International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2013): 6.14 | Impact Factor (2014): 5.611

Fastest Iteration Method for Estimation of Solution of Nonlinear Function $f(x) = 0$ (Bennie's Method)

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Abstract: ***There has been a considerate progress and achievement in development of mathematical models and different formulas of which they are used to describe behavior of different systems. Several Numerical methods were used for estimations of fixed points, roots, and series expansion. However in many cases when roots were evaluated using the fixed point method, the number of iteration required to end process by inferring the final answer was very long, sometimes it took more than 1000 iterations inferring the final answer .Fixed point method has been found not to converge in some functions, $f(x) = 0$ thus disclosed failure. In this paper we introduced the fastest method of iteration (Bennie's iteration) for Estimation of roots of a functions and square roots with zero error-bounds and high convergence speed. Fixed point iteration and slop finding had accelerated the establishment of the algorithm. With this approach mathematicians found easier and straightforward to evaluate roots of a function manually. Moreover the algorithm resolved all problems of which fixed point method found to diverge. In addition to that this paper deduced the newton's method from the proposed Bennie's method (scheme).***

Keywords: Numerical methods, fastest iteration method, high convergence speed, small bound error, fixed point method

1. Introduction

The numerical methods are powerful algorithms used for estimation of solutions for many nonlinear single variable systems or functions, but still we have found that to some of functions these methods were unable to work due to theoretical limitations. [1]And therefore mathematicians have been suggesting the alternative methods for high accuracy.

This paper was firstly aimed to solve the problem of slow convergence speed revealed in fixed point method. Secondly this paper is intended to find solutions of all functions where the condition $|g\acute{}(x)| < 1$ on [a b] is not met for a chosen starting point of $x_0 \in [a\ b]$, for which a fixed point $x \in [a, b]$. Those functions such that $|g\acute{}(x)| \geq 1$, (for which fixed point failed to converge)

Thirdly this paper is intended to give a scheme which gives small error bound estimated to zero, further more in this research paper we will develop a new scheme calledfastest iteration (Bennie's method) for finding square rootsand deduce newton's method

2. Related Works

Despite having different works related to methods ofsolving function $f(x) = 0$, for nonlinear function of single variable, we have common methods which are frequently used, these are fixed point method, newton's method, regular falsi method, secant method, bisection method, Muller's method, and deflation method [2]

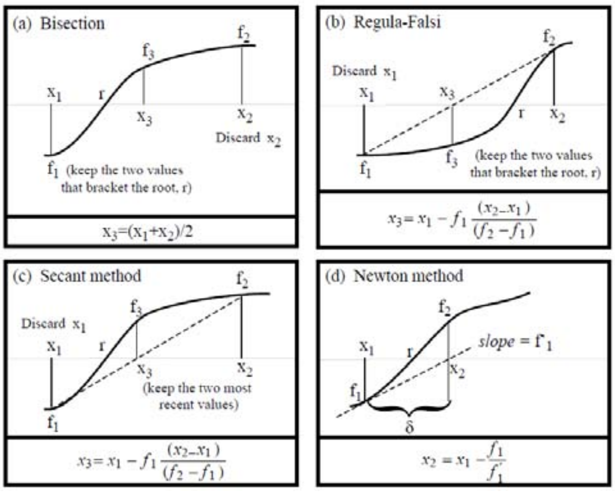

Figure 1: bisection, Regula falsi, Newton Method, Secant Method demonstration [2].

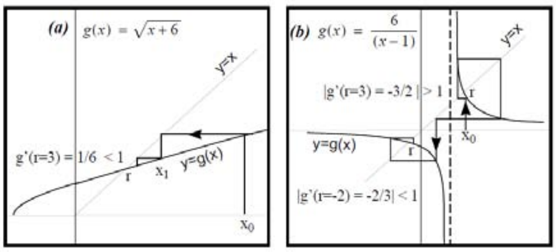

Figure 2: Fixed Method Demonstration [2]

However in this research we analyze fixed point method and Newton's Method as related works. And these works will be used as the reference, motivating factors for foundation of Bennie's method.

A fixed point Method can be defined as an algorithm of the form $x_{n+1} = g(x_n)$ (1)

Which used for solving $(x) = 0$, where n is the number of iteration we can go. The technique used is first to change $f(x) = 0$ to a form $x = g(x)$[2].

