Performance Analysis of RPCA Algorithm for Segregation of Singing Voice from Polyphonic Music

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Abstract: There are numerous real world applications for singing voice segregation from mixed audio. Using Robust Principal Component Analysis which is a compositional model for segregation, which decomposes the varied source of audio signal into low rank and sparse components, where it is assumed that musical accompaniment as low rank subspace since musical signal model is repetitive in character while singing voices can be treated as moderately sparse in nature within the song. Performance evaluation of the algorithm is verified by various performance measurement parameter such as source to distortion ratio (SDR), source to artifact ratio (SAR), source to interference ratio (SIR) and Global Normalized source to Distortion Ratio (GNSDR).

Keywords: Robust Principle Component Analysis (RPCA), Augmented Lagrange Multiplier (ALM), low rank matrix, sparse matrix, Singing voice separation

1. Introduction

Robust Principal Component Analysis (RPCA) technique is extensively used in the domain of image processing for image segmentation, surveillance video processing, batch image alignment etc. This technique has received recent prominence in the field of audio separation for the application of singer identification, musical information retrieval, lyric recognition and alignment.

A song usually comprise of mixture of human vocal and musical instrumental audio pieces from string and percussion instruments etc. Our area of interest is segregating vocal line from music which is complex and vital musical signal element from song, thus we can treat musical as intrusion or noise with respect to singing voice. Human auditory system has incredible potential in separating singing voices from background music accompaniment. Human beings are capable to derive semantic information of the audio perceived which comprises of multiple time and frequency overlapping sources, even when interfering energy is nearby of the target sources. This task is natural and effortless for humans, but it turns out to be difficult for machines [15].

Compositional model based RPCA has emerged as a potential method for singing voice separation based on the idea that low rank subspace can be assumed to consist of repetitive musical accompaniment, whereas the singing voice is relatively sparse in time frequency domain. Basic audio voice separation systems can be divided into two categories that are supervised system and unsupervised system. In supervised systems, training data is required to train the system. On the contrary, in unsupervised systems, no prior training or particular feature extraction is required.

Excessive study has been carried out for speech separation, but only a lesser number of researches are devoted to separating singing voice from music accompaniment. Singing voice bears many similarities to speech. They both consist of voiced and unvoiced sounds. A well-known difference is the presence of an additional formant, termed as singing formant, in the frequency range of 2000Hz-3000Hz in singing; this singing formant makes the voice of a singer to stand out from the accompaniment. Likewise if we see singing voice mainly consists of voiced parts while that of speech primarily consists of unvoiced parts. If we analyze the pitch information of singing voice we come across that there are abrupt fluctuations in the pitch in between while in contrast the pitch of natural speech smoothly changes in an utterance. Likewise singing voice has much wider frequency ranges while the pitch range of normal speech is between 80–400Hz [1].

The challenges for singing voice separation from background music accompaniment are as follows. In general, the auditory scene created by a musical composition can be viewed as a multi-source environment, where diverse sound sources from several classes of instruments are momentarily active, some of them only sparsely. The music sources may be of different instrumental type (therefore exhibiting altered timbral sensations), may be played at various pitches and loudness, and even the spatial location of a given sound source may differ with respect to time. Regularly individual sources repeat during a musical piece, one or the other way in a different musical context or by revisiting already established phrases. Therefore, the scene can be regarded as a time-varying schedule of source activity containing both novel and recurring patterns, representing changes in the spectral, temporal, and spatial complexity of the mixture. Moreover the singing voice has varying pitch frequency for male and female singer which may at some instant overlap with background frequency pattern of musical instruments. To solve these challenges a compositional model designed using a novel technique Robust Principal Component Analysis (RPCA) is proposed using Augmented Lagrange Multiplier (ALM) as a
optimization algorithm for better convergence. Robust Principal Component Analysis [2], which is a matrix factorization algorithm for solving low rank matrix and sparse matrix. Here in our proposed system we assume that the music accompaniment lie in low rank subspace while the singing voice is relatively sparse due to its more variability within the song.

The paper is organized as follows: Section 2 describes the proposed algorithm for singing voice separation. Section 3 presents the evaluation results for the separated audio and Section 4 concludes the paper.

2. Proposed Algorithm

We have a input audio which is superimposition of singing voice and background musical instruments which can be considered in terms data matrix (audio signal) which is combination of low rank component (musical accompaniment) and sparse components (singing voice). We suppose the above statement we are leveraging on the fact that such data have low intrinsic dimensionality like as they lie on numerous low dimensional subspace, are sparse also in some basis [8]. We execute separation of singing voice as follows seen in figure 1. 

