The Closed and Completely Closed U-BG-Ideals with Respect to Element of a U-BG-BH-Algebra

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Abstract: ***In this paper, we introduce the notions of closed and completely closed U-BG-ideals with respect to element of a U-BG-BH-algebra, we state and prove some theorems and examples which determine the relationships among these notions and some types of ideals of a BH-algebra***.

Keywords: BG-algebras, BH-algebras, ideal, U-BG-ideal, closed and completely closed U-BG-ideals with respect to element of a BH-algebra

1. Introduction

The notion of a BH-algebra and ideal of a BH-algebra introduced in [8] by Y. B. Jun, E. H. Roh and H .S. Kim in 1998. In 2008, C. B. Kim and H. S. Kim introduced the notion of a BG-algebra [1]. In 2011 H. H. Abass and H. M. Saeed introduced the notion of a closed ideal with respect to an element of a BH-algebra [3]. In 2012 H.H. Abass and H. B. Dahham introduced the notion of a completely closed ideal with respect to an element of a BH-algebra [2]. In 2014 H.H. Abbass and L.S. Mahdi introduced the notion of a new class of a BH-algebra and closed and completely closed U-BG-ideal [4]. In this paper, we introduce the notions of closed and completely closed U-BG-ideals with respect to element of a U-BG-BH-algebra as a new type of ideals of a U-BG-BH-algebra. Also, we give some properties of these notions and we link then with some other types of ideals of U-BG-BH-algebra.

2. Preliminaries

In this section, we give some basic concepts about a BG-algebra, BH-algebra, U-BG-BH-algebra, (BCA part, ideal**,** BH-ideal, U-BG-ideal, closed and completely closed U-BG-ideal) of a BH-algebra and some other concepts that we need in our work.

Definition 1 (see [1]) A **BG-algebra** is a non-empty set X with a constant 0 and a binary operation " * " satisfying the following axioms:

i. $x*x=0$
ii. $x*0=x$
iii. $(x*y)*(0*y)=x$, for all $x,y\in X$.

Definition 2 (see [8]). A **BH-algebra** is a nonempty set X with a constant 0 and a binary operation"*" satisfying the following conditions:

i. $x*x=0$, $\forall x\in X$.
ii. $x*y=0$ and $y*x=0$ imply $x = y$, for all $x, y\in X$.
iii. $x*0 =x$, for all $x\in X$.

Definition 3 (see [4]). A **BH-algebra** X is called is a **U-BG-BH-algebra** if there exists a proper subset **U** of X, such that:

i. $0\in$ **U** , $|$**U**$|\geq 2$.
ii. **U** is a BG-algebra.

Definition 4 (see [3]). Let X be a BH-algebra. Then the set $X_+=\{x\in X:0*x=0\}$ is called the **BCA-part** of X.

Definition 5 (see [2]). A BH-algebra X is said to be **0-commutative** if: $x*(0*y)=y*(0* x)$ for all $x , y \in X$.

Definition 6 (see [5]). Let X be a BH-algebra. Then the set $med(x)=\{x\in X: 0* (0* x) =0\}$ is called the **medial part** of X.

Definition 7 (see [4]). Let X be a BH-algebra. Then the set **G(X)**$=\{x\in X:0*x = x\}$ is called **G-part** of X.

Remark 1 (see [7]). Let X and Y be BH-algebras.A mapping $f:X\to Y$ is called a homomorphism if $f(x*y)=f(x)*f(y)$, for all $x,y\in X$. a homomorphism f is called a monomorphism(resp., epimorphism) if it is injective (resp., surjective). A bijective homomorphism is called (isomorphism). Two BH-algebras X and Y are said to be isomorphic, written $X\cong Y$, if there exists an isomorphism $f:X\to Y$. For all homomorphism $f:X \to Y$, the set $\{x\in X : f(x)=0\}$ is called the kernel of f, denoted by Ker(f), and the set$\{f(x):x\in X\}$is called the image of f, denoted by f(x). Notice that $f(0)=0'$, for all homomorphism f, and $f^{-1}(Y) =\{x\in X:f (x) = y, \text{ for some } y\in Y\}$

Definition 8 (see [7]). Let X be a BH-algebra. For a fixed $a\in X$, we define a map $R_a:X\to X$ such that $R_a(x)=x*a$ for all $x\in X$, and call R_a a **right map** on X. The set of all right maps on X is denoted by R(X). A left map L_a is defined by a similar way, and the set of all left maps on X is denoted by L(X).

