Mean Labeling of Some Graphs

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Abstract: ***A graph G with p vertices and q edges is a mean graph if there is an injective function f from the vertices of G to {0,1,2,….q} such that when each edge uv is labeled with $\frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is evenand $\frac{f(u)+f(v)+1}{2}$if $f(u)+f(v)$ is odd then the resulting edges are distinct. In this paper we investigate mean labeling of shadow graph of bistar and comb and splitting graph of comb.***

Keywords: Mean labeling, comb , splitting graph, shadow graph, bistar

1. Introduction

By a graph $G = (V(G), E(G))$ with p vertices and q edges we mean a simple, connected and undirected graph. In this paper a brief summary of definitions and other information is given in order to maintain compactness. The term not defined here are used in the sense of Harary [3].

A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain conditions. A useful survey on graph labeling by J. A. Gallian (2014) can be found in [2].
Somasundaram and Ponraj [4] have introduced the notion of mean labeling of graphs.

In this paper we investigate mean labeling of shadow graph of bistar and comb and splitting graph of comb.

Definition 1.1 : A graph G with p vertices and q edges is a mean graph if there is an injective function f from the vertices of G to {0,1,2,….q} such that when each edge uv is labeled with $\frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd then the resulting edges are distinct.

Definition1.2: A Comb is a caterpillar in which each vertex in the path is joined to exactly one pendent vertex.

Definition1.3: The Splitting graph of G,S(G) is obtained from G by adding to each vertex v of G a new vertex v' so that v' is adjacent to every vertex that is adjacent to v in G.

Definition1.4: The Shadow graph $D_2(G)$ of a connected graph G is obtained by taking two copies of G, sayG' and G'' then joining each vertex u' in G' to the nighbours of the corresponding vertex u'' in G''.

Definition1.5: A bistar is the graph obtained by joining the apex vertices of two copies of star $K_{1,n}$ by an edge.

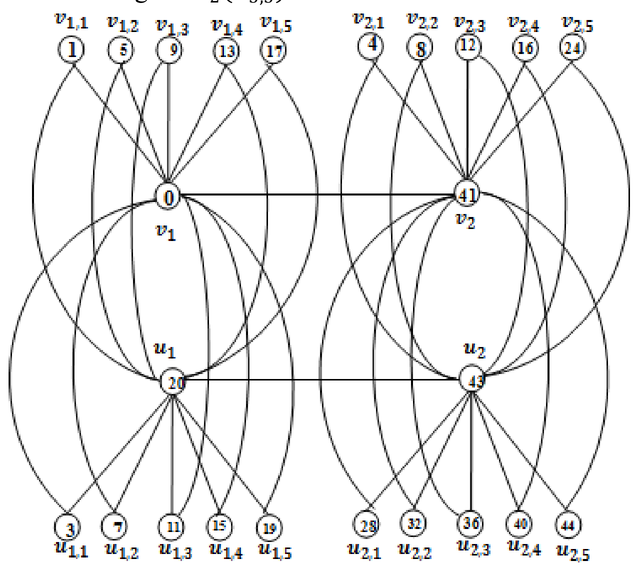
2. Results on Mean Labeling

Theorem2.1: The graph $D_2(B_{n,n})$ has mean labeling.

Proof: Consider two copies of $B_{n,n}$.
Let $\{v_1, v_2, v_{1,j}, v_{2,j}, 1 \le j \le n\}$ be the vertices of first copy of $B_{n,n}$.
Let $\{u_1, u_2, u_{1,j}, u_{2,j}, 1 \le j \le n\}$ be the vertices of second copy of $B_{n,n}$ where v_1, v_2 and u_1, u_2 are the respective apex vertices.
Let $D_2(B_{n,n})$ b the shadow graph of the two copies of $B_{n,n}$.
Define $f: V(D_2(B_{n,n}) \rightarrow \{0,1,2,\dots.q\}$ by
$f(v_1) = 0$
$f(v_2) = 8n + 1$
$f(u_1) = 4n$
$f(u_2) = 8n + 3$
$f(v_{1,j}) = 4j - 3 \ if\ 1 \le j \le n$
$f(v_{2,j}) = 4j \ if\ 1 \le j \le n - 1$
$f(v_{2n}) = 4n + 4$
$f(u_{1,j}) = 4j - 1 \ if\ 1 \le j \le n$
$f(u_{2,j}) = 4(n + j + 1) \ if\ 1 \le j \le n$
Thus the induced edge labels are distinct. Hence the graph $D_2(B_{n,n})$ has mean labeling.

Example2.2:

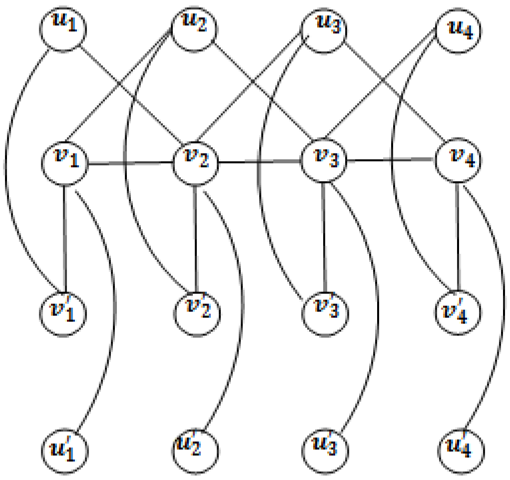
Mean labeling for $D_2(B_{5,5})$



Theorem2.3: The split graph of comb has mean labeling.

