Special Theory of Relativity: A Review

Basant Chhotray

Dhelua, Mahatab Road, Kansbahal-770034, India

Abstract: ***Thomas young through his double-slit experiment gave the wave character (frequency and wavelength) To light while Einstein in his photo-electric equation gave it the particle character (Energy and momentum). The dual character of light thus experimentally established. Louis de Broglie showed that the matter, which exists in the concrete form (particle form) also exists in the wave form. In other words matter behaves like a particle as well as like a wave. This lead to the generalization of matter. Newton's doctrine in regard to space:"Absolute space in its own nature without relation to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute space; which our senses determine by its position to bodies; and which is commonly taken for immovable space. The absolute space is the supporter of all relative spaces as well as matter.***

Keywords: Galileo, matter, Michelson-Morfley experiment, absolute space.

1. Introduction

Galileo put forth the principle of relativity, which was valid for mechanics. At his time, the mechanics was representing the whole physics, and which dealt with the "material objects" of the external world. The notion of matter, at that time was confined to the objects, consist of whole basic elements - Air, Fire, Water and Earth (Greek philosophers are of the view that the universe is made of these four basic elements) .

After Maxwell, the electromagnetic field became a physical reality, existing along with the material objects in the external world. At this point the concept of matter was generalized, which admitted the two forms of matter e.g. the particle form and the field form, co-exist side by side. On the perspective of this generalization of matter , the "space" must be considered as the fifth basic element of the universe. (Hindu philosophers have recognized five elements – Space, Air, Fire, Water and Earth, as the basic elements of this perceivable universe).

So the principle of relativity, which is valid for mechanics, must valid for electrodynamics too. But the principle seemed fail on electrodynamics (The propagation of light wave does not retain it‟s form, when it is transformed from one inertial frame to an another inertial frame, under Galilian transformation laws. The failure may be ascribed to either i) The generalization is done in the wrong way or ii) The transformation laws are inadequate. Keeping the former intact (the generalization is right) and forcing the principle of relativity on the electrodynamics, it brought about a radical change in the notions of space, time and motion.

2. The Velocity of Light, "c" is Independent of the Motion of the Source

2.1 Theory

In the heliocentric reference frame, K, the velocity of the earth round the sun is, $v \approx 30*10^8$ mm. / sec. and that of light is, $c=3 \times 10^8$ mtr. / sec. Consider any two points A‟ ans A on the surface of the earth, the point A, vertically bellow the point A‟ and the distance between themis 30 mtr, as shown in the figure-1, below.

K

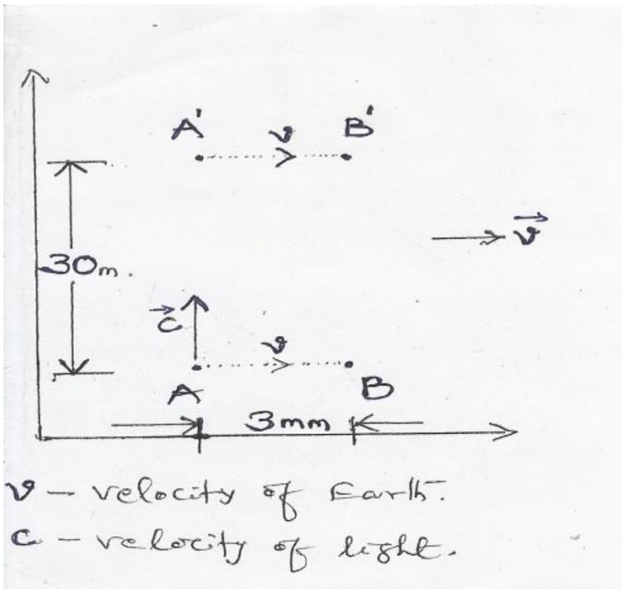

Figure 1

In the time interval, Δ t = 10^{-7} sec, the point A‟, would suffer a displacement, ˡ (=A‟B‟)=3mm in the direction of the motion of the earth, in the frame K. In the same time interval, the light when sent from the point A towards A‟ would have traveled a distance, d=30mtr.

Let us attach a reference frame K‟ on the earth.

If we assume that the velocity of light is independent of the source (the motion K‟), then the light pulse sent from the point A, must miss the point A‟ (by the time the light reaches the position of A‟, the point A‟ would have been shifted to B‟ in K due to the motion of K‟).