1) We compute Short-Time Fourier Transform (STFT) of the targeted audio signal where signal is represented in time frequency domain. In the separation method, STFT of the input audio signal is calculated using overlapping hamming window with N=1024 samples at sampling rate of 16Khz.2) After calculation of STFT, RPCA is applied by means of Augmented Langrange Multiplier (ALM) which solves the computational problem of RPCA [2]. After applying RPCA we get two output matrices ‘L’ low rank matrix and ‘S’ sparse matrix. Binary frequency mask is later applied for quality of separation result. 3) Inverse Short Time Fourier transform (ISTFT) is latter applied, in order to obtain the waveform of the estimated results followed by evaluation of the results.

![Figure 1: Proposed System](image)

A. Robust Principal Component Analysis (RPCA)

In real world data if n m-dimensional data vectors is put in the form of a matrix \( A \in \mathbb{R}^{m \times n} \), where \( A \) should have a rank \( \leq \min \left( m, n \right) \), which means that there are some linearly independent columns [5]. The objective is to obtain low rank approximation of \( A \) in the existence of noises and outliers. The classical principal component analysis approach which assume the given high dimensional data lie near a much lower dimensional subspace [11]. The method seeks a rank \( - r \) estimate \( M \) of the matrix \( A \) by solving,

\[
\min_{X} ||A - M||_F \\
\text{Subject to rank}(X) \leq r, \quad (1)
\]

where \( ||A|| \) denotes the largest singular value decomposition value of \( A \) or the spectral norm. The above problem can be solved via singular value decomposition (SVD), by using \( r \) largest singular values. But PCA is subtle to outliers and performance declines under bulky corruption. To solve this issue, robust PCA (RPCA) [2, 6] is used to render PCA robust to outliers and gross corruption.

A data matrix \( M \in \mathbb{R}^{m \times n} \) can be uniquely and exactly be decomposed into a low rank component \( A \) and a sparse component \( E \), also retrieval of low rank matrix by convex programming. The convex optimization problem can be put forth as follows in terms of objective function and a constraint function [8].

\[
\text{minimize}||A||_*, + \lambda ||E||_1 \\
\text{subject to } A + E = M \quad (2)
\]

where \( ||\cdot||_* \) denote the nuclear norm i.e the sum of singular values and \( ||\cdot||_1 \) denote the L1-norm that is the sum of the absolute values of matrix entries is an valuable surrogate for L0 pseudo norm, the number of non-zero entries in the matrix \( \lambda \) is the trade off parameter between the rank of \( A \) and sparsity of \( E \) [6].

\[
\lambda_k = k \sqrt{\max\{m,n\}} \quad (3)
\]

where \( \lambda > 0 \) is a regularization parameter and for \( k=1 \) we get best quality separation result and the results are tested for different values of \( k \). Proficient optimization scheme the Augmented langrange multiplier method is used for solving the above RPCA problem which has higher convergence property. ALM algorithm is iterative converging scheme which works by repeatedly minimizing the rank of \( A \) and \( E \) matrices simultaneously [4]. ALM is optimization technique for noise reduction.

The ALM function is defined as follows

\[
\begin{align*}
L(A, E, \lambda, Y, \mu) &= ||A||_* + \lambda ||E||_1 + \langle Y, A + E - M \rangle + \frac{\mu}{2} ||A + E - M||_F^2 \\
\end{align*}
\]

(4)

Where \( \lambda \in \mathbb{R}^{m \times n} \) is the langrange multiplier of the linear constraint that allows removing the equality constraint, \( \mu > 0 \) is a penalty parameter for the violation of the linear constraint, \( \langle \cdot, \cdot \rangle \) implies the standard trace inner product and \( ||\cdot||_F \) is a frobenious norm. Thus the augmented langrange multiplier gives us two segregated matrices that is the low rank matrix \( A \) and the sparse matrix \( E \) respectively. To acquire the wave forms of the projected components the phase of the original signal is appended with two separated matrices.
For better separation outcomes masking can be applied to the separation results of ALM that are low rank A and sparse E matrices by using binary time frequency masking [6]. We need to accurately segregate the components as singing voice mostly lines the music accompaniment during beat instances in order to match with rhythmic structure of the song and hence we apply masking for enhanced separation outcomes.