Or if satisfies $x^* = g(x^*)$,then x^* is called a fixed point of$g(x)$(2)

A fixed point method is guaranteed to work if $|g'(x)| < 1$ on [a b], for which $x \in [a, b]$

Under this condition $x_{n+1} = g(x_n)$ is saidto converge if as $n \to 0 \lim x_{n+1} = \lim g(x_n) = x$ [1]-[3]-[5].

Fixed point method was found to be the most popular method which was definitely dependent of the series of iteration, the series had Stop criteria which states if $if\ x_{n+1} = x_n$ under a certain number of digit accuracy, where $x_{n+1} = g(x_n)$ then the above x_{n+1} is considered to be an estimated solution and iteration process should stop there [1]-[2]-[5]
For a given$f(x) = 0$, in order to use fixed method, then we rewrite $f(x) = 0$ in terms of $x = g(x)$,

Then $x_{n+1} = g(x_n)$ Becomes an iteration generating function, with starting point being$x_1 = g(x_0)$, where $x_0$is a chosen starting point, $x_1$is a first value generated. [1]- [5].

The following examples used to demonstrate slow convergence of fixed point method
Solve for $f(x) = 0$, in the following functions of which$(a)\ f(x) = 2(x-1)^{\frac{1}{2}} - x$, with starting point $x_0 = 2.5$, (b)$f(x) = e^{-x} - x$ Starting with $x_0 = 0.5$

The following examples used to demonstrate failure of fixed point method (c) $f(x) = -4 - x + 4x - 1/2x^2$starting with $x_0 = 1.9$ [1].
$(d) f(x) = 2(x-1)^{1/2} - x$, $x \in [1\ 2]$ Starting with x_0=1.5, $(e) f(x) = 1 - x - \frac{x^2}{4}, x \in [-3 - 1\ ]$ Starting with x_0= -2.05,$(f) f(x) = e^{-2x}(x-1)$
, $x \in [0\ 2\ ]$Starting with x_0= 0.99 [1].
Solution for $(a)\ f(x) = 2(x-1)^{\frac{1}{2}} - x$, with starting point $x_0 = 2.5$ the function $f(x) = 0$ has to be rewritten to$x = g(x)$
We get $2(x-1)^{\frac{1}{2}} - x = 0 \to x = 2(x-1)^{\frac{1}{2}}$
Where$g(x) = 2(x-1)^{\frac{1}{2}}$, and iteration generating function will be$x_{n+1} = g(x_n) = 2(x_n - 1)^{\frac{1}{2}})$ [1].

Table 1: Results of 1003 iterative procedure

n	x_n	$g(x_n)$
0	2.5	2.449489743
1	2.449489743	2.37309514
2	2.37309514	2.34358284
⋮	⋮	⋮
1001	2.00398714	2.00398317
1002	2.00398317	2.00397921

$$x_{1002} = 2.00397921$$
$$lim x_n = 2$$
The sequence converges too slowly to $x = 2$
(More than 1000 iteration)[1].

Absolute and relative Error Considerations – this is the difference between consecutive terms

$$|x_{1002} - x_{1001}| = |2.00397921 - 2.00398327| = 0.00000396$$
$|x - x_{1002}|$=|2.00000000-2.00397921|=0.00397921
This observation shows that the closeness of consecutive terms does not guarantee that accuracy has been achieved, since the difference
$|x - x_{1002}|$ =0.00397921 which is used to approximate the value of x is larger compared to the difference $|x_{1002} - x_{1001}|$ =0.00397921.but it is usually the only criterion available and often used to terminate an iterative procedure. [1].
$(b)\ f(x) = e^{-x} - x$ Starting with $x_0 = 0.5$
Rewrite to $x = g(x)$, we obtain to $x = e^{-x}$
Then iteration generating function is
$$x_{n+1} = g(x_n) = e^{-x_n}$$

Table 2: Results of 23 iterations

n	x_n	$g(x_n)$
0	0.5	0.606531
1	0.606531	0.545239
2	0.545239	0.579703
⋮	⋮	⋮
9	0.566409	0.567560
⋮	⋮	⋮
20	0.567145	0.567142
21	0.567142	0.567144
22	0.567144	0.567143