3. Experiment

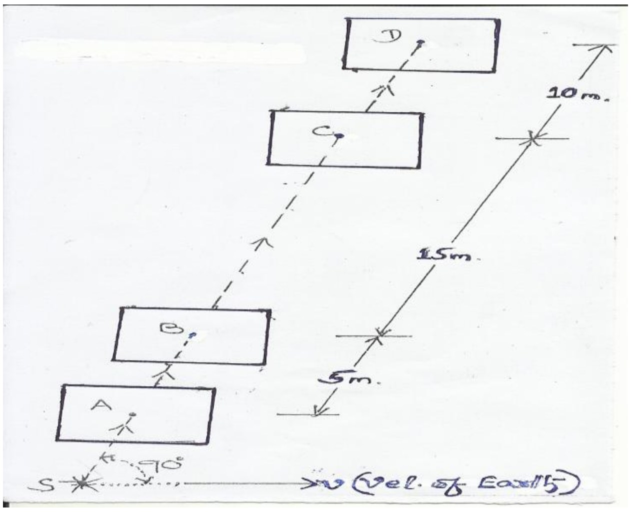

Figure 2

Take four identical cardboards, with small hole at the centre of each. Place them on the same level and on the same straight light, at right angles to the direction of the velocity, v , of the earth, as shown in the figure - 2.

The distance between A & B is 5.0 m, between B & C is 15.0 m, and between C & D is 10.0 m.

The holes on the cardboards at A & B are of diameters 1mm each and that of at C & D are of 2 mm each.

S is the source of light. The light pulse from S is supposed to be observable at C, but not at D. If either D is shifted by 3.0 mm, in the opposite direction of v or if the whole arrangement is rotated through 90 degree about S, the light pulse from S, may be observable at D.

For this short interval of time ($\Delta t = 10^{-7}$ sec.) the earth can be considered as an inertial reference frame. It therefore follows that, the velocity of light, c, is independent of the source i;e of any inertial system. It is the law of light. So we can"t assume it as a postulate.

When "C" is independent of the source :

If a wave is travelling in a space, with a velocity c, in an inertial coordinate system, K , then the propagation equation for such a wave is of the form,

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)f=0$$

Where $f = f\,(x\,,y\,,z\,,t\,)$ is a wave function and

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)$$

is known as D"Alembert"s operator, denoted by $\boxdot^2$.

In the system K",

$$\left(\frac{\partial^2}{\partial x'^2}+\frac{\partial^2}{\partial y'^2}+\frac{\partial^2}{\partial z'^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t'^2}\right)f=0$$

Where $f = f\,(x'', y'', z'', t'')$.

Consider the Galilean transformation laws,

$x = x'' + v\,t$ „

$y = y$ „; $z = z$ „and

$t = t$ „ .

where "v" is the velocity of K" with respect to K , in the +ve direction of X - axis of K .

Now,

$$\frac{\partial f}{\partial x'}=\frac{\partial f}{\partial x}\frac{\partial x}{\partial x'}+\frac{\partial f}{\partial y}\frac{\partial y}{\partial x'}+\frac{\partial f}{\partial z}\frac{\partial z}{\partial x'}+\frac{\partial f}{\partial t}\frac{\partial t}{\partial x'}$$

$$\Rightarrow \frac{\partial f}{\partial x'}=\frac{\partial f}{\partial x}+0 \text{ (by Galilian transformation law)}$$

$$\frac{\partial}{\partial x'}\equiv\frac{\partial}{\partial x}$$

Similarly ,

$$\frac{\partial}{\partial y'}\equiv\frac{\partial}{\partial y},\ \frac{\partial}{\partial z'}\equiv\frac{\partial}{\partial z}$$

$$\text{and } \frac{\partial}{\partial t'}\equiv v\frac{\partial}{\partial x}+\frac{\partial}{\partial t}$$

$$\text{Hence, } \boxdot'^2\equiv\frac{\partial^2}{\partial x'^2}+\frac{\partial^2}{\partial y'^2}+\frac{\partial^2}{\partial z'^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t'^2}$$

$$\equiv\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\left(v^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial t^2}+2v\frac{\partial^2}{\partial x\partial t}\right)$$

$$\Rightarrow \boxdot''^2=\boxdot^2-\frac{v^2}{c^2}\frac{\partial^2}{\partial x^2}-\frac{2v}{c^2}\frac{\partial^2}{\partial x\partial t} \quad \text{---(A) .}$$

The D"Alembert"s operator does not remain invariant.