Definition 9 (see [2]). A BH-algebra X is called an associative BH-algebra if: $(x*y) *z= x* (y*z)$, for all $x , y , z \in X$.

Definition 10 (see [8]). Let I be a nonempty subset of a BH-algebra X. Then I is called an **ideal** of X if it satisfies:

i. $0\in I$.
ii. $x*y\in I$ and $y\in I$ imply $x\in I$, for all$\in X$

Definition 11 (see [2]). Let X be a BH-algebra and I be a subset of X. Then I is called a **BH-ideal** of X if it satisfies following conditions:

i. $0\in I$,

ii. $x*y\in I$ and $y\in I \Rightarrow x\in I$,
iii. $x\in I$ and $y\in X\Rightarrow x*y\in I$, $I*X\subseteq I$.

Definition 12 (see [3]). An ideal I of a BH-algebra X is called a **closed ideal** of X if : $0*x\in I$, for all $x\in I$.

Definition 13 (see [2]). An ideal I of a BH-algebras is called a **completely closed ideal** if $x*y\in I$, for all $x,y\in I$.

Definition 14 (see [3]). A nonempty subset I of a BH-algebra X is called a **p-ideal** of X if:
i. $0 \in I$,
ii. $(x*z)*(y*z) \in I$ and $y \in I$ imply $x\in I$, for all $x, y, z \in X$.

Remark 2 (see [3]). Let X be a BH-algebra. Then every p-ideal is an ideal of X.

Definition 15 (see [3]). A nonempty subset I of a BH-algebra X is called a **q-ideal** of X if:
i. $0 \in I$,
ii. $x*(y*z) \in I$ and $y \in I$ imply $x*z\in I$, for all $x, y, z \in X$.

Definition 16 (see[6]). A nonempty subset I of a BH-algebra X is called a **strong-ideal** of X if:
i. $0 \in I$,
ii. $(x*y)*z \in I$ and $y \in I$ imply $x*z\in I$, for all $x, y, z \in X$.

Remark 3 (see [6]). Let X be a BH-algebra. Then every strong-ideal is an ideal of X.

Definition 17 (see [4]). A nonempty subset I of a U-BG-BH algebra X is called a **U-BG-ideal** of X related to U if it satisfies:
i. $0\in I$.
ii. $x*y\in I \Rightarrow x\in I$, for all $x\in U$ and $y\in I$.

Definition 18 (see [4]). A U-BG-ideal I of a U-BG-BH-algebra X is called a **closed U-BG-ideal** of X if:$0*x\in I$, for all $x\in I$, and called a **completely closed U-BG-ideal** of X if: $x*y\in I$, for all $x, y\in I$.

Proposition 1(see[4]). Let X be a U-BG-BH-algebra. Then:
1) every ideal of X is a U-BG-ideal of X.
2) if $\{ I_i, i\in\lambda\}$ be a chain of U-BG-ideals of X. Then $\bigcup_{i\in\lambda} I_i$ is a U-BG-ideal of X.
3) If U_1 and U_2 are a BG- algebra ,which are properly contained in X such that $U_1\subseteq U_2$,Then every U_2-BG-ideal is U_1-BG-ideal.
4) I be a U-BG- ideal such that $x*y\notin I$, for all $x \notin I$ and $y\in I$. Then I is an ideal of X.
5) every BH-ideal is a completely closed U-BG-ideal of X.

Proposition 2 (see [4]). Let $f:(X,*,0) \rightarrow (Y,*',0')$ be a U-BG-BH-epimorphism then.
i. If I a U-BG-ideal of X. Then f(I) is a f(U)-BG-ideal of Y.
ii. if I is a U-BG-ideal of Y. Then $f^{-1}(I)$ is a $f^{-1}(U)$-BG-ideal in X.
iii. ker(f) is a U-BG-ideal of X.

3. The Main Results

In this section, we introduce the concepts closed and completely closed U-BG-ideals with respect to element of U-BG-BH-algebra. Also, we state and prove some propositions and examples about these concepts.