The sequence is converging, such that
$lim_{n\to\infty} x_n = 0.567143$, which is the solution for $e^{-x} - x = 0$[1]– [5].
$(c)\ f(x) = -4 - x + 4x - (1/2)x^2$ Starting with $x_0 = 1.9$, rewrite to $x = g(x)$,
For$-4 - x + 4x - (1/2)x^2 = 0$, we obtain
$$x = -4 + 4x - \left(\frac{1}{2}\right)x^2$$
Iteration generating function
$$x_{n+1} = g(x_n) = -4 + 4x_n - \left(\frac{1}{2}\right)x_n^2$$

Table 3: Results of 6 iterations

n	x_n	$g(x_n)$
0	1.9	1.795
1	1.795	1.5689875
⋮	⋮	⋮
5	−5.52988294	−41.4093344

It is clearly observed the results obtained by iterative procedure diverges from the correct solution of a function$f(x) = 0$, which is $x = 2\ or\ x = 4$.The divergence is due to the starting point chosen x_0 does not make the satisfaction condition such that$|g'(x_0)| < 1$ since
$$g(x) = -4 + 4x - \left(\frac{1}{2}\right)x^2\ and\ g'(x) = 4 - x, \to\ g'(1.9) = 2.1 > 1$$
Therefore the fixed point method cannot work for this case [1].However the question remains that if the fixed point method cannot work does it mean that there is no solution for this$f(x) = 0$, the answer is no.
$(d) f(x) = 2(x-1)^{\frac{1}{2}} - x$, $x \in [1\ 2]$ Starting with x_0=1.5, the function $f(x) = 0$ has to be rewritten as

$x = g(x)$, we get $2(x-1)^{\frac{1}{2}} - x = 0 \to x = 2(x-1)^{\frac{1}{2}}$
Where $g(x) = 2(x-1)^{\frac{1}{2}}$, and iteration generating function will be$x_{n+1} = g(x_n) = 2(x_n - 1)^{\frac{1}{2}}$

Table 4: Results of 5 iterations for problem(d)

n	x_n	$g(x_n)$
0	1.5	1.41421356
1	1.41421356	1.28718851
2	1.28718851	1.07179943
3	1.07179943	0.53590832
4	0.53590832	$2(-0.46409171)^{1/2}$

Since $x_5 = g(x_4)$ lies outside the domain of $g(x)$, the term $g(x_5)$ cannot be computed.It gives the complex value. [1].Nevertheless we know that the solution of $f(x) = 0$ is available $x = 2$ .fixed point method was able to infer the answer when a starting point was $x_0 = 2.5 in\ (a)$since

$$g(\acute{x})' = (x-1)^{-1/2} at\ x_0 = 2.5$$
$$\to g(\acute{x_0}) = g(\acute{2.5}) = 0.81649658 < 1$$

But with$x_0 = 1.5$,the fixed point does not guarantee the solution due to $|g(\acute{x})| = (x-1)^{-1/2} at\ x_0 = 1.5$

$$\to |g(\acute{x_0})| = |g(\acute{1.5})| = 1.41421356 > 1$$

Therefore *since* $g(\acute{x})$ is greater than 1, the fixed point method guarantee not to work. [4]−[2]

$(e) f(x) = 1 - x^2/4$,$x \in [-3 - 1]$ Starting with $x_0 = -2.05$, $x = g(x) = 1 + x - \frac{x^2}{4}$
$g(x)' = 1 - x/2$ On $x \in [-3 - 1]$,

$$x_{n+1} = g(x_n) = 1 + x_n - x_n^2/4$$

Table 5: Results of 8 iteration of problem(e)

n	x_n	$g(x_n)$
0	−2.05	−2.10062500
1	-2.10062500	.−2.20378135
2	.−2.20378135	−2.41794441
3	−2.41794441	−2.87955819
⋮	⋮	⋮
7	−17.61661519	−94.20289788

The sequence will not converge to $x = -2$ as one solution for $f(x) = 0 = 1 - x^2/4$ since
$|g(x)'| \geq \frac{3}{2}$ on [-3 -1], [1].For this function for any value $x \in [-3 - 1]$ .The $|g(x)'| \geq 3/2$ implies that the fixed point method will definitely unable to find the solution for such case, though the solution exit.