To make the principle of relativity valid for the laws of physics, the transformation laws must be modified.

From the two additional terms in eq. (A), the former implies the linearity of the transformation laws, while the latter implies that the space and time coordinates are inter related i.e

$t = t\,(x'', t'')$ and $t'' = t''(x\,,t\,)$

Let us consider the transformation laws in the following form :

$x = x\,(x'', t\,„)$; $y = y$ „; $z = z$ „and $t = t\,(x'', t\,„)$.

$$\frac{\partial f}{\partial x'}=\frac{\partial f}{\partial x}\frac{\partial x}{\partial x'}+\frac{\partial f}{\partial y}\frac{\partial y}{\partial x'}+\frac{\partial f}{\partial z}\frac{\partial z}{\partial z'}+\frac{\partial f}{\partial t}\frac{\partial t}{\partial x'}$$

$$=\frac{\partial f}{\partial x}\frac{\partial x}{\partial x'}+\frac{\partial f}{\partial t}\frac{\partial t}{\partial x'}$$

$$\Rightarrow\frac{\partial}{\partial x'}\equiv\frac{\partial x}{\partial x'}\frac{\partial}{\partial x}+\frac{\partial t}{\partial x'}\frac{\partial}{\partial t}$$

Similarly,

$$\frac{\partial}{\partial y'} \equiv \frac{\partial}{\partial y} \quad ; \quad \frac{\partial}{\partial z'} \equiv \frac{\partial}{\partial z}$$

and $$\frac{\partial}{\partial t'} \equiv \frac{\partial x}{\partial t'}\frac{\partial}{\partial x} + \frac{\partial t}{\partial t'}\frac{\partial}{\partial t}$$

Hence $\boxdot'^2 = \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \frac{\partial^2}{\partial z'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2}$

=

$$\left(\frac{\partial x}{\partial x'}\frac{\partial}{\partial x} + \frac{\partial t}{\partial x'}\frac{\partial}{\partial t}\right)^2 + \left(\frac{\partial}{\partial y}\right)^2 + \left(\frac{\partial}{\partial z}\right)^2 - \frac{1}{c^2}\left(\frac{\partial x}{\partial t'}\frac{\partial}{\partial x} + \frac{\partial t}{\partial t'}\frac{\partial}{\partial t}\right)^2$$

$$\Rightarrow \boxdot'^2 = \left[\left(\frac{\partial x}{\partial x'}\right)^2 - \frac{1}{c^2}\left(\frac{\partial x}{\partial t'}\right)^2\right]$$

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} +$$

$$\left[\left(\frac{\partial t}{\partial x'}\right)^2 - \frac{1}{c^2}\left(\frac{\partial t}{\partial t'}\right)^2\right]\frac{\partial^2}{\partial t^2}$$

$$+ 2\left(\frac{\partial x}{\partial x'}\frac{\partial t}{\partial x'} - \frac{1}{c^2}\frac{\partial x}{\partial t'}\frac{\partial t}{\partial t'}\right)\frac{\partial^2}{\partial x \partial t}$$

The conditions for invariance ,

$$\left(\frac{\partial x}{\partial x'}\right)^2 - \frac{1}{c^2}\left(\frac{\partial x}{\partial t'}\right)^2 = 1 \qquad --1$$

$$\left(\frac{\partial t}{\partial x'}\right)^2 - \frac{1}{c^2}\left(\frac{\partial t}{\partial t'}\right)^2 = -\frac{1}{c^2} \qquad --2$$

$$\frac{\partial x}{\partial x'}\frac{\partial t}{\partial x'} - \frac{1}{c^2}\frac{\partial x}{\partial t'}\frac{\partial t}{\partial t'} = 0 \qquad -3$$

From eq. [3] ,

$$\frac{\partial x}{\partial x'}\frac{\partial t}{\partial x'} - \frac{1}{c^2}\frac{\partial x}{\partial x'}\frac{\partial x'}{\partial t'}\frac{\partial t}{\partial t'} = 0$$

$$\Rightarrow \frac{\partial t}{\partial x'} + \frac{v}{c^2}\frac{\partial t}{\partial t'} = 0 \qquad --4$$

Where $\frac{\partial x'}{\partial t'} = -v$, the velocity of K , with respect to K".

