

# Minimum Cut in the Project Network to Solve Time Cost Trade - Off Problem Using Renewable and Non Renewable Resources

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**Abstract:** *The time cost trade – off problem is one of the most important aspects of construction of project planning , scheduling and controlling. There are trade-offs between time and cost to complete the activities of a project. In general, the less expensive the resources used, the longer it takes to complete an activity. Using critical path method the overall project cost can be reduced by using less expensive resource for non-critical activities without impacting the project duration. Here a new solution procedure is introduced to solve the trade – off problems by using minimum cut or maximal flow concept. We find the minimum cuts for a acyclic graph which represents the critical path of the project by using the Karger’s fast algorithm of randomized minimum cut. The minimum cuts so obtained are used to trace the project activities which are to be reduced, in order to reduce the total project duration. The concepts of renewable and non-renewable resources are used to minimize the project duration. The applicability of the proposed solution procedure is illustrated by an example.*

**Keywords:** Project network, randomized minimum cut, maximal flow, renewable resources and non- renewable resources

## 1. Introduction

A project is made up of a group of interrelated work activities constrained by a specific scope, budget, and schedule to deliver capital assets needed to achieve the strategic goals of an Agency. All projects must have a beginning and an end. The management of construction projects requires knowledge of modern management as well as an understanding of the design and construction process. Construction projects have a specific set of objectives and constraints such as a required time frame for completion. While the relevant technology, institutional arrangements or processes will differ, the management of such projects has much in common with the management of similar types of projects in other specialty or technology domains such as aerospace, pharmaceutical and energy developments. Project management is the art of directing and coordinating human and material resources throughout the life of a project by using modern management techniques to achieve predetermined objectives of scope, cost, time, quality and participation satisfaction.

A project can be modeled as a directed acyclic graph, the network is constructed, where the activities are represented by arcs incident on a node and precede the activities or arcs emanating from the node. Where project network consist of  $n$  nodes, node 1 refers as starting node and  $n$  as finishing node. An activity cost is also considered with respect to the precedence relations and duration of the project. These curves represent the cost of performing the activity as a function of time, thus specifying the cost of resources utilization for that activity as presented in (1). In general, the function may be non-linear or discrete but, we assume a linear and continuous cost function. Here cost is considered as a non renewable resources and availability of renewable resources per period has been neglected and as mentioned in (2) that the quality of the complete project is also affected by project crashing. Hence the problem that considers project

time as renewable and cost and quality as non-renewable resources.

In this paper, a solution procedure is developed using minimal cut or maximal flow to identify the activities in the project network which are to be reduced in order to reduce the total project duration is presented. In section 2, we present the problem with notations used. In section 3, we present solution procedure. In section 4, we present a numerical example for our solution procedure.

## 2. Problem Formulation

Let  $G = (V, E)$  be a multi graph (an acyclic graph) with  $n$  vertices and  $m$  edges. Computing the cut with minimum number of edges, find a set which is subset of  $V$  i.e.  $S \subseteq V$  such that  $(S \times (V - S) \cap E)$  is as small as possible, where  $S$  is non-empty. To contract two vertices say  $P_1$  and  $P_2$  are replaced by new vertex say  $P$ . The set of edges incident on  $P$  are then the union of the sets of edges incident on  $P_1$  and  $P_2$ . Edges from  $P_1$  and  $P_2$  that have same end point say  $Q$  are not merged, rather multiple instances of those edges created. Self loop edges are removed and rest of the graph remains unchanged. Graph  $G$  with edge  $(P, Q)$  contracted is denoted by  $G / (P, Q)$ . And then Karger’s randomized algorithm proposed by (3) is used to locate the minimal cut of a multi graph representing the critical network and then reducing the duration of the activities on the cut until required amount of reduction in the total project’s duration is achieved. The advantages of the solution procedure are of double. First, it does not rely on graphically locating the cuts in the network. Second, it utilizes a fast algorithm to locate the minimal cut and utilizes an algorithm that finds the minimum cut in only  $O(n^2 \log n)$  time. Karger’s algorithm uses a fundamental operation called Contraction or collapsing of graph vertices.

### 3. Notations

$X_{ij} = 1$ , capacity of minimum cut  
 = 0 otherwise

Interpreting the flows  $X_{ij}$  as capacities for a minimum cut problem, one wants to know whether the capacity of a minimum cut in the network is less than 1.

