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Modelling and Analysis of Extended Spiking Neural P systems with Astrocytes using Petri nets

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Abstract

Spiking Neural P (SN P) system characterizes the movement of spikes among neurons. Extended Spiking Neural P systems with Astrocytes (ESNPA systems) incorporates the functioning of astrocytes (star-shaped glial cells spanning around neurons), which play an important role on the functioning and interaction of neurons. In this paper we propose a methodology for translating ESNPA systems to Petri nets and analyze its properties using Petri nets.

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Keywords: Extended Spiking Neural P system with Astrocytes, Petri net with guard.

1 Introduction

Spiking Neural P systems (in short SN P systems) were introduced in 2006 [3], as a class of P systems which characterize the signal transmission of neurons. SN P systems have a pictorial representation of a directed graph whose nodes represents the neurons and arcs represents the synapses. A symbol 'a' denotes the spike (electrical impulse) in the neuron.

An extended variant of SN P systems with astrocytes was considered in [1], where an astrocyte can sense at the same time, the traffic of spikes along several neighboring synapses. Astrocytes have two kinds of roles; one excitatory and the other inhibitory. The use of astrocytes defined adds a new degree of non–determinism to the functioning of the system, by the branching to the non–deterministic choice of the surviving spike.

Petri net is a powerful mathematical tool which incorporates graphical representation of a dynamic behaviour of systems. They were invented by Carl Adam Petri in 1962. A Petri net consists of four elements: places, transitions, directed arcs and tokens. The directed arcs are connected from places to transitions and from transitions to places. Petri nets are widely used as a model of concurrency which allows to represent the occurrence of independent events and parallelism, occurrence of simultaneous events. These have been used to model computer networks, communication systems, logistic networks, work flows, command and control systems. Petri nets have a number of properties which allows them for precise modelling and analysis of system behaviour and visualization of system state changes.

Due to the similarity of the graphical structure, the translation of Spiking Neural P systems into models of Petri nets was first mentioned in [4]. In [4] a variant of SN P system with anti-spikes are studied which contains two types of objects spikes and anti-spikes, corresponding to the inhibitory impulses from neurobiology and describe a methodology to model and simulate SN P systems with anti-spikes using Petri nets was proposed.

In this work, we are motivated with the idea of modelling and analysis of Extended Spiking Neural P systems with Astrocytes using a class of Petri nets, having guards for transitions and coloured tokens.

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2 Prerequisites

For basic results of formal language theory we can refer [6] and the results in membrane computing we refer [2]. Here we recall the definition of Extended Spiking Neural P systems with Astrocytes in [5].

Definition 2.1. *An Extended Spiking Neural P systems with Astrocytes (ESNPA system), of degree* $m \geq 1, l \geq 1$, *is a construct of the form*

$$\Pi = (O, \sigma_1, \ldots, \sigma_m, syn, ast_1, \ldots, ast_l, out), \text{ where}$$

- $O = \{a\}$ *is the singleton alphabet (a is called spike);*
- $\sigma_1, \ldots, \sigma_m$ *are neurons of the form* $\sigma_i = (n_i, R_i), 1 \leq i \leq m$ *where:*
  - *a)* $n_i \geq 0$ *is the initial number of spikes contained in* σ_i;
  - *b)* R_i *is a finite set of extended rules of the form:*

    $$E/a^c \to a^p$$

    where E *is a regular expression over* a, *and* $c \geq 1, p \geq 1$ *with* $c \geq p$.
- $syn \subseteq \{1, 2, \ldots, m\} \times \{1, 2, \ldots, m\}$ *with* $(i, i) \notin syn$ *for* $1 \leq i \leq m$ *(synapses between neurons);*
- $ast_1, \ldots, ast_l$ *are astrocytes, of the form* $ast_i = (syn_{ast_i}, t_i)$, *where* $1 \leq i \leq l$, $syn_{ast_i} \subseteq syn$ *is the set of synapses controlled by the astrocyte* $ast_i, t_i \in \mathbb{N}$ *is the threshold of the astrocyte* ast_i.
- $out \in \{1, 2, \ldots, m\}$ *indicate the out put neuron.*

The rules $E/a^c \to a^p$ *with* $p \geq 1$ *are called (extended) spiking rules, and they are applied as follows. If the neuron* σ_i *contains* k *spikes, and* $a^k \in L(E), k \geq c$, *then rule* $E/a^c \to a^p \in R_i$ *can be applied. This means consuming (removing)* c *spikes (leaving* $k - c$ *spikes in neuron* σ_i*), the neuron is fired, sending* p *spikes out along all outgoing synapses. These spikes then reach the neighboring neurons, unless they are intercepted by one of the astrocytes. If* $L(E) = \{a^c\}$, *then the rule is written in the simplified form* $a^c \to a^p$.