From the eqs. [2] &[4] :

$$\frac{v^2}{c^4}\left(\frac{\partial t}{\partial t'}\right)^2 - \frac{1}{c^2}\left(\frac{\partial t}{\partial t'}\right)^2 = -\frac{1}{c^2}$$

$$\Rightarrow \frac{\partial t}{\partial t'} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \qquad --5$$

$$\Rightarrow t = \frac{t'}{\sqrt{1-\frac{v^2}{c^2}}} + c(x') \qquad --6$$

Where c (x „) is a constant of integration and is a function of x „only.

Putting the value of $\frac{\partial t}{\partial t'}$ from eq.[5], in eq.[2], we get

$$\frac{\partial t}{\partial x'} = \frac{\frac{v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow t = \frac{\left(\frac{v}{c^2}\right)x'}{\sqrt{1-\frac{v^2}{c^2}}} + c'(t') \qquad --7$$

Where c „(t „) is a constant of integration and is a function of t „only.

By inspecting the two equations [6]&[7] , we find

$$\Rightarrow t = \frac{t' + \left(\frac{v}{c^2}\right)x'}{\sqrt{1-\frac{v^2}{c^2}}} \qquad --8$$

Consider the eq.[1].

$$\left(\frac{\partial x}{\partial x'}\right)^2 - \frac{1}{c^2}\left(\frac{\partial x}{\partial t'}\right)^2 = 1$$

$$\Rightarrow \left(\frac{\partial x}{\partial x'}\right)^2 - \frac{1}{c^2}\left(\frac{\partial x}{\partial t}\frac{\partial t}{\partial t'}\right)^2 = 1$$

$$\Rightarrow \left(\frac{\partial x}{\partial x'}\right)^2 - \frac{1}{c^2}\left((+v)^2\frac{1}{1-\frac{v^2}{c^2}}\right) = 1 \ (By\ Eq.\ 5\ )$$

($\frac{\partial x}{\partial t}$ = + v , the velocity of K" w.r.t K)

$$\Rightarrow \frac{\partial x}{\partial x'} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \qquad --9$$

$$\Rightarrow x = \frac{x'}{\sqrt{1-\frac{v^2}{c^2}}} + c''(t') \qquad --10$$

Where c „"(t „) is a constant of integration and is a function of t „only.

Putting the eq.[9] , in eq.[1], we get

$$\frac{1}{c^2}\left(\frac{\partial x}{\partial t'}\right)^2 = \frac{1}{1-\frac{v^2}{c^2}} - 1$$

$$\Rightarrow \quad \frac{\partial x}{\partial t'} = \frac{v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow x = \frac{vt'}{\sqrt{1-\frac{v^2}{c^2}}} + c'''(x') \qquad -- (11)$$

Where $c''''(x'')$ is a constant of integration and is a function of x'' alone.
By inspecting eq.[10]&[11] , we get

$$x = \frac{x'+vt'}{\sqrt{1-\frac{v^2}{c^2}}} \qquad -- (12)$$

Thus the modified transformation laws becomes ,

$$x = \frac{x'+vt'}{\sqrt{1-\frac{v^2}{c^2}}} \quad y = y' \;;\quad z = z' \quad and$$

$$t = \frac{t'+\left(\frac{v}{c^2}\right)x'}{\sqrt{1-\frac{v^2}{c^2}}}$$

Which makes the equation ,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) f = 0$$

Coordinate independent.
The reverse transformation law (modified),

$$x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}} \;;\; y' = y \,; z' = z \;\; and$$

$$t' = \frac{t-\left(\frac{v}{c^2}\right)x}{\sqrt{1-\frac{v^2}{c^2}}}$$

4. Michelson – Morley Experiment

4.1 CASE – I

When the velocity of light is **not independent** of the source.