$t_{ij}$  = the duration an activity (i,j)

$r_{ij}$  = the amount of the renewable resource r

$P_r$  is the price of the renewable resource r

$C_{ij}^*$  = cost of the activity (i,j) =  $t_{ij} \times r_{ij} \times P_r$  + the cost of non renewable resources

$C_{ij}$  = resource utilization rate for activity ij (cost slope)

$$= \frac{\text{crashcost} - \text{normalcost}}{\text{normaltime} - \text{crashtime}}$$

where, crash time < normal time

=  $\infty$ , Otherwise

$q_{ij}$  = quality of activity (i,j)

$N_i = \{E_i, E_i+1, \dots, L_i\}$ , where  $E_i$  is the earliest time of occurrence for event i and  $L_i$  is the latest time of occurrence for event i.

$y_{iu} = 1$  if event i occurs in time u

= 0 otherwise

$M_T$  = maximum time to complete the project

$B_{max}$  = maximum available budget

### 4. Solution Procedure

The general contraction algorithm of Karger for reducing the graph until it has only t-vertices. The contraction algorithm is used by itself to find a minimum cut in a graph, to improve the algorithm performance it seems practical to partially contract the graph, and then switch to a possibly slower algorithm with better chance of success on what remains as proposed by D.R.Karger. The same concept is implemented in the solution of time cost trade off problem. A multi graph is used to represent the activity- on – arc network. The multi arcs between any pair of nodes, say node V and U represents the corresponding resource utilization. At each iteration of the proposed algorithm, a multi graph is constructed using the critical activities that are found from the original network with respect to the cost  $C_{ij}$ . The multi graph is the input to the fast contract procedure to obtain the minimum cut. The duration of activities in the cut are reduced by the amount given below.

$$t = \min \left\{ (T_{cur} - T_{req}), \min [ (t_{ijn} - t_{ijc}), \forall i, j \in Z_1 ], (T_{cur} - l_n) \right\}$$

In which  $t$  is the amount by which activities in the cut are reduced in time  $T_{req}$  is the time required to complete the project and it is the length of the current critical path. The current duration of activity (i,j) is  $t_{ijn}$ .  $Z_1$  is the set of activities in the minimum cut and  $l_n$  is the length of the longest non-critical path. Activity durations are updated and new critical activities are determined. In an iterative manner, the activities in the minimum cuts are crashed until the required completion time is obtained. Now let us use the concept of renewable and non-renewable resources for the above multi graph approach for time cost trade-off problems i.e., by finding cost  $C_{ij}^*$  using the formula below to construct the multi graph for identifying the minimum cuts.

$C_{ij}^*$  = cost of the activity (i,j) =  $t_{ij} \times r_{ij} \times P_r$  + the cost of non renewable resources

Construction of steps are as usual except the cost calculations. We reprocess the project data based on the original activity time which is renewable resource and cost non-renewable resource. Here we consider renewable resource per period is 4 ( i.e.,  $r_{ij} = 4$ ) and price per period is 6 ( i.e.,  $P_r = 6$ ) and we calculate the cost as per the requirement.

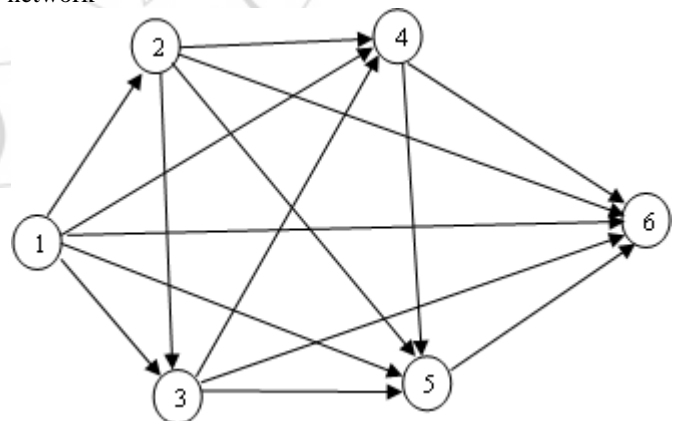
### 5. Numerical Example

Our solution procedure is illustrated by the following example. Let us consider that the completion time of the project in this example should be reduced to 20 days with minimum additional cost.

**Table 1: Problem Description**

| Activity | Normal |      | Crash |      | Cost slope $C_{ij}$ |
|----------|--------|------|-------|------|---------------------|
|          | Time   | Cost | Time  | Cost |                     |
| 1-2      | 9      | 30   | 4     | 60   | 6                   |
| 1-3      | 16     | 24   | 8     | 40   | 2                   |
| 1-4      | 6      | 8    | 2     | 40   | 8                   |
| 1-5      | 6      | 24   | 2     | 40   | 4                   |
| 1-6      | 7      | 40   | 3     | 60   | 5                   |
| 2-3      | 8      | 22   | 4     | 50   | 7                   |
| 2-4      | 9      | 8    | 5     | 20   | 3                   |
| 2-5      | 5      | 10   | 3     | 18   | 4                   |
| 2-6      | 8      | 64   | 4     | 100  | 9                   |
| 3-4      | 3      | 6    | 1     | 10   | 2                   |
| 3-5      | 10     | 16   | 8     | 22   | 3                   |
| 3-6      | 8      | 32   | 4     | 80   | 12                  |
| 4-5      | 7      | 40   | 3     | 92   | 13                  |
| 4-6      | 9      | 24   | 5     | 80   | 14                  |
| 5-6      | 4      | 50   | 1     | 74   | 8                   |