$(f) f(x) = e^{-2x}(x-1)$,$x \in [0\ 2]$ starting with x_0= 0.99. From $f(x) = 0$, Implies
$0 = e^{-2x}(x-1) \to\ x = x + e^{-2x}(x-1) = g(x)$,

Therefore;the iterative sequence is

$$x_{n+1} = g(x_n) = x_n + e^{-2x_n}(x_n - 1)$$

Table 6: Results of 5 iterations of (f) with x_0= 0.99

n	x_n	$g(x_n)$
0	0.99	0.98861931
1	0.98861931	0.98704364
2	0.98704364	0.98524416
3	0.98524416	0.98318736
4	0.98318736	0.98083420

The sequence does not converge to 1, since $|g(x)'|$=1.14083062 at $x = 0.99$, but what if we start with $x_0 = 2$?

Table 7: Results of 2 iterations of (f) with $(x_0$= 2)

n	x_n	$g(x_n)$
0	2	2.01831564
1	2.01831564	2.036295889

We satisfy ourselves that the iterative procedure diverges from $x = 1$, since the $|g(x)'| = 1.01216201$ at $x = 2$
Which is the value of which the fixed point method will not work [1].

3. Proposed Algorithm

The main purpose of this research is to reveal the efficient algorithm for estimating solutions of single variable nonlinear functions $f(x) = 0$,it intends to resolve and cover all problems associated with functions of which fixed point method failed to give the solutions i.e. $|g(\acute{x})| \geq 1$, furthermore the scheme intends to reduce the number of iterations inferring the answer, moreover the algorithm will subsequently give an additionaliteration(Bennie's iteration) for finding the square roots of any given value A.In addition to that the paper will deduce the newton's method from the proposed Bennie's method (scheme)

A. Foundation Of The Idea/Concept

The concept to the foundation of the algorithm based on the idea of fixed point iteration, and the slopof the curve concept which were together linked to generate the algorithm, and this can be well understood geometrically below

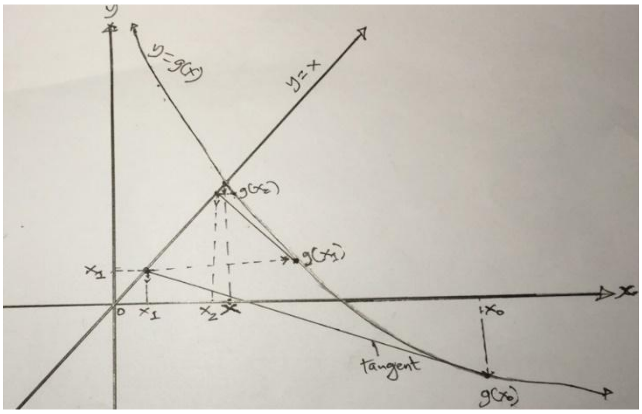

Figure 3: geometrical presentation of Bennie's method

Geometrically a fixed points of a function $y = g(x)$, Are points of intersection of $y = g(x)$ and $y = x$ [1].It is observed that the points from $x_0, x_1, and\ x_2$ converges to x Where x is fixed point or intersection point of $y = g(x)$ and $and\ y = x$,from the graph we ca derive the a formula

Slop of $g(x) at\ x_0 = \frac{\Delta y}{\Delta x} = \frac{x_1 - g(x_0)}{x_1 - x_0} = g(x_0)'$

$$x_1 - g(x_0) = (x_1 - x_0) g(x_0)'$$

$$x_1 - g(x_0) = x_1 g(x_0)' - x_0 g(x_0)'$$

$$x_1 - x_1 g(x_0)' = g(x_0) - x_0 g(x_0)'$$

$$x_1(1-g(x_0)') = g(x_0) - x_0 g(x_0)'$$

$$x_1 = g(x_0) - x_0 g(x_0)'/(1-g(x_0)')$$

$$x_1 = \frac{g(x_0) - x_0 g(x_0)'}{1-g(x_0)'} \quad (3)$$

This is the first term of iterative sequence, and therefore the n^{th}General iterative procedure will be

$$x_{n+1} = \frac{g(x_n) - x_n g(x_n)'}{1-g(x_n)'} = B(x_n) \quad (4)$$

Where $g(x_n)' \neq 1$

The stop criteria is when value of $x_{n+1}$is the same to x_n since

$$lim_{n\to\infty} x_n = x, \text{then} lim_{n\to\infty} x_{n+1} = x \quad (5)$$

The iteration sequence $x_{n+1} = B(x_n)$ we propose to name it as Bennie's iteration