Definition 19. Let I be a U-BG-ideal of a BH-algebras X and $b\in X$. Then I is called a **closed U-BG-ideal with respect to b**(denoted by **b-closed U-BG-ideal**) if $b*(0*x)\in I$, for all $x\in I$.

Example 1: Consider the U-BG-BH- algebras X, where X={0,1,2,3}and U={0 ,1,2}with binary operation * defined by:

*	**0**	1	2	3
0	0	1	2	3
1	1	0	1	3
2	2	2	0	3
3	3	3	3	0

Then U-BG-ideal I={0,1} is a 1-closed U -BG-ideal, Since $1*(0*0)=1*0=1\in I$ and $1*(0*1) =1*1=0\in I$. But it is not 2-closed U-BG-ideal since $2*(0*1) = 2*1=\notin I$.

Proposition 3. Let X be a U-BG-BH-algebra and I be a b-closed ideal of X. Then I is a b-closed U-BG-ideal of X.

Proof: Suppose that I is a b-closed ideal of X. We have I is an ideal of X. By proposition1(i), I is a U-BG-ideal of X. Now, if $x\in I$. Then $b*(0*x)\in I$. Therefore, I is a b-closed U-BG-ideal of X. ■

Proposition 4. Let X be a U-BG-BH-algebra and I be a b-closed U-BG-ideal such that $x*y\notin I$, for all $x \notin I$ and $y \in I$. Then I is a b-closed ideal of X.

Proof: Let I be a b-closed U-BG-ideal of X. Then I is a U-BG-ideal of X. By proposition1(iv), I is an ideal of X. Now, if $x\in I$, then $b*(0*x)\in I$. Therefore, I is a b-closed ideal of X .■

Proposition 5. Let X be a U-BG-BH-algebra. If I is a BH-ideal and X is 0-commutative, then I is b-closed U-BG-ideal of X, for all $b\notin I$.

Proof: Let $b \notin I$, $x \in I$, $b* (0 *x) = x*(0* b)$. Since X is 0-commutative, since I is a BH-ideal, we have $0*b\in I$ and $x*(0*b)\in I$. Hence $b*(0*x)\in I$. Therefore, I is b-closed U-BG-ideal of X .■

Proposition 6. Let U_1 and U_2 be two BG-algebras contained in a U-BG-BH-algebra X such that $U_1 \subseteq U_2$. Then every b-closed U_2-BG-ideal of X is a b-closed U_1-BG-ideal of X.

Proof: Let I be b-closed U_2-BG-ideal of X. Imply that I is a U_2-BG-ideal of X.
By proposition1(iii), I is a U_1-BG-ideal of X. Now, suppose that $x \in I$, $b\in X$, $b * (0 * x) \in I$. Since I is b-closed U_2-BG-ideal of X. Therefore, I is a b-closed U_1-BG-ideal of X.■

Proposition 7. Let X be a U-BG-BH-algebra. If G(X)=X, then every U-BG-ideal of X is a 0-closed U-BG-ideal of X.

Proof : Assume that I is a U-BG-ideal of X and $x\in I$. Hence $0 * (0 * x) = x\in I$, since G(X) =X. Then I is a 0-closed U-BG-ideal of X .■

Proposition 8. Let $f:(X,*,0)\to(Y,*',0')$ be a U-BG-BH-epimorphism. If I is a b-closed U-BG-ideal of X, then f(I) is a f(b)-closed U-BG-ideal of Y.

Proof: Let I is a b-closed U-BG-ideal of X. Then I is a U-BG-ideal of X. By proposition 2(i), f (I) is a U-BG-ideal of Y. Let y∈f (I). Then there exists x∈I such that y = f (x), f(b) $*'$ $(0'$ $*'$ y) =f (b)$*'$(f (0)$*'$f (x)) =f (b*(0*x))∈f (I), since b * (0 * x)∈I. Therefore, f(I) is a f (b)-closed-U-BG-ideal of Y.■

Proposition 9. Let X be a U-BG-BH-algebra. If I is a U-BG-ideal of X such that I subset of med(X). Then I is a 0-closed U-BG-ideal of X.

Proof: Assume that I subset of med (X) and x∈I. Hence 0*(0*x)=x. Imply that 0*(0*x)∈I. Therefore, I is a 0-closed U-BG-ideal of X.■

Proposition 10. Let X be a U-BG- BH-algebra. If I is a U-BG-ideal and $L_b\circ L_0\in L(I)$, then I is a b-closed U-BG-ideal of X.

