Mean Labeling of Some Graphs

N. Revathi

Department of Mathematics, Seethalakshmi Ramaswami College, Tiruchirappalli – 620002

Abstract: A graph G with p vertices and q edges is a mean graph if there is an injective function f from the vertices of G to \{0,1,2,.....q\} such that when each edge uv is labeled with \((\frac{f(u)+f(v)}{2})\) if \(f(u) + f(v)\) is even and \((\frac{f(u)+f(v)+1}{2})\) if \(f(u) + f(v)\) is odd then the resulting edges are distinct. In this paper we investigate mean labeling of shadow graph of bistar and comb and splitting graph of comb.

Keywords: Mean labeling, comb, splitting graph, shadow graph, bistar

1. Introduction

By a graph \(G = (V(G),E(G))\) with p vertices and q edges we mean a simple, connected and undirected graph. In this paper a brief summary of definitions and other information is given in order to maintain compactness. The term not defined here are used in the sense of Harary [3].

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. A useful survey on graph labeling by J. A. Gallian (2014) can be found in [2].

Somasundaram and Ponraj [4] have introduced the notion of mean labeling of graphs.

In this paper we investigate mean labeling of shadow graph of bistar and comb and splitting graph of comb.

Definition 1.1: A graph G with p vertices and q edges is a mean graph if there is an injective function f from the vertices of G to \{0,1,2,.....q\} such that when each edge uv is labeled with \((\frac{f(u)+f(v)}{2})\) if \(f(u) + f(v)\) is even and \((\frac{f(u)+f(v)+1}{2})\) if \(f(u) + f(v)\) is odd then the resulting edges are distinct.

Definition 1.2: A Comb is a caterpillar in which each vertex in the path is joined to exactly one pendant vertex.

Definition 1.3: The Splitting graph of G, \(S(G)\) is obtained from G by adding to each vertex \(v\) of G a new vertex \(v'\) so that \(v'\) is adjacent to every vertex that is adjacent to \(v\) in G.

Definition 1.4: The Shadow graph \(D_2(G)\) of a connected graph G is obtained by taking two copies of G, say \(G'\) and \(G''\) and then joining each vertex \(u\) of \(G'\) to the neighbours of the corresponding vertex \(u'\) in \(G''\).

Definition 1.5: A bistar is the graph obtained by joining the apex vertices of two copies of star \(K_{1,n}\) by an edge.

2. Results on Mean Labeling

Theorem 2.1: The graph \(D_2(B_{n,n})\) has mean labeling.

Proof: Consider two copies of \(B_{n,n}\).

Let \(\{v_1,v_2,v_1j,v_2j, 1 \leq j \leq n\}\) be the vertices of first copy of \(B_{n,n}\).

Let \(\{u_1, u_2, u_1j, u_2j, 1 \leq j \leq n\}\) be the vertices of second copy of \(B_{n,n}\) where \(v_1, v_2\) and \(u_1, u_2\) are the respective apex vertices.

Let \(D_2(B_{n,n})\) be the shadow graph of the two copies of \(B_{n,n}\).

Define \(f: V(D_2(B_{n,n})) \rightarrow \{0,1,2,.....q\}\) by

\[
\begin{align*}
 f(v_1) &= 0 \\
 f(v_2) &= 8n + 1 \\
 f(u_1) &= 4n \\
 f(u_2) &= 8n + 3 \\
 f(v_{1j}) &= 4j - 3 \text{ if } 1 \leq j \leq n \\
 f(v_{2j}) &= 4j \text{ if } 1 \leq j \leq n - 1 \\
 f(v_{2n}) &= 4n + 4 \\
 f(u_{1j}) &= 4j - 1 \text{ if } 1 \leq j \leq n \\
 f(u_{2j}) &= 4(n + j + 1) \text{ if } 1 \leq j \leq n
\end{align*}
\]

Thus the induced edge labels are distinct. Hence the graph \(D_2(B_{n,n})\) has mean labeling.

Example 2.2:

Mean labeling for \(D_2(B_{5,5})\)

Theorem 2.3: The split graph of comb has mean labeling.

Proof: Let \(\{v_i, 1 \leq i \leq n\}\) and \(\{v'_i, 1 \leq i \leq n\}\) be the vertices of comb in which \(\{v_i, 1 \leq i \leq n\}\) are the pendent vertices.

Let \(\{u_i, 1 \leq i \leq n\}\) and \(\{u_i, 1 \leq i \leq n\}\) be the newly added vertices.
Define $f: v(G) \rightarrow \{0,1,2,\ldots,q\}$ by

\[
f(v_i) = 6i - 4 \quad \text{if } i \text{ is odd}
\]

$$f(v_i) = 6i - 3 \quad \text{if } i \text{ is even}
$$

$$f(v_1) = 1
$$

$$f(v_3) = 15
$$

Let $f(v_i) = 6i - 3$ if $i$ is odd $i \neq 1,3$

$$f(u_i) = 6i - 4 \quad \text{if } i \text{ is even}
$$

$$f(u_i) = 6i - 7 \quad \text{if } i \text{ is odd}
$$

$$f(u_1) = 3
$$

$$f(u_1) = 11
$$

Hence the above defined function provides mean labeling for split graph of comb.

**Example 2.6:**
Mean labeling for $D_2(comb)$

\[
f(v_i) = 8i - 8 \quad \text{if } i \text{ is odd}, 1 \leq i \leq n
\]

$$f(v_i) = 8i - 7 \quad \text{if } i \text{ is even}, 1 \leq i \leq n
$$

$$f(u_i) = 8i - 4 \quad \text{if } i \text{ is odd}, 1 \leq i \leq n
$$

$$f(u_i) = 8i - 5 \quad \text{if } i \text{ is even}, 1 \leq i \leq n
$$

Thus the induced edge labels are distinct.

Hence the above defined function provides mean labeling for split graph of comb.

**References**


Author Profile

N. Revathi received the M.Sc., M.Phil., degrees in mathematics from Seethalakshmi Ramaswami College, Trichy and working as an assistant professor in mathematics at Seethalakshmi Ramaswami College from 2008 to till now. Currently doing Ph.d in labeling of graphs.