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Design and Analysis of a Fuzzy Matrix model for Decision making

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Abstract: ***This paper mainly deals with the design of a TOPSIS technique for order preference by similarity to an ideal solution. This multiple criteria method identifies the solution from a finite set of alternatives. The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and farthest distance from the negative ideal solution. This paper analyzes the newspaper reading habits of college students using multicriteria decision making problems with interval data.***

Keywords: Normalized decision matrix, Positive ideal solution, Negative ideal solution, normalized decision matr**ix**

1. Introduction

Decision making problem is the process of finding the best option from all of the feasible alternatives. In almost all such problems the multiplicity of criteria for judging the alternatives is pervasive. That is, for many such problems the decision maker wants to solve a multiple criteria decision making (MCDM) problem. In classical, decision theory can be characterized by a set of decision alternatives, a set of nature, a relation assigning to each pair of a decision and state and a result and finally the utility function which orders the results according to their desirability. The alternative that leads to the outcome yielding the highest utility is chosen i.e. the decision making problem becomes an optimization problem of maximizing the expected utility. When probabilities of the outcomes are not known, or may not even be relevant and outcome for each action are characterized only approximately i.e. decisions are made under uncertainties, this is the prime domain for fuzzy-decision making.

The research work done on fuzzy multi-criteria decision making analysis has been one of the fastest growing areas in the field of decision making and operations research in the real world scenario since a couple of decades. A fuzzy model of decision making was initially suggested by Bellman and Zadeh in 1970 in which relevant goals and constraints are expressed in terms of fuzzy sets. A fuzzy model group decision was proposed by Blin and Whinston in 1973 and again by Blin in 1974. In classical MCDM methods, the ratings and the weights of the criteria are known precisely. Technique of order performance by similarity to ideal solution (TOPSIS), one of known classical MCDM method, was first developed by Hwang and Yoon in 1981, for solving a MCDM problem. It was based upon the concept that the chosen alternative should have the shortest distance from the positive ideal solution and farthest from the negative ideal solution. In 1982 a similar concept has also been pointed out by Zeleny In the process of TOPSIS, the performance ratings and the weights of the criteria are given as exact values. In 2002 Chen extended the concept of TOPSIS to develop a methodology for solving multi-person-multi-criteria decision-making problems in fuzzy environment. A MCDM problem can be concisely expressed in matrix format as

$$\begin{array}{c c} & \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{pmatrix} \end{array}$$

$W = [w_1, w_2, \dots, w_n]$

where $A_1, A_2, \dots, A_m$ are possible alternatives among which decision makers have to choose, $C_1, C_2, \dots, C_n$ are criteria with which alternative performance are measured, X_{ij} is the rating of alternative A_i with respect to criterion C_j, W_j is the weight of criterion C_j.

Main steps of multiple criteria decision making problem are

1) Establishing system evaluation criteria that relate system capabilities to goals.
2) Developing alternative systems for attaining the goals (generating alternatives).
3) Evaluating alternatives in terms of criteria (the values of the criterion functions).
4) Applying a normative multi-criteria analysis method.
5) Accepting one alternative as "optimal" (preferred).
6) If the final solution is not accepted, gather new information and go into the next iteration of multi-criteria optimization.

Steps (i) and (v) are performed at the upper level, where decision makers have the central role, and the other steps are mostly engineering tasks. For step (iv), a decision maker should express his/her performances in terms of the relative importance of criteria, and one approach is to introduce criteria weights. This weights in MCDM do not have a clear economic significance, but their use provides the opportunity to model the actual aspects of decision making (the preference structure).

Under many conditions, exact data are inadequate to model real-life situations. For example, human judgments including preferences are often vague and cannot estimate his preference with an exact numerical data, therefore these data may have some structures such as bounded data, ordinal data, interval data and fuzzy data. In this paper, by considering the fact that, in some cases, determining precisely the exact value of the attributes is difficult and that, as a result of this, their values are considered as intervals, therefore we extended the concept of TOPSIS to develop a methodology for solving MCDM problems with interval data.

2. TOPSIS Method

TOPSIS (technique for order preference by similarity to an ideal solution) method is presented in Chen and Hwang, with reference to Hwang and Yoon. TOPSIS is a multiple criteria method to identify solutions from a finite set of alternatives. The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and farthest distance from the negative ideal solution.