Proof: Let I be a U-BG-ideal of X and x∈I. Since $L_b\circ L_0\in L(I)$, $L_b\circ L_0(x) = L_b(0*x) = b*(0*x)\in I$. Therefore, I is a b-closed U-BG-ideal.■

Proposition 11. Let X be a U-BG-BH-algebra and I be a U-BG-ideal. If I subset of X_+ , then I is a b-closed U-BG-ideal of X, for all b∈I.
Proof : Assume that b∈I and I subset of X_+, then b*(0*x)=b*0=b∈I, since I⊆ X_+. Therefore, I is a b-closed U-BG-ideal of X.■

Proposition 12. Let X be a U-BG-BH-algebra. If I is a U-BG- ideal of X and $L_0(x) = 0$, for all x∈I, then I is a b-closed U-BG-ideal ,for all b∈I.

Proof: Let I be a U-BG- ideal and x∈I. Since $L_0(x) = 0$, (0*x) = 0. Thus x∈X_+ . Hence I⊆ X_+ . By proposition 11. I is a b-closed U-BG-ideal of X, for all b∈I.■

Definition 20. Let I be a U-BG- ideal of a U-BG-BH-algebra X and b∈X. Then I is called a **completely closed U-BG-ideal with respect to b** (denoted by **b-completely closed U-BG-ideal**) if b*(x*y)∈I, for all x,y∈I.

Example 2. Consider a U-BG-BH-algebra X in example (1). The U-BG-ideal I={0,2} is a 2-completely closed U-BG-ideal of X, since 2 * (0 * 0) =2∈I, 2 * (0 * 2) = 0 ∈ I, 2 * (2 * 0) = 0∈ I, 2*(2*2)=2 ∈I. But it is not 1-completely closed U-BG- ideal, since: 1 * (0 * 2) = 1*2=1∉ I.

Proposition 13. Let X be a U-BG- BH-algebra and I be b-completely closed ideal of X. Then I is a b-completely closed U-BG-ideal of X.

Proof: Let I be b-completely closed ideal of X. Then I is an ideal of X**.** By proposition 1(i), I is a U-BG- ideal of X. Assume that x,y∈I. Imply that b * (x * y) ∈ I. Therefore, I is a b-completely closed U-BG-ideal of X. ■

Proposition 14. Let X be U-BG- BH-algebra and I be a b-completely closed U-BG-ideal such that x*y∉ I for all x∉ I and y∈I. Then I is a b-completely closed ideal of X.

Proof:
Let I be b-completely closed U-BG-ideal of X and x,y∈I. Then I is U-BG- ideal of X. By proposition 1(iv) I is an ideal of X Hence b*(x*y)∈I, Since I is a b-completely closed U-BG-ideal of X. Completes The proof.■

Proposition 15. Let X be a U-BG-BH-algebra. Then every b-completely closed U-BG-ideal is a b-closed U-BG-ideal of X.

Proof: Let I be b-completely closed U-BG-ideal of X. Then I be U-BG- ideal of X, and b*(x*y)∈I, for all x,y∈I, by putting x=0, we obtain b*(0*y)∈I. Then I be b-closed U-BG-ideal of X. ■

Remark 4. The converse of proposition 15 is not correct in general as in the following example.

Example 3: Consider a U-BG-BH- algebra X={0,1,2,3,4} with binary opera tion "*" defined as follows:

*	0	1	2	3	4
0	0	2	1	0	0
1	1	0	2	2	1
2	2	1	0	1	2
3	3	0	1	4	2
4	4	3	1	2	0

where U={0,1,2}. The subset I={0,3,4} is 0-closed U-BG-ideal of X but not a 0-completely closed U-BG-ideal of X, since 0*(3*4)=0*2=1∉I.

Theorem 1. Let X be a U-BG-BH-algebra and let I be a U-BG- ideal of X. Then I is a completely closed U-BG-ideal if and only if I is a b-completely closed U-BG-ideal, for all b∈I.