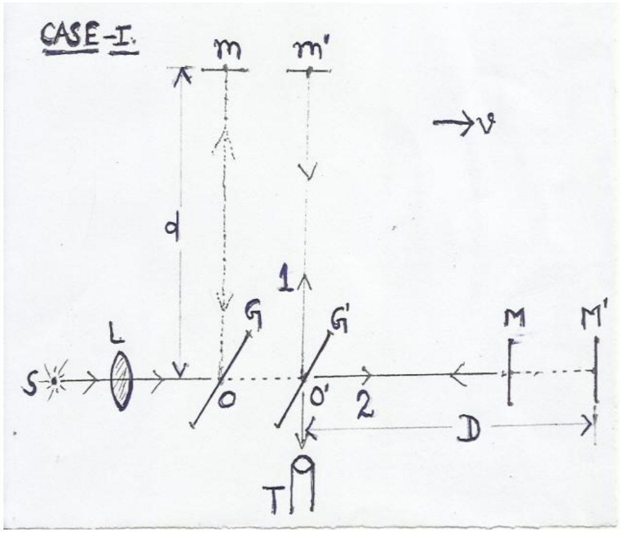

Figure 3

Refer the fig-3.

In this case, the distance between the mirrors G and M will remain fixed for the light, since the light is supposed to have an initial velocity " v " (the velocity of the earth).

The light travels towards M with velocity (c – v) and back to O" with velocity (c + v).

Thus the time required for the light to cover the distance O M" O", as shown in the figure,

$$t_2 = \left(\frac{D}{c+v}\right) + \left(\frac{D}{c-v}\right)$$

$$\Rightarrow t_2 = \left(\frac{2D}{c}\right)\left(\frac{1}{1-B^2}\right) \text{ where } D = OM \text{ , } B = \frac{v^2}{c^2}.$$

Since the mirrors G , m and the light have the same initial velocity , v , the time taken by the light to travel the distance O"m"O",

$$t_1 = \frac{2d}{c} \text{ where } d = O"m".$$

The time difference in the propagation of beams 2 & 1 ,

$$\Delta t = t_2 - t_1$$

$$\Rightarrow \Delta t = \left(\frac{2}{c}\right)\left(\frac{D}{1-B^2} - d\right) \text{------------- (A)}$$

When the interferometer is rotated through 90°, the paths followed by the beams 2 & 1 get interchanged. Hence the time difference ,

$$\Delta t'' = t'_2 - t'_1$$

$$\Rightarrow \Delta t' = \left(\frac{2}{c}\right)\left(D - \frac{d}{1-B^2}\right)$$

Now the time difference between the two sets of fringes following the rotation,

$$\Delta\tau = \Delta t' - \Delta t = -\left(\frac{2}{c}\right)(D + d)\left(\frac{B^2}{1-B^2}\right)$$

Neglecting 3rd and higher orders of (v / c),

$$\Delta\tau \approx -\left(\frac{2}{c}\right)(D+d)\left(\frac{v^2}{c^2}\right)[B = v/c]$$

The path difference for the two sets of fringes ,

$$\Delta X \approx (\Delta\tau)c \approx -2(D+d)\left(\frac{v^2}{c^2}\right)$$

The negative sign implies that the path difference is in the opposite sense to that obtained before.

4.2 CASE – II

When the velocity of light is independent of the source.

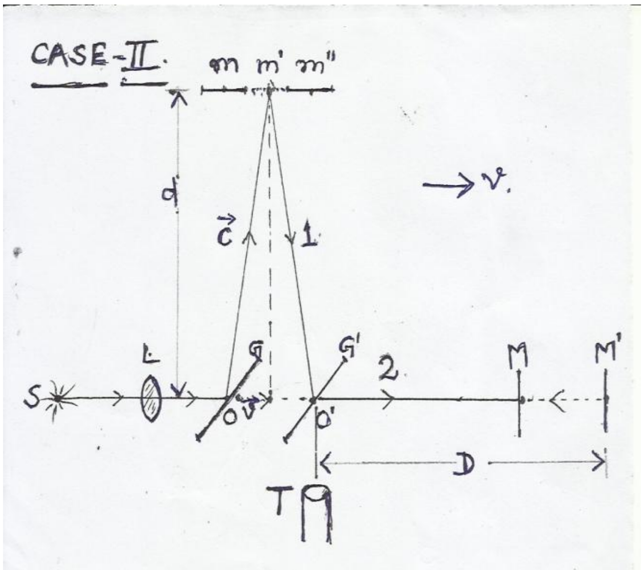

Figure 4

Refer the fig-4.

In this case, the distance between the mirrors G and M i.e OM, which is to be travelled by the light is not fixed. The distance would be longer for light, when it travels in the direction of motion of the earth and shorter in the reverse case.

