of some other fields. So, it has generated due to interference of some other sources. The amplitude of any gravitational waves incident on Earth is small. And such several fields of different amplitudes constitute stronger waves for that planet.

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1. Introduction

It is to see that the amplitude of any gravitational waves incident on Earth is small. A strong gravitational wave should have $h_{\mu\theta} = 0$, and we should expect amplitudes like this only near the source, where the Newtonian potential would be of order 1. For a source of mass M, this would be at distance of order M from it. As with all radiation fields, the amplitude of the gravitational waves falls off as r^{-1} far from the distance. So, if the Earth is a distance R from a source of mass M, the largest amplitude waves as we should expect are of order M/R. For a formation of a $10M_{\odot}$ black hole in a supernova explosion in a nearby galaxy 10^{23} m away, this is about 10^{-17} . This is in fact an upper limit in this case, and less violent events will lead to very much smaller amplitudes.

An approximate calculation of wave generation. Our subject is to solve equation

$$\left(\frac{\partial^2}{\partial t^2} + \nabla^2\right) h_{\mu\theta} = -16\pi T_{\mu\theta} \dots\dots\dots(1.1)$$

Here, we will make some simplifying but realistic assumptions. We assume that the time dependent part of $T_{\mu\theta}$ is in sinusoidal oscillation with frequency Ω , i.e the real part of

$$T_{\mu\theta} = S_{\mu\theta} (x^i) e^{-i\Omega t} \dots\dots\dots(1.2)$$

And that the region of space in which $S_{\mu\theta} \neq 0$ is small compared with $2\pi/\Omega$, the wave length of a gravitational wave of frequency Ω . The first assumption is not much of a restriction, since a general time dependence can be reduced to a sum over sinusoidal motions by Fourier analysis. Besides, many interesting astrophysical sources are roughly periodic pulsating stars, pulsars, binary systems. The second assumption is called the slow motion assumption, since it implies that the typical velocity inside the source region, which is Ω times the size of that region, should be much less than 1. All but the most powerful sources of gravitational waves probably satisfy this condition.

Let us look for a solution for $h_{\mu\theta}$ of the form

$$h_{\mu\theta} = B_{\mu\theta} (x^i) e^{-i\Omega t} \dots\dots\dots(1.3)$$

Putting this and equation (1.2) into equation (1.1) gives

$$(\nabla^2 + \Omega^2) B_{\mu\theta} = -16\pi S_{\mu\theta} \dots\dots\dots(1.4)$$

It is important to bear in mind as we proceed that the indices on $h_{\mu\theta}$ in equation (1.1) play almost no role. We shall regard each component $h_{\mu\theta}$ as simply a function on Minkowski space, satisfying the wave equation.

2. Gravitation as a Consequence of Other Fields

Let us return of a theory of gravitation. In general we expect that there should be two schools of thought about what to do with the new phenomenon. These are

- (a) That gravitation is a new field
- (b) That gravitation is a consequence of something that we already know, but we have not calculated correctly.

Let us take the second case and see whether it has any possibilities. The fact of a universal attraction might remind us of the situation in molecular physics. We know that a;; molecules attract one another by a force which at long distance goes like $1/r^6$. This is universal well known from the fact that all substances may be made to condense by cooling them sufficiently. One possibility is that gravitation may be some attraction due to similar fluctuations in something, perhaps having to do with charge.

It may consider whether gravitational forces might not come from the virtual exchange of a particle which is already known, such as the neutrino. After all, superficially it has the right qualities, since it is a neutral particle of zero mass, so that its interaction would go like $1/r$ and its interaction will be very weak.

3. Emitted Energy in the Form of Gravitational Radiation

The energy emitted in the form of gravitational radiation will be calculated by a system whose energy momentum tensor can be expressed as a Fourier integral,

$$T_{\mu\theta} (x,t) = \int_0^\infty d\omega T_{\mu\theta} (x,\omega) e^{-i\omega t} + \text{c.c} \dots\dots\dots(3.1)$$

Here, c.c means the complex conjugate of preceding term The above (3.1) equation can be expressed as a sum of Fourier components,

$$T_{\mu\theta} (x,t) = \sum_{\omega} e^{-i\omega t} T_{\mu\theta} (x,\omega) + \text{c.c} \dots\dots\dots(3.2)$$

We first do the calculation for a single Fourier component,

$$T_{\mu\theta}(x,t) = T_{\mu\theta}(x,\omega)e^{-i\omega t} + c.c \dots\dots\dots(3.3)$$

And then will return to the more general systems described by (3.1) and (3.2)

The conservation equation for $T_{\mu\theta}(x,t)$ is

$$\frac{\partial}{\partial x^n} T_{\mu\theta}(x,t) = 0$$

Applying this to (3.3) becomes

$$\frac{\partial}{\partial x^i} T_{i\theta}(x,\omega) - i\omega T_{0\theta}(x,\omega) = 0 \dots\dots\dots(3.4)$$

If the energy momentum tensor is a sum of individual Fourier components as in (3.2), then the field $h_{\mu\theta}$ in the wave zone will look like a sum of the plane wave

$$h_{\mu\theta}(x,t) = e_{\mu\theta}(x,\omega)e^{ik_\mu x^\mu} + c.c \dots\dots\dots(3.5)$$

The gravitational energy momentum tensor will then be given by a double sum over these Fourier components, but all cross terms drop out when we average over a time interval long compared with the longest “beat period”, that is, the reciprocal of the shortest frequency difference.

The gravitational radiation produced by the collisions occurring in a gas can be determined by the summing up the radiated energies per collision given by the following equations

$$\frac{dE}{d\omega} = \frac{G}{2\pi} \sum_{NM} n_N n_M \frac{1+\beta_{NM}^2}{\beta_{NM} \sqrt{1-\beta_{NM}^2}} \ln\left(\frac{1+\beta_{NM}}{1-\beta_{NM}}\right) \dots\dots\dots(3.6)$$

Where β_{NM} the relative speed of particles N and M

$$\text{And } \beta_{NM} = \sqrt{1 - \frac{m_N^2 m_M^2}{(P_N - P_M)^2}} \dots\dots\dots(3.7)$$

For non-relativistic two body elastic scattering this reduces to

$$\frac{dE}{d\omega} = \frac{8G}{5\pi} \mu^2 v^4 \sin^2 \theta \dots\dots\dots(3.8)$$

Where μ the reduced mass, v is the relative velocity, and θ is the scattering angle in the centre of mass reference frame.

4. Conclusions

Hence, we conclude that gravitational waves are the consequence of the interference of some other fields. For a planet, the amplitude of any gravitational waves incident on Earth is small. And such several fields of different amplitudes constitute stronger waves. Gravitational forces might not come from the virtual exchange of a particle which is already known, such as the neutrino. After all, superficially it has the right qualities, since it is a neutral particle of zero mass, so that its interaction would go like $1/r$ and its interaction will be very weak.

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Author Profile



Dr. Pradeep Kumar ,received PhD in 1995 from BRA Bihar University, Muzaffarpur, Bihar, an associate professor & head of the department of Mathematics M.S College, Motihari, Bihar, worked two UGC, MRP on the topics “ Parameterized Post Newtonian Formalism: Gravitation & Application” (2001-03) and “ Super Energy of Gravitational Waves” (2011-13), worked as PhD guide, published several research papers, recently “ Gravitational Spiral Waves” vol 2 issue 6th June 2013, “Intensive Energy of gravitational waves”, vol 2 7th July 2013 , “ Quantum Effects in Gravitation, vol 2 ,10th oct 2013,and Gravitational Lens, vol 3, issue 9, sept 2014 all published in www.ijsr.net