An astrocyte can sense the spike traffic along the neighboring synapses. For an astrocyte ast_i, *suppose that there are* k *spikes passing along the neighboring synapses* syn_{ast_i}. *If* $k > t_i$, *then the astrocyte* ast_i *has an inhibitory influence on the neighboring synapses, and the* k *spikes are suppressed (that is the spikes are removed from the system). If* $k < t_i$, *then the astrocyte* ast_i *has an excitatory influence on the neighboring synapses, all spikes survive and pass to the destination neurons. If* $k = t_i$, *then the astrocyte* ast_i *non-deterministically chooses an inhibitory or excitatory influence on the neighboring synapses.*

There is a possibility that two or more astrocytes control the same synapse. In this case, if all these astrocytes have excitatory influence on the synapses, then the spikes along this synapse can survive and pass to the destination neurons. If one of these astrocytes has inhibitory influence on the synapse, then the spikes along the synapse are suppressed and removed from the system.

A configuration of the system is described by the number of spikes present in each neuron. Then the initial configuration is defined by the number of initial spikes $n_1, \ldots, n_m$. Using the rules, one can define transitions among configurations. Any sequence of transitions starting from the initial configuration is called a computation. A computation halts when it reaches a configuration where no rule can be used.

Petri nets are graphical and mathematical modeling tools for representing of a system in which multiple independent activities in progress at the same time. The extensions of Petri nets with weighted arcs, coloured tokens, time, add features to model probabilistic behaviour. Time can be associated with places, transitions or arcs.In coloured Petri nets, by adding colours to tokens, Petri nets are enhanced with the new feature such as reducing the size of the models when modelling large systems with Petri nets. A transition can fire with respect to each of its colours.In [4], Petri net with guard was introduced, which is an extension of the coloured timed Petri net to accommodate the features of spiking neurons of SN P system. In that Petri net the tokens are of single colour representing spike in an SN P system. The definition in [4] is given below.

Definition 2.2 (Petri net with guard)**.** *A Petri net with guard is represented by*

$$\mathcal{NL} = (P, T, F, W, G, \mathcal{M}_0)$$

where

- $P = \{p_0, p_1, \ldots, p_m\}$ *is a finite, non–empty set of places.*
- $T = \{t_1, t_2, \ldots, t_n\}$ *is a finite, non–empty set of transitions.*
- $F \subseteq (P \times T) \cup (T \times P)$ *is a set of directed arcs which connects places with transitions and transitions with places.*
- $W : F \to N$ *is a weight function.*
- G *is guard function that maps each transition* t_i *to boolean expression, which specifies an additional constraint which must be fulfilled before the transition is enabled.*
- $\mathcal{M}_0$*:* $P \to \mathbb{N}$ *is the initial marking.*

A marking $\mathcal{M}$ of $\mathcal{NL}$ is a function from the set of places of $\mathcal{NL}$ into the set of non-negative integers. The initial marking $\mathcal{M}_0 = \{n_1, n_2, \ldots, n_m\}$, each n_i is the number of tokens initially associated with each place p_i and m is the number of places. Submarking of a Petri net $\mathcal{NL}$ is the marking of some of the places of $\mathcal{NL}$. Motivated from [4], we investigate that the traffic of spikes in ESNPA system can be translated to the passing of tokens in Petri net with guard. We use coloured Petri net with guard and it is used as a computational model for ESNPA system.