Proof: Let I be a completely closed U-BG-ideal and x,y∈I. Then x*y∈I, since I is completely closed U-BG-ideal. Now, if b∈I, then b*(x*y)∈I, since b, x*y∈I. Therefore, I is a b-completely closed U-BG- ideal of X.
Conversely**,** Let x,y∈I. Then I is x- completely closed U-BG-ideal. Imply that x*(y*0) = x * y∈ I, since 0∈I and y*0=y, by definition 2(iii). Therefore I is a U-BG-completely closed ideal of X .■

Proposition 16. Every U-BG-ideal in U-BG-BH-algebra X is not a b-completely closed U-BG-ideal of X, for all b∉I.

Proof: Suppose that I be U-BG- ideal of X and b∉I. Now, b*(0*0)=b*0=b∉I
Hence I is a not b-completely closed U-BG-ideal of X .■

Proposition 17. Let X be an associative a U-BG-BH-algebra. Then every p-ideal of X is a b-completely closed U-BG-ideal of X, for all b∈I.

Proof : Let I be a p-ideal of X. By remark 2, we have I is an ideal of X. By proposition 1(i)**,** I is a U-BG-ideal of X. Now, let x,y∈I. Using definition 2(i) and (iii), we obtain x = x*0 =

x * (y* y) = x*((y *0)* (y*0) = x * ((y *0)) * (y*0) =((x *y)* 0) * (y*0), since X is an associative U-BG-BH-algebra. By putting x=x*y, y=0 and z=y in definition 14(ii), we have x*y∈I. So, I is a completely closed U-BG-ideal of X. By theorem 1, I is a b-completely closed U-BG-ideal of X. for all b∈I.■

Proposition 18. Let X be a U-BG-BH-algebra. Then every q-ideal is a b-completely closed U-BG-ideal of X, for all b∈I.

Proof : Let I be a q-ideal of X. By definition 15(i) and (ii), we get 0∈I and x*(y*z)∈I, y∈I. By putting z=0, we get x*0=x∈I. Then I is an ideal of X. Using proposition 1(i), I is a U-BG-ideal of X. Now, let x,y∈I. Using definition 2(i) and (iii), x=x*0=x*(y*y). By putting x=x, y=y, z=y in definition 15(ii), we get x*y∈I. Hence, I is a completely closed U-BG-ideal of X. Using theorem 1, I is a b-completely closed U-BG-ideal of X, for all b∈I.■

Proposition 19. Let X be an associative a U-BG-BH-algebra. Then every strong-ideal is a b-completely closed U-BG-ideal of X, for all b∈I.

Proof : Is similar to proof of a Proposition 17.

Proposition 20. Let X be a U-BG-BH-algebra. If I is a BH-ideal, then I is b-completely closed U-BG-ideal of X, for all b∈I.

Proof: It follows from proposition1 (v) and theorem 1.

Proposition 21. Let X be a U-BG-BH-algebra. If I is a U-BG-ideal and $L_b∘L_x(I)$ subset of I, for all x∈I, then I is a b-completely closed U-BG-ideal of X.

Proof: Let I is a U-BG-ideal and x,y∈I, b*(x*y) = $L_b(x*y)$ = $L_b∘L_x(y)$∈ I, since $L_b∘L_x(I) \subseteq I$. Hence, I is a b-completely closed U-BG-ideal of X.■

Proposition 22. Let{I_i , i∈λ} be a family of b-completely closed U-BG-ideals of a U-BG-BH-algebra X. Then $\bigcap_{i\in\lambda} I_i$ is a b-completely closed U-BG-ideal of X.

Proof: Straightforward.

Remark 5. Let X be a U-BG-BH-algebra and let I,J be a b-completely closed U-BG ideals of X. Then I∪J is not necessary a b-completely closed U-BG ideal of X.

Example 4: Consider the U-BG-BH-algebra X={0,1,2,3,4} with binary operation "*" defined as follows.

*	0	1	2	3	4
0	0	1	2	0	0
1	1	0	1	2	2
2	2	2	0	1	1
3	3	1	2	0	2
4	4	3	1	2	0

where U={0,1,2}. I={0,3} and J={0,4} are two 0-completely closed U-BG-ideals of X, but I∪J ={0,3,4} is not a 0-U-BG-ideal of X. Since 0*(3 *4) = 0 * 2 = 2∉ I∪J.