In the first step we define the project using an activity-on-arc network



**Figure 1: Project Network**

Determine the critical activities

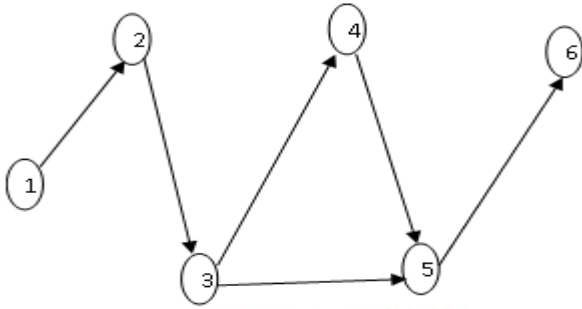


Figure 2: Critical Path

Establish the multi graph with respect to the cost which represents the critical activities

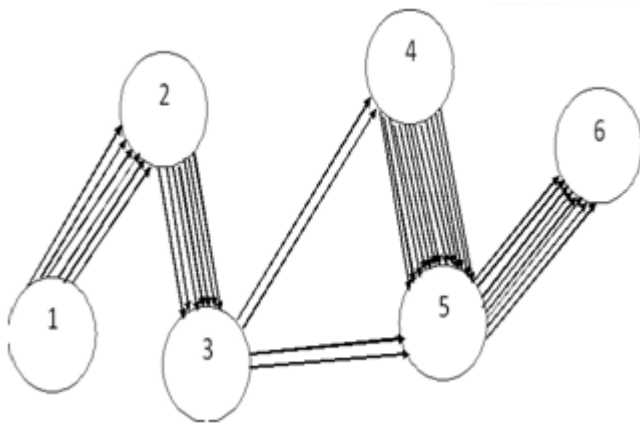


Figure 3: Multi graph constructed using  $C_{ij}$

Now using Karger's Algorithm, the critical activities in the minimum cut are identified, i.e., finding the minimum number of edges in a multi graph the deletion of which the graph disconnected. Therefore the critical activities identified from above minimum cut are the critical activities 3 – 4 and 3 – 5. Calculate the value of  $t$  and deduct the value of  $t$  from the activities identified.

That is,

$$t = \min \{ (T_{cur} - T_{req}), \min [ (t_{ij-cur} - t_{ijc}), \forall i, j \in S_1 ], (T_{cur} - l_n) \}$$

$$t = \min \{ (31 - 20), \min [ (3 - 1, 10 - 8), (31 - 27) ] \}$$

$$t = \min \{ 11, 2, 4 \}$$

Therefore,  $t = 2$

Reduce the duration of each critical activity in the minimum cut by 2 units. i.e., duration of critical activities 3 – 4 and 3 – 5 by 2 units. Now construct the network with the new time durations for the above said activities, identify the critical activities and critical path. Repeat the procedure till the project is deliverable at  $T_{req}$  i.e., 20 days for the above said example. Now let us consider the renewable and non renewable resources and calculate the revised cost values  $C_{ij}^*$  and proceed as usual till the project duration reach the required time i.e., 20 days

## 6. Conclusion

In this paper firstly, the application of a new solution procedure to the problem of time cost trade-off is presented and the karger's algorithm is used to identify the minimum

cut in an iterative manner and the activities in the minimum cuts are crashed till the project required completion time is obtained. Secondly, the renewable and non-renewable resources are considered and cost value is found with respect to renewable and non-renewable resources and the rest of the solution procedure is as usual. The difference between these two approaches are presented.

## References

- [1] Hamed R.Tareghian, Hassan Taheri, An application of randomized minimum cut to the project time/cost tradeoff problem, Applied Mathematics and Computation, 173(2006) 1200-1207
- [2] N.Ravi Shankar, M.M.K.Raju, P.Himabindu, Discrete time, cost and quality trade off problem with renewable and non renewable resources, International journal of computational science and mathematics, 2010, 285-290
- [3] D.R.Karger, Random sampling in graph Optimization problems, Ph.D Thesis, Stanford University, Stanford, CA 94305, 1994
- [4] J.R.Kelly Jr., Critical path planning and scheduling mathematical basis, Operations Research, 9(3)(1961) 296-320.
- [5] D.R. Fulkerson, A network flow computation for project cost curves, Management Science 7(1961) 167- 178