Binary time frequency masking \( J_m \) as follows:

\[
J_m(m,n) = \begin{cases} 
1, & |E(m,n)| > \text{gain} \times |A(m,n)| \\
0, & \text{otherwise}
\end{cases}
\]

(5)

After application of time frequency masking it is applied to the original audio signal \( M \) in order to obtain the separation matrix as singing voice and music respectively.

3. Evaluation Results

We have worked on this project using MIR-1K database, comprising of male singer and another half of female singer with a sample rate of 16KHz and the duration of the audio clip is 10-14 seconds. We create three clips, first consisting of mixed song, second consisting of singing voice and third consisting of musical accompaniment from the stereo database by converting it to mono channel using Audacity software, for the evaluation of the results. The separated audio files are compared with these files.

For the separation and evaluation purpose, spectrograms of each mixture is computed for input audio signal and separated audio signals i.e. the singing voice and music accompaniment. We have taken audio clips consisting of two or more musical instruments in the background and studied its impact on separation. Figures 2 and 3 show the spectrograms for respective audio signal separately for different values of \( k \) (of \( \lambda_k \)) and on merging the spectrogram of singing voice and music accompaniment we get spectrogram of mixed song. For construction of spectrogram results the low rank and sparse matrix and multiplied by the initial phase of the audio signal. We can examine the varying pitch pattern of separated vocal from song in the spectrograms obtained. In figure 1 spectrogram consists of larger voiced part than that of figure 2 spectrogram for singing voice.

The value of \( \lambda_k = \frac{k}{\max_{m,n} |E(m,n)|} \) which is a tradeoff parameter with respect to rank of A (low rank component) and with the scarcity of E (sparse matrix). From investigational outcomes it has been observed that if the matrix E is sparser which means that there is less interference in the matrix E (sparse matrix) but due to this deletion of original components may result in artifacts which is undesirable for the proposed system. If E matrix is less sparse, then audio signal will contain, then the signal contains less artifacts which implies that there is more interference from the sources which exist in matrix E. Thus from this we can say that matrix E (sparse matrix) is sparser with higher \( \lambda_k \) value and vice versa. We can spot this difference for value of \( k \) (of \( \lambda_k \)) = \{0.1, 0.25, 0.50, 0.75, 1, 2, 3, 4\} from the above array we can notice that for values above 1 in the array separation does not take place.

For evaluation of the performance separation results in terms of Source to Interference Ratio (SIR), Source to Artifacts Ratio (SAR) and Source to Distortion Ratio (SDR) with help of BSS-EVAL metrics [15, 13]. We also evaluate the performance in terms of Global Normalized Source to Distortion Ratio which takes into account the re-synthesized singing voice (\( v \)), original clean voice (\( v \)) and the mixture (\( x \)).

The Source to Distortion Ratio (SDR)

\[
SDR = 10 \log_{10} \frac{\|target\|^2}{\|\text{interf} + \text{enoise} + \text{eartif}\|^2}
\]

(4)

The Source to Interference Ratio (SIR)

\[
SIR = 10 \log_{10} \frac{\|target\|^2}{\|\text{interf}\|^2}
\]

(5)

The Source to Noise Ratio (SNR)

\[
SNR = 10 \log_{10} \frac{\|target + \text{interf}\|^2}{\|\text{enoise}\|^2}
\]

(6)
The Source to Artifact Ratio (SAR)

\[
\text{SAR} = 10 \log_{10} \frac{\| \text{target} \|_2}{\| \text{artifact} \|_2} + \| \text{einterf} \|_2 + \| \text{enoise} \|_2
\]  
(7)

The Normalized SDR (NSDR) is defined as

\[
\text{NSDR} (\mathbf{v}, \mathbf{v}, \mathbf{x}) = \frac{\sum_{i=1}^{N} \text{SDR}(\mathbf{\tilde{v}}, \mathbf{v}, \mathbf{x})}{\sum_{i=1}^{N} w_i}
\]  
(8)

\[
\text{GNSDR}(\mathbf{\tilde{v}}, \mathbf{v}, \mathbf{x}) = \frac{\sum_{i=1}^{N} \text{NSDR}(\mathbf{\tilde{v}}, \mathbf{v}, \mathbf{x})}{\sum_{i=1}^{N} w_i}
\]  
(9)

Whereas target \( \mathbf{v} \) is an tolerable distortion of the target source \( \mathbf{s}_i(t) \), einterf \( (t) \) is an impermissible deformation of the sources which accounts for the interferences of the undesirable sources, e_noise \( (t) \) is an allowed deformation of the perturbing noise (but not the sources), and e_artif \( (t) \) is an “artifact” term that may parallel to artifacts of the separation algorithm such as musical noise, etc. or simply to distortions encouraged by the separation algorithm that are not allowed.