B. Adjustment Wheng$(\mathbf{x})' = 1$

If the Bennie's Method does not work when for a chosen starting point x_0 gives$g(x_0)' = 1$, then for a given point x_0 an increase of ΔB can be added to make a new initial point such that ,a starting point will be $x_0 + \Delta B$

Where$\Delta B \leq 0.1$, and ΔB is called a Bennie's increase for initial value x_0

4. Testing of the Algorithm and Results

In this research we are going to apply how this developed iteration formula converge to an answer very fast with almost zero error bounds. We will see how it guarantee answers even to the functions with $|g(x)'| > 1$ .We are going to repeat finding solution for $f(x) = 0$ for all examples done by fixed point method but this time by Bennie's method.

$(a)\ f(x) = 2(x-1)^{\frac{1}{2}} - x$, with starting point $x_0 = 2.5$, rewrite in terms of $x = g(x)$, then we get

$x = g(x) = 2(x-1)^{\frac{1}{2}}$, and $g(x)' = (x-1)^{-1/2}$

$$B(x) = \frac{g(x) - xg(x)'}{1-g(x)'}$$

$$= \frac{2(\mathrm{x}-1)^{\frac{1}{2}} - \mathrm{x}(\mathrm{x}-1)^{-1/2}}{1-(\mathrm{x}-1)^{-1/2}}$$

Then

$$x_{n+1} = B(x_n) = \frac{2(x_n-1)^{\frac{1}{2}} - x_n(x_n-1)^{-\frac{1}{2}}}{1-(x_n-1)^{-\frac{1}{2}}}$$

Table 8: Results of 12Bennie's iterations $B(x_{11})$ for (a)

n	x_n	$B(x_n)$
0	2	2.22474487
1	2.01831564	2.10668192
	2.10668192	2.051989506
⋮	⋮	⋮
8	2.001586102	2.000792238
9	2.000792238	2.00039604
10	2.00039604	2.000198001
11	2.000198001	2.000098995

It is clearly seen the sequence converge to $x = 2$ and it converges as fast as 100 times fixed point method iteration's convergence speed,

Absolute and relative Error Considerations

$|x_{12} - x_{11}|$=|2.000098995-2.000198001|

=0.00009906

$|x - x_{12}|$=|2.00000000 − 2.000098995|

=0.000098995

This observation demonstrations that the closeness of consecutive terms guarantee that accuracy has been achieved

$(b)\ f(x) = e^{-x} - x$ Starting with $x_0 = 0.5$

Rewrite to$x = g(x)$, we have $x = g(x) = e^{-x}$

$$g(x)' = -e^{-x}$$

Then

$$B(x) = \frac{e^{-x} - x(-e^{-x})}{1-(-e^{-x})}$$

$$x_{n+1} = B(x_n) = \frac{e^{-x_n} - x_n(-e^{-x_n})}{1-(-e^{-x_n})}$$

Table 9: Results of 4 iterations of (b)

n	x_n	$B(x_n)$
0	0.5	0.566311003
1	0.566311003	0.567143165
2	0.567143165	0.56714329
3	0.56714329	0.56714329

We stop at$B(x_3)$ since is the same tox_3, and the sequence converges very fast to $x = 0.56714329$, contrary to normal fixed point iteration method which converged at $23^{th}(x_{23})$ iteration sequence

$(c)\ f(x) = -4 - x + 4x - (1/2)x^2$ Starting with $x_0 = 1.9$, rewriting to $x = g(x)$, we obtain $x = -4 + 4x - (1/2)x^2$, and $g(x)' = 4 - x$

$$B(x) = \frac{\left(-4 + 4x - \left(\frac{1}{2}\right)x^2\right) - x(4-x)}{1-(4-x)}$$

$$x_{n+1} = B(x_n) = \frac{\left(-4 + 4x_n - \left(\frac{1}{2}\right){x_n}^2\right) - x_n(4-x_n)}{1-(4-x_n)}$$

Table 10: Results of 4 iterations for (c)

n	x_n	$B(x_n)$
0	1.9	1.99545455
1	1.99545455	1.999989716
2	1.999989716	2.00000000
3	2.00000000	2.00000000