Let t'_2 be the time in which the light reach the mirror M .
The path travelled by the light = c. t'_2
In this time t'_2 the mirror has travelled a distance = v. t'_2 (= MM').

If D = OM , then,

$$ct'_2 = D + v\,.\,t'_2$$

$$\Rightarrow t'_2 = \frac{D}{c-v}$$

Similarly for the return journey,

$$ct''_2 = D - v\,.\,t''_2$$

$$\Rightarrow t''_2 = \frac{D}{c+v}$$

The total time taken after reflection , to reach O'' ,

$$t_2 = t'_2 + t''_2$$

$$\Rightarrow t_2 = \left(\frac{2D}{c}\right)\left(\frac{1}{1-B^2}\right)$$

The time to travel Om''O'' ,

$$t_1 = \left(\frac{2d}{c}\right)\left(\frac{1}{\sqrt{1-B^2}}\right)$$

The time difference ,

$$\Delta t = t_2 - t_1 = \frac{2}{c\sqrt{1-B^2}}\left(\frac{D}{\sqrt{1-B^2}} - d\right)$$

On rotation the time difference

$$\Delta t' = \frac{2}{c\sqrt{1-B^2}}\left(D - \frac{d}{\sqrt{1-B^2}}\right)$$

Thus the time difference for the two sets of fringes ,

$$\Delta\tau = \Delta t' - \Delta t \approx -\left(\frac{D+d}{c}\right)\left(\frac{v^2}{c^2}\right)[B = v/c]$$

The corresponding path difference ,

$$\Delta X'' = (\Delta\tau)\,c \approx (D+d)\left(\frac{v^2}{c^2}\right)$$

The path difference exists (though they differ in magnitude) whether "c" the velocity of light is dependent or independent of the source. From the Michelson – Morley experiment, it cannot be concluded that the velocity of light is independent of the source or not.

To find the reason of " no shift " of fringes, the following experiment should be performed :

a) After getting the first set of fringes, keeping the telescope T, fixed , rotate the arms through 90° and observe the fringes.
b) After getting the first set of fringes, keeping the telescope T , and the arms of the interferometer fixed i) interchange the path and observe , ii) change the path difference between the interfering rays arbitrarily and observe.

In a given pattern of fringes, if the path difference between the interfering rays is changed, the fringes shift. But it is doubtful that, the shifting of two sets of fringes with respect to each other would be possible following a path difference between them. (The two sets of fringes are independent of each other !)

The shifting may be anticipated if the path difference (between the individual interfering rays) in the former pattern is made different from the one, that in the latter pattern. (Some of the laws of nature may be rigid on magnitude only).

5. Conclusion

The velocity of light, c , is same in all inertial systems. This is an exceptional phenomenon, which contradicts the existing notions of physics. The reality, that lies behind the light phenomenon does not communicate with our presentday notions. Notions need to be modified. (It is something like this : if a man does not know “French”, can he communicate with the French speaking people).

Since light has wave nature, on the formal basis, like the sound wave , an hypothetical medium may be imagined for the propagation of light wave, which is absolutely immobile.

“ Imagination “ is a stage through which knowledge has to pass (in electrostatics, a unit point charge is imagined to solve the boundary value problems). Physics has discovered many spaces e.g Minkowski Space, Euclidean Space, Riemannian space etc. However there is one general space, without attributes, on which all these spaces are superimposed. This general space, Newton has called it as “ Absolute space “ , which one cannot negate (can we say that , this world is full of “ names” only and no man exists). The immobile hypothetical medium may be, for the sake of knowledge, replaced with this absolute space.

References

[1] V.A. Ugarov, “SPECIAL THEORY OF RELATIVITY”, (Translated from the Rusian by Yuri Atanom) MIR publisher, MOSCOW, 1979.“Supplement, section-ll, page-338”.
[2] Francis A. Jenkins, Harvey E. White, “Fundamentals of optics” 4th Edition, 1987 McGRAW-HILL BOOK COMPANY “Section 19.13, page-416”.
[3] D.P.Gribanov, “ALBERT EINSTEIN‟S philosophical views and the theory of Relativity” (Translated from the Rusian by H.CAMPBELL CREIGHTON, M.A (oxon)) Progress publishers, MOSCOW, 1987.

Author Profile

Basant Chhotray has recived his B.Sc. degree in physics from Sambalpr Univercity in 1987.