Suppose $A_1, A_2, \ldots, A_m$ are m possible alternatives among which decision maker have to choose, $C_1, C_2, \ldots, C_n$ are criteria with which alternative performance are measured, x_{ij} is the rating of alternative A_i with respect to criterion C_i and is not known exactly and only we know $x_{ij} \in [x_{ij}^L, x_{ij}^U]$. A MCDM problem with interval data can be concisely expressed in matrix format as

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{pmatrix} C_1 & C_2 & \cdots & C_n \\ [x^L{}_{11}, x^U{}_{11}] & [x^L{}_{12}, x^U{}_{12}] & \cdots & [x^L{}_{1n}, x^U{}_{1n}] \\ [x^L{}_{21}, x^U{}_{21}] & [x^L{}_{22}, x^U{}_{22}] & \cdots & [x^L{}_{2n}, x^U{}_{2n}] \\ \cdots & \cdots & \cdots & \cdots \\ [x^L{}_{m1}, x^U{}_{m1}] & [x^L{}_{m2}, x^U{}_{m2}] & & [x^L{}_{mn}, x^U{}_{mn}] \end{pmatrix}$$

$W = [w_1, w_2, \ldots, w_n]$,
where w_j is the weight of criterion C_j.

2.1 Algorithm

STEP 1: Calculate the normalized decision matrix. The normalized values $\bar{n}^L{}_{j1}$ and $\bar{n}^U{}_{j1}$ are calculated as

$$\bar{n}^L{}_{j1} = \frac{x_{ij}^L}{\sqrt{\sum_{j=i}^m (x_{ij}^L)^2 + (x_{ij}^U)^2}} \quad j=1,\ldots,m, i=1,\ldots,n \quad (1)$$

$$\bar{n}^U{}_{j1} = \frac{x_{ij}^U}{\sqrt{\sum_{j=i}^m (x_{ij}^L)^2 + (x_{ij}^U)^2}}, j=1,\ldots,m, i=1,\ldots,n \quad (2)$$

Now interval $[\bar{n}^L{}_{ij}, \bar{n}^U{}_{ij}]$ is a normalized interval of $[x^L{}_{ij}, x^U{}_{ij}]$. The purpose of normalization is to preserve the property that the ranges of normalized interval numbers belong to [0,1].

STEP 2: Considering the difference importance of each criterion, construct the weighted normalized interval decision matrix as

$$\bar{v}_{ij}^L = w_i \bar{n}_{ij}^L, \text{where}, j=1,\ldots,m, i=1,\ldots,n, \quad (3)$$

$$\bar{v}_{ij}^U = w_i \bar{n}_{ij}^U, \text{where} j=1,\ldots,m, i=1,\ldots,n \quad (4)$$

where w_i is the weight of the ith attribute or criterion and $\sum_{i=1}^n w_i = 1$

STEP 3: Determine the positive ideal solution and negative ideal solution as

$$\bar{A}^+ = \{\bar{v}_1^+, \ldots, \bar{v}_n^+\} = \{(max_j \bar{v}_{ij}^U / i \in I), (min_j \bar{v}_{ij}^L / i \in J)\}, \quad (5)$$

$$\bar{A}^- = \{\bar{v}_1^+, \ldots, \bar{v}_n^+\} = \{(max_j \bar{v}_{ij}^L / i \in I), (min_j \bar{v}_{ij}^U / i \in J)\}. \quad (6)$$

where I is associated with benefit criteria, and J is associated with cost criteria.

STEP4: Calculate the separation of each alternative from the positive ideal solution, using the n-dimensional Euclidean distance as

$$\bar{d}_j^+ = \left\{\sum_{i \in I} (\bar{v}_{ij}^L - \bar{v}_i^+)^2 + \sum_{i \in J} (\bar{v}_{ij}^U - \bar{v}_i^+)^2\right\}^{\frac{1}{2}} j=1,\ldots,m \quad (7)$$

Similarly, the separation from the negative ideal solution can be calculated as

$$\bar{d}_j^- = \left\{\sum_{i \in I} (\bar{v}_{ij}^U - \bar{v}_i^-)^2 + \sum_{i \in J} (\bar{v}_{ij}^L - \bar{v}_i^-)^2\right\}^{\frac{1}{2}} j=1,\ldots,m \quad (8)$$

STEP5: Calculate the relative closeness coefficient to determine the ranking order of all alternatives. The relative closeness of the alternative A_j with respect to $\bar{A}^+$ is defined as

$$\bar{R}_j = \frac{\bar{d}_j^-}{(\bar{d}_j^+ + \bar{d}_j^-)} j=1,\ldots,m \quad (9)$$

Obviously, an alternative A_j is closer to the $\bar{A}^+$ and farther from $\bar{A}^-$ as $\bar{R}_j$ approaches to 1. Therefore, according to the closeness coefficient, we can determine the ranking order of all alternatives and select the best one among a set of feasible alternatives.