We stop here since $B(x_3)$ *and* x_3 are the same, implies that the sequence converges to $x = 2$, which means $x = 2$ is a solution for $-4 - x + 4x - \left(\frac{1}{2}\right)x^2 = 0$ for $(x) = 0$. It has been clearly seen that Bennie's method is more efficient, it is capable to help to fix fixed point method problem when$|g(x)'| > 1$

$(d) f(x) = 2(x-1)^{1/2} - x$, $x \in [1\ 2]$ Starting with x_0=1.5, rewrite into $x = g(x)$,

Then$g(x) = 2(x-1)^{1/2}$ and $g(x)' = (x-1)^{-1/2}$

$$B(x) = \frac{(2(x-1)^{1/2}) - x(x-1)^{-1/2}}{1-(x-1)^{-1/2}},$$

Iteration sequence

$$x_{n+1} = (x_n) = \frac{\left(2(x_n-1)^{\frac{1}{2}}\right) - x_n(x_n-1)^{-\frac{1}{2}}}{1-(x_n-1)^{-\frac{1}{2}}}$$

Table 11: Results of 11 iterations for (d)

n	x_n	$B(x_n)$
0	1.5	1.707107
1	1.707107	1.8408964
2	1.8408964	1.9170040
⋮	⋮	⋮
10	1.999932332	1.99966161

The sequence converges to $x = 2$ after further calculations, this verifies that, Bennie's iteration sequence converges to an answer from either side and for whatever chosen starting point, contrary to fixed point method which failed to work for this case

$(e) f(x) = 1 - x^2/4$,$x \in [-3 - 1\,]$ Starting with $x_0 = -2.05$, Weget $x = g(x) = 1 + x - \frac{x^2}{4}$, and

$$g(x)' = 1 - \frac{x}{2}, B(x) = \frac{(1 + x - \frac{x^2}{4}) - x(1 - \frac{x}{2})}{1-(1-\frac{x}{2})}$$

Iterative procedure will be

$$x_{n+1} = B(x_n) = \frac{\left(1 + x_n - \frac{{x_n}^2}{4}\right) - x_n\left(1 - \frac{x_n}{2}\right)}{1-\left(1-\frac{x_n}{2}\right)}$$

Table 12: Results of 4 iterations for (e)

n	x_n	$B(x_n)$
0	-2.05	-2.0006096
1	-2.0006096	-2.00000009
2	-2.00000009	-2.00000000
3	-2.00000000	-2.00000000

Clearly seen the sequence converges to $x = -2$ since $x_3 = x_4$, and the closeness of consecutive terms guarantee the accuracy has been achieved

$|\,x_3 - x_2| = |-2.00000000 - 2.00000009| = 0.00000009$

$|\ x\ -\ x_4\ |$=|-2.00000000-2.00000000|=0.00000000we have seen Bennie's method converges to a solution in this case, which was seen to diverge in the fixed point method.

$(f) f(x) = e^{-2x}(x-1)$, $x \in [0\ 2\,]$ starting with $x_0 = 0.99$, we have $x = g(x) = x + e^{-2x}(x-1)$, and $g(x)' = 1 + e^{-2x} - 2(x-1)e^{-2x}$,Therefore

$$B(x) = \frac{\left(x + e^{-2x}(x-1)\right) - x(1 + e^{-2x} - 2(x-1)e^{-2x})}{1-(1+e^{-2x} - 2(x-1)e^{-2x})}$$

Iterative procedure

$$x_{n+1} = B(x_n) = \frac{\left(x_n + e^{-2x_n}(x_n-1)\right) - x_n(1 + e^{-2x_n} - 2(x_n-1)e^{-2x_n})}{1-(1+e^{-2x_n} - 2(x_n-1)e^{-2x_n})}$$

Table 13: Results of 4 iterations for (f).

n	x_n	$B(x_n)$
0	0.99	-2.0006096
1	0.999803921	-2.00000009
2	0.999999923	1.00000000
3	1.00000000	1.00000000

We stop there since $x_3 = B(x_3)$,Observations justifies that the sequence converges to $x = 1$ which is the solution of$e^{-2x}(x-1) = 0$, contrary to fixed point method which failed to work under the same starting point x_0= 0.99