Proposition 23. Let{I_i ,i∈λ} be a chain of a b-completely closed U-BG-ideals of a U-BG-BH-algebra X. Then $\bigcup_{i\in\lambda} I_i$ is a b-completely closed U-BG-ideal of X.

Proof: Since each I_i is a b-completely closed U-BG-ideal of X, then I_i is a U-BG-ideal of X, by proposition 1(ii), we have $\bigcup_{i\in\lambda} I_i$ is a U-BG-ideal of X, for all i∈λ. Now, let x , y ∈ $\bigcup_{i\in\lambda} I_i$. Then there exist I_i, I_k∈{ $I_i\}_{i\in\lambda}$, such that x ∈ I_i ,y∈I_k Thus, either $I_i \subseteq I_k$ or $I_k \subseteq I_i$, If $I_i \subseteq I_k$, then x, y∈I_k, imply that b * (x * y) ∈ I_k. Similarly, if $I_k \subseteq I_i$. Therefore, b * (x * y) ∈ $\bigcup_{i\in\lambda} I_i$ This completes the proof. ■

Proposition 24. Let f : X → Y be a BH-homomorphism. Then ker(f) is a b-completely closed ideal of X, for all b∈ker(f).

Proof: Let f : X → Y be a BH-homomorphism. By Proposition 2(iii) ker(f) is a U-BG-ideal. Let x,y∈ ker(f). Then f(x * y) = f(x) * f(y) = 0*0 = 0. Then x * y ∈ ker(f) . Therefore, ker(f) is a completely closed U-BG-ideal. By theorem 1, ker(f) is a b-completely closed ideal, for all b∈ ker(f). ■

Proposition 25. Let f:(X,*,0)→(Y,$*'$,$0'$) be a U-BG-BH-epimorphism such that f^{-1}(U) is a BG-algebra of X. If I is a b-completely closed U-BG-ideal of Y, then f^{-1}(I) is a c-completely closed f^{-1}(U)-BG-ideal of X, where b=f (c) and c ∈ X.

Proof : Let I be a b-completely closed U-BG-ideal of Y. Then I is a U-BG-ideal of Y. By proposition 2(ii), we have, f^{-1}(I) is a f^{-1}(U)-BG-ideal of X. Now, let x , y∈f^{-1}(I). Then f(c* (x * y)) = f(c)$*'$(f(x)$*'$ f(y))∈f(I). Hence, c*(x*y)∈ f^{-1} (I). This completes the proof.■

Proposition 26. Let f:(X,*,0)→(Y,$*'$,$0'$) is a U-BG- BH-epimorphism. If I is a b-completely closed U-BG-ideal of X , then f(I)is a f(b)-completely closed U-BG-ideal of Y.

Proof: Straightforward.

Remark 6. The relationships among types of ideals of U-BG-BH-algebra are explained in figure 1.

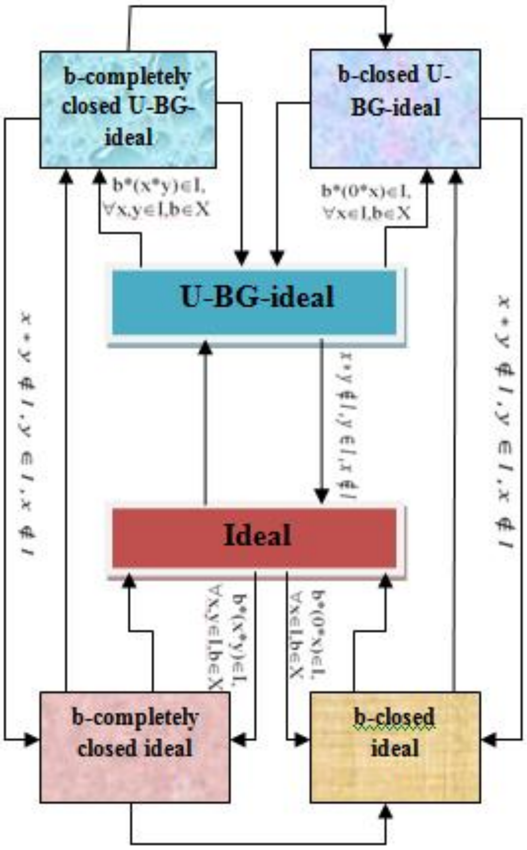

Figure 1: The relationships among types of ideals of U-BG-BH-algebra

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