In sum, an algorithm to determine the most preferable choice among all possible choices, when data is interval, with extended TOPSIS approach is given in the following:

Step 1: Establishing system evaluation criteria that relate system capabilities to goals (identification the evaluation criteria).

Step 2: Developing alternative systems for attaining the goals (generating alternatives).

Step 3: Evaluating alternatives in terms of criteria (the values of the criterion functions which are intervals).

Step 4: Identifying the weight of criteria.

Step 5: Construct the interval decision matrix and the interval normalized decision matrix (using the formulas (1) and (2)).

Step 6: Construct the interval weighted normalized decision matrix (using the formulas (3) and (4)).

Step 7: Determine positive ideal solution and negative ideal solution (identification of $\bar{A}^+$ and $\bar{A}^-$, using the formulas (5) and (6).

Step 8: Calculate the separation of each alternative from positive ideal solution and negative ideal solution, respectively (identification of $\bar{d}_j^+$ and $\bar{d}_j^-$, using the formulas (7) and (8).

Step 9: Calculate the relative closeness of each alternative to positive ideal solution (identification of $\bar{R}_j$, using the formula (9)).

Step 10: Rank the preference order of all alternatives according to the closeness coefficient.

3. Numerical Calculations

This section studies the application of the TOPSIS method for decision-making problems with interval data. The aim of this study is to analyze the newspaper reading habits of college students based on the data collected from them and to find out their preference in reading the following choices of news:

1.International News, 2.Interest in politics,3.Local News, 4.Advertisement, 5.Scientific Articles, 6.Literary News, 7.Forecast, 8.Review of Books, 9.Film world news and 10.Sports column. The following four criteria were identified as the evaluation criteria to find out the reading choice of students. That is,C_1 – Highly, C_2.Frequently, C_3 – Occasionally and C_4 –Rarely.
Note that steps 1,2 and 3 are done.

Step4: Suppose that the vector of corresponding weight of each criteria is W=[0.125,0.125,0.125,0.125]

Step5: The interval decision matrix and interval normalized decision matrix are shown in tables 1 and 2, respectively.

Table 1: The interval decision matrix

A_i	C_1		C_2		C_3		C_4	
	x^L_{1j}	x^U_{1j}	x^L_{2j}	x^U_{2j}	x^L_{3j}	x^U_{3j}	x^L_{4j}	x^U_{4j}
A_1	12	24	40	52	18	32	2	8
A_2	13	35	15	31	9	27	10	23
A_3	20	64	14	32	3	8	5	12
A_4	4	8	8	32	21	40	21	46
A_5	27	56	21	44	7	12	1	4
A_6	7	16	20	44	19	38	9	18
A_7	8	16	19	44	16	32	11	24
A_8	11	24	5	32	17	35	13	25
A_9	1	4	9	16	10	40	23	56
A_{10}	13	22	25	58	4	12	14	24

Table 2: The interval normalized decision matrix

A_i	C_1		C_2		C_3		C_4	
	$\bar{n}^L_{j1}$	$\bar{n}^U_{j1}$	$\bar{n}^L_{j2}$	$\bar{n}^U_{j2}$	$\bar{n}^L_{j3}$	$\bar{n}^U_{j3}$	$\bar{n}^L_{j4}$	$\bar{n}^U_{j4}$
A_1	0.1073	0.2142.	0.2801.	0.36422	0.1722	0.3061.	0.2023	0.08092
A_2	0.1160	0.3124	0.1050.	0.21733	0.0861	0.2583	0.1011	0.23265
A_3	0.1785	0.5713	0.0906	0.22413	0.0287	0.0765	0.0505	0.12138
A_4	0.0357	0.0714	0.0560	0.22413	0.2009	0.3827	0.2124	0.46531
A_5	0.2410	0.4999	0.1470.	0.30819	0.0669	0.1148	0.0101	0.40461
A_6	0.0624	0.1484	0.1400	0.30819	0.1817	0.3635	0.0910	0.18207
A_7	0.0714	0.1428	0.1330	0.30819	0.1530	0.3061	0.1112	0.24277
A_8	0.0982	0.2142	0.1050	0.22413	0.1626	0.3348	0.1315	0.25288
A_9	0.0089	0.0357	0.0630	0.11206	0.0956	0.3827	0.2326	0.56646
A_{10}	0.1160	0.1964	0.1751	0.40625	0.0382	0.1148	0.1416	0.24277