**Table 1:** Results for different values of \( k \) (of \( \lambda_k \)) for song1

<table>
<thead>
<tr>
<th>k (of ( \lambda_k ))</th>
<th>SDR</th>
<th>SIR</th>
<th>SAR</th>
<th>GNSDR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.0927</td>
<td>0.1031</td>
<td>29.2480</td>
<td>0.0177</td>
</tr>
<tr>
<td>0.25</td>
<td>0.1926</td>
<td>0.2662</td>
<td>20.8194</td>
<td>0.1176</td>
</tr>
<tr>
<td>0.50</td>
<td>0.6007</td>
<td>0.8967</td>
<td>14.9970</td>
<td>0.5257</td>
</tr>
<tr>
<td>0.75</td>
<td>1.3295</td>
<td>2.0241</td>
<td>11.7475</td>
<td>1.2545</td>
</tr>
<tr>
<td>1.00</td>
<td>2.7360</td>
<td>5.0676</td>
<td>7.7281</td>
<td>2.6610</td>
</tr>
<tr>
<td>1.25</td>
<td>3.5370</td>
<td>12.6074</td>
<td>4.3434</td>
<td>3.4620</td>
</tr>
<tr>
<td>1.50</td>
<td>3.3043</td>
<td>19.9380</td>
<td>1.0480</td>
<td>1.2293</td>
</tr>
<tr>
<td>1.75</td>
<td>-1.5218</td>
<td>34.6055</td>
<td>-1.5193</td>
<td>-1.5968</td>
</tr>
<tr>
<td>2</td>
<td>-3.7622</td>
<td>45.0948</td>
<td>-3.7620</td>
<td>-3.8372</td>
</tr>
<tr>
<td>3</td>
<td>-11.4609</td>
<td>56.5936</td>
<td>-11.4608</td>
<td>-11.5358</td>
</tr>
<tr>
<td>4</td>
<td>-17.2891</td>
<td>45.0206</td>
<td>-17.2890</td>
<td>-17.3644</td>
</tr>
</tbody>
</table>

**Table 2:** Results for different values of \( k \) (of \( \lambda_k \)) for song2

<table>
<thead>
<tr>
<th>k (of ( \lambda_k ))</th>
<th>SDR</th>
<th>SIR</th>
<th>SAR</th>
<th>GNSDR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.0115</td>
<td>0.0387</td>
<td>25.0416</td>
<td>0.0397</td>
</tr>
<tr>
<td>0.50</td>
<td>1.3941</td>
<td>2.2545</td>
<td>10.8753</td>
<td>1.4223</td>
</tr>
<tr>
<td>0.75</td>
<td>2.4416</td>
<td>5.2213</td>
<td>6.8367</td>
<td>2.4699</td>
</tr>
<tr>
<td>1.00</td>
<td>2.5535</td>
<td>8.8622</td>
<td>4.2416</td>
<td>2.5818</td>
</tr>
<tr>
<td>1.25</td>
<td>1.3038</td>
<td>12.3903</td>
<td>1.8993</td>
<td>1.3320</td>
</tr>
<tr>
<td>1.50</td>
<td>-0.4133</td>
<td>14.7784</td>
<td>-0.1377</td>
<td>-0.3851</td>
</tr>
<tr>
<td>1.75</td>
<td>-2.1408</td>
<td>17.4368</td>
<td>-2.0150</td>
<td>-2.1126</td>
</tr>
<tr>
<td>2</td>
<td>-3.6884</td>
<td>20.0092</td>
<td>-3.6267</td>
<td>-3.6602</td>
</tr>
</tbody>
</table>

Better separation results are obtained for value of \( k \) (of \( \lambda_k \)) less than 1.5 as seen in table 1 and 2 respectively. Greater the value of SDR, SAR, SIR and GNSDR better separation quality is achieved, the above values can be compared for various values of \( k \) (of \( \lambda_k \)) in the above table and graph.

4. Conclusion

In this paper, singing voice separation from music accompaniment is tried using technique known as Robust Principal Component Analysis. We have optimized separation algorithm using Augmented Lagrange Multiplier for various values of \( \lambda_k \) where in quality separation outcomes are proven for values less than 2 and the results are justified through spectrogram outcomes and can be verified perceptually through segregated audio.

**References**