5. Bennie's Iteration for Finding Square Roots

In this paper description of how finding of square root of a value based on Bennie's iteration is given, suppose

If $\sqrt{A} = x$, we rewrite as $A = x^2 \rightarrow f(x) = x^2 - A = 0$,then we form$x = g(x) = f(x) + x$=$x^2 - A + x$
Simplifying $x = g(x) = x^2 + x - A$ and $g(x)' = 2x + 1$
Applying Bennie's formula

$$B(x) = \frac{g(x) - xg(x)'}{1 - g(x)'}$$

$$B(x) = \frac{(x^2 + x - A) - x(2x+1)}{1-(2x+1)}$$

When we simplify more

$$\text{We obtain } x = B(x) = \frac{x + \frac{A}{x}}{2} \qquad (6)$$

Bennie's iteration for finding square root

$$x_{n+1} = B(\,x_n) = \frac{x_n + \frac{A}{x_n}}{2} \qquad (7)$$

It is clearly seen, that the Bennie's iteration and Newton's iteration for finding square roots yields' the same results since they have the same form of iteration
$\sqrt{A} = x$,Then $A = x^2 \rightarrow x^2 - A = f(x) = 0$

And$f(x)' = 2x$, using newton's iteration formula

$$g(x) = x - \frac{f(x)}{f(x)'} = x - \frac{x^2 - A}{2x} = \frac{x + \frac{A}{x}}{2} \qquad (8)$$

Bennie's iteration for finding square root

$$x_{n+1} = g(\,x_n) = \frac{x_n + \frac{A}{x_n}}{2} \qquad (9)$$

6. Deduction of Newton's Method from Bennie's Method

Newton's method can be deduced from Bennie's method, based on the fact that $g(x) = f(x) + x$
Therefore $g(x)' = f(x)' + 1$

From

$$B(x) = \frac{g(x) - xg(x)'}{1 - g(x)'}$$

Substituting $g(x)$ and $g(x)'$ in terms of $f(x)$and$f(x)'$,we get

$$B(x) = \frac{(f(x) + x) - x(f(x)' + 1)}{1 - (f(x)' + 1)}$$

Simplifying we obtain

$$B(x) = x - \frac{f(x)}{f(x)'} \qquad (10)$$

Which is the newton's iteration for finding roots

7. Conclusion

Algorithm was developed based on the idea of fixed point iteration and slop of tangent concept for which derivative of the function was found at a particular point to generate the complete algorithm. The algorithm was then tested to more than 100 problems related to the finding of fixed points, roots of nonlinear single variable functions and square roots of any value A, the algorithm was found to work for all problems tested and therefore offer 100% accuracy among the tested problems some of problems were which fixed point method failed to work, butfound to be solved precisely with this algorithm. However the algorithm can be used to deduce the newton's method iteration which proves the stability and powerfulness of the algorithm with the accuracy as newton's method. In advantage to that the algorithm was found to converge as faster as Newton's method, furthermore the algorithm was as powerful as it gave very small error approaching to negligible which was therefore found advantageous over fixed point method, bisection method and regular falsi.

8. Acknowledgement

Benedictor Alexander sincerely conveys his remarkable thanks to his Mother Bonita Menderad for her inspiration and hearty support on matters concerning education and intellectual life. He also thanks Mr. Juma Mashenene, Mr. Jamal Firmat Banzi and Lameck Mjalifu Sondo all from Tianjin University of Technology and Education for their Editing and idea support toward accomplishment of this paper.

References

[1] John Mathews and Kurtis D.Fink"Numerical Methods Using MATLAB Fourth Edition" 2008 (book style)
[2] "Commonwealth Scholarship Tenable in the United Kingdom for the year 2015 " (book Chapter style)
[3] "AMS 147 Computational Methods and Applications" Copyright by Hong Yun Wang, UCSC. (Conference style)
[4] NumericalAnalysis,"05fixed_point_iteration"EE, NCKU, Tien-Hao Chang (Darby chang), (technical report style)
[5] CS 412: Introduction to Numerical Analysis, 09/14/10." Lecture 3.solving equations using fixed point Iterations" Scribes:
[6] Yun Peng Li ,Mark Cowl Ishan ,Nathanael Fillmore.(lecture 3,by Professor Amos Ron),(lecture style)

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