Table-3: The interval weighted normalized decision matrix

A_i	C_1		C_2		C_3		C_4	
	$\bar{v}^L_{j1}$	$\bar{v}^U_{j1}$	$\bar{v}^L_{j2}$	$\bar{v}^U_{j2}$	$\bar{v}^L_{j3}$	$\bar{v}^U_{j3}$	$\bar{v}^L_{j4}$	$\bar{v}^U_{j4}$
A_1	0.01339	0.02678	0.03502	0.04552	0.02152	0.03827	0.00252	0.01011
A_2	0.01450	0.03905	0.01313	0.02714	0.01076	0.03229	0.01264	0.02908
A_3	0.02231	0.07142	0.01225	0.02801	0.00358	0.00956	0.00632	0.01517
A_4	0.00446	0.00892	0.00700	0.02801	0.02511	0.04783	0.02655	0.05816
A_5	0.03013	0.06249	0.01838	0.03852	0.00837	0.01435	0.00126	0.00505
A_6	0.00781	0.01855	0.01751	0.03852	0.02272	0.04544	0.01137	0.02275
A_7	0.00892	0.01855	0.01663	0.03852	0.01913	0.03827	0.01390	0.03034
A_8	0.01227	0.02678	0.01313	0.02801	0.01581	0.00418	0.01643	0.03161
A_9	0.00111	0.00446	0.00787	0.01400	0.01195	0.04783	0.02908	0.07058
A_{10}	0.01450	0.02455	0.02188	0.05078	0.00478	0.01435	0.01770	0.03034

Table 4: Distance of each alternative from positive ideal Solution

$\bar{d}^+_1$	$\bar{d}^+_2$	$\bar{d}^+_3$	$\bar{d}^+_4$	$\bar{d}^+_5$	$\bar{d}^+_6$	$\bar{d}^+_7$	$\bar{d}^+_8$	$\bar{d}^+_9$	$\bar{d}^+_{10}$
0.11246	0.10390	0.13248	0.1117	0.09468	0.10451	0.01080	0.10456	0.10587	0.10795

Table 5: Distance of each alternative from negative ideal solution

$\bar{d}^-_1$	$\bar{d}^-_2$	$\bar{d}^-_3$	$\bar{d}^-_4$	$\bar{d}^-_5$	$\bar{d}^-_6$	$\bar{d}^-_7$	$\bar{d}^-_8$	$\bar{d}^-_9$	$\bar{d}^-_{10}$
0.10588	0.06137	0.092	0.0818	0.07623	0.06415	0.05322	0.0629	0.22606	0.06505

Table 6: Closeness coefficient and ranking

Alternatives	$\bar{R}_j$	*Rank*
A	0.4849353	2
A	0.3713377	9
A	0.4098737	5
A	0.4243100	4
A	0.446020	3
A	0.3803598	6
A	0.33014101	10
A	0.37567707	8
A	0.69234727	1
A	0.3760209	7

From table-6, we conclude that the student's choice of reading news is as below: 1.Film world news 2.InternationalNews, 3.Scientific Articles, 4.Advertisement, 5.Local News, 6.Literary News, 7.Sports column, 8.Books Review ,9.Interest in politics and 10.Weather.

Thus from the above calculation, we infer that the students are giving high priority to read film related news and their next preference goes to read international news and scientific articles etc,. Also it shows that they are not interested to know much about politics and weather conditions. The main reason is that because of the latest availability of e-resources the students can get the required information where ever they are without spending much time in reading newspaper.

4. Conclusion

Since multicriteria decision problems generally involve uncertainty it is important to incorporate different types of uncertainty in any proposed solution. Under many conditions, exact data are inadequate to model real-life situations. In this paper by considering the fact that in some cases, determining precisely the exact value of the attributes is difficult and that, as a result of this, their values are considered as intervals, therefore, we have applied the extended TOPSIS method to develop a methodology for solving MCDM problems with interval data and it is effective in tackling complex, ill-defined and human-oriented decision problems.

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