Proof: Let $\{v_i, 1 \le i \le n\}$ and $\{v_i', 1 \le i \le n\}$ be the vertices of comb in which $\{v_i', 1 \le i \le n\}$ are the pendent vertices
Let $\{u_i, 1 \le i \le n\}$ and $\{u_i', 1 \le i \le n\}$ be the newly added vertices.

The ordinary labeling for split graph of comb



Define $f: v(G) \to \{0,1,2, \dots . q\}$ by

$f(v_i) = 6i - 4 \; if \; i \; is \; odd$

$= 6i - 3 \; if \; i \; is \; even$

$f(v_1') = 1$

$f(v_3') = 15$

Let $f(v_i') = 6i - 3 \; if \; i \; is \; odd \; i \neq 1,3$

$= 6i - 4 \; if \; i \; is \; even$

$f(u_i) = 6i - 6 \; if \; i \; is \; odd$

$= 6i - 7 \; if \; i \; is \; even$

$f(u_1') = 3$

$f(u_3') = 11$

$f(u_i') = 6i - 7 \; if \; i \; is \; oddi \neq 1,3$

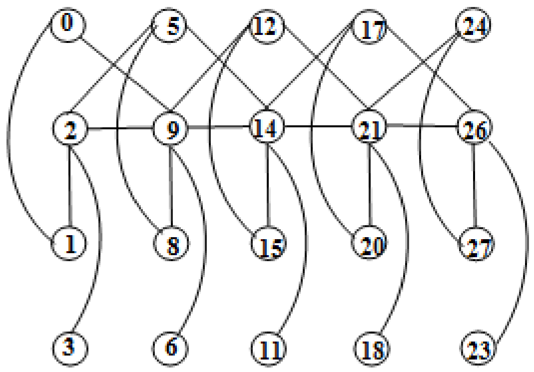
$= 6i - 6 \; if \; i \; is \; even$

Thus the induced edge labels are distinct.

Hence the above defined function provides mean labeling for split graph of comb.

Example 2.4:

Mean labeling for split graph of comb



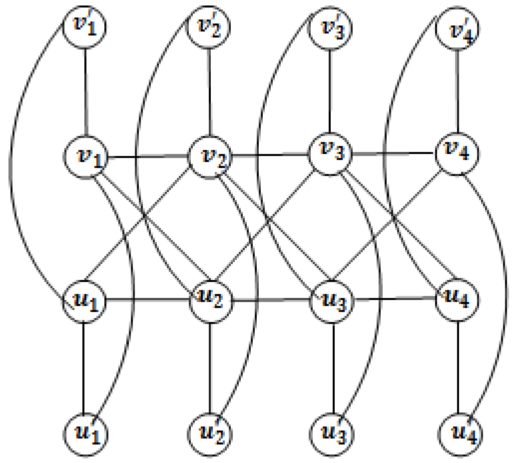
Theorem2.5:$D_2(comb)$ admits mean labeling.

Proof: Consider two copies of comb G_1 and G_2.

Let $\{v_i, 1 \le i \le n\}$ and $\{v_i', 1 \le i \le n\}$ be the vertices of comb G_1

Let $\{u_i, 1 \le i \le n\}$ and $\{u_i', 1 \le i \le n\}$ be the vertices of comb G_2

The ordinary labeling for $D_2(comb)$ as



Let G be the shadow graph of comb

Define$f: v(G) \to \{0,1,2, \dots , q\}$ by

$f(v_i) = 8i - 8 \; if \; i \; is \; odd , 1 \le i \le n$

$= 8i - 7 \; if \; i \; is \; even, 1 \le i \le n$

$f(v_i') = 8i - 7 \; if \; i \; is \; odd , 1 \le i \le n$

$= 8i - 8 \; if \; i \; is \; even, 1 \le i \le n$

$f(u_i) = 8i - 4 \; if \; i \; is \; odd , 1 \le i \le n$

$= 8i - 5 \; if \; i \; is \; even , 1 \le i \le n$

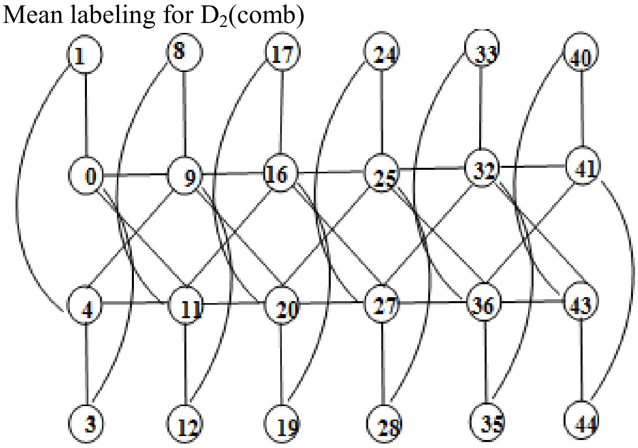
$f(u_i') = 8i - 5 \; if \; i \; is \; odd , 1 \le i \le n$

$= 8i - 4 \; if \; i \; is \; even, 1 \le i \le n$

Hence the above function provides mean labeling for shadow graph of comb.

Example2.6:

Mean labeling for D_2(comb)



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Author Profile

N. Revathi received the M.Sc., M.Phil., degrees in mathematics from Seethalakshmi Ramaswami College, Trichy and working as a assistant professor in mathematics at Seethalakshmi Ramaswami College from 2008 to till now .Currently doing Ph.d in labeling of graphs .