2.1 ESNPA system to labelled Petri net

Let $\Pi = (O, \sigma_1, \ldots, \sigma_m, syn, ast_1, \ldots, ast_l, out)$, be an ESNPA system. Then the corresponding labelled Petri net is a construct

$$K = (V, NL_\Pi, \zeta), NL_\Pi = (P, T, F, W, G, M_o)$$

where

1. $V = \{0, 1\}$ is an alphabet.
2. The components of NL_Π are defined as
   a) The set of places P is defined as
   $P = \{p_1, p_2, \ldots, p_m, q_1, \ldots, q_l, p_0\} \cup$
   $\{p_{is}/R_i$ has more than one rule $1 \leq i \leq m\} \cup$
   $\{p'_{qx}, q'_x, q_{xs}/$ the passing of spikes from σ_i is influenced by the ast_x, $1 \leq i \leq m, 1 \leq x \leq l\}$
   b) The set of transitions T is defined as $T = T_1 \cup T_2 \cup \ldots \cup T_m \cup T_{qx}$ where T_i is the set of transitions corresponding to each neuron σ_i, $1 \leq i \leq m$ and $T_{qx} = \{t''_{qx_1}, t''_{qx_2}, t''_{qx_3}, t'_{qxd}, t_{c1}, t_{c2}/$passing of spikes to σ_d is influenced by ast_x for $1 \leq x \leq l\}$.
   c) The set of directed arcs F is defined as $F \subseteq (P \times T) \cup (T \times P)$ whose elements are from places to transitions and transitions to places.

d) The initial marking of each place is defined as

$$\mathcal{M}(p_i) = n_i \text{ where } n_i \geq 0,$$

$$\mathcal{M}(p_{is}) = 1 \text{ for } 1 \leq i \leq m.$$

$$\mathcal{M}(q_x) = k_x \text{ where } k_x > 0,$$

$$\mathcal{M}(q_{xs}) = 1 \text{ for } 1 \leq x \leq l.$$

where all other places are initially kept empty. Also n_i's and k_x's are one coloured tokens.

e) (i) If $ij : E/a^r \to a^s$ is a spiking rule of σ_i and the passing of spikes is not influenced by ast_x, then T_i contains a distinct transition $t_{ij} = t_i$ with following connectivity: $W(p_i, t_i) = r, W(t_i, p_j) = s$, $W(p_{is}, t_i) = W(t_i, p_{is}) = 1$
$G(t_{ij})$: if ($\mathcal{M}(p_i) \in \Psi(L(E))$ and $\mathcal{M}(p'_{qx}) = 0$) then return true else return false.

(ii) If $ij : E/a^r \to a^s$ is a spiking rule of σ_i and passing of spikes to σ_d is influenced by astrocyte of threshold k and $s \neq k$, then add new places p'_{qx}, q'_x and transitions $t_{ij}, t'_{qxd}, t''_{qx_1}$ and a sink transition t_{c_1} to T for $1 \leq i \leq m, 1 \leq x \leq l$ with following connectivity
$W(p_i, t_{ij}) = r$, $W(t_{ij}, p'_{qx}) = (i, s)$, $W(t_{ij}, q_x) = (i, s)$,
$W(p_{is}, t_{ij}) = W(t_{ij}, p_{is}) = 1$
$G(t_{ij})$: if ($\mathcal{M}(p_i) \in \Psi(L(E))$ and $\mathcal{M}(p'_{qx}) = 0$) then return true else return false.
If ($G(t''_{qx_1}) : (\mathcal{M}(p'_{qx}) > 0, s < k$)
$W(q_x, t''_{qx_1}) = (i, s), W(t''_{qx_1}, q'_x) = (i, s)$
If ($G(t'_{qxd}) :$ ($\mathcal{M}(p'_{qx}) > 0, \mathcal{M}(q'_x) > 0$)
$W(p'_{qxd}, t'_{qxd}) = \mathcal{M}(p'_{qx}), W(q'_x, t'_{qxd}) = \mathcal{M}(q'_x)$,
$W(t'_{qxd}, p_d) = s$.
else return false.
else
the place q'_x remains empty. Therefore this inhibits the firing of transition t'_{qxd} and inhibits the passing of tokens. Then tokens are accumulated in p'_{qx} and there are excess tokens in q_x, which should be removed for the next firing of any transition t_{ij}. Then for removing accumulated tokens from p'_{qx} and restoring the actual threshold k of astrocyte place q_x, we add a sink transition t_{c_1} with following connectivity:
$W(p'_{qx}, t_{c_1}) = \mathcal{M}(p'_{qx}), W(q_x, t_{c_1}) = (i, s)$ with guard function
$G(t_{c_1})$: if ($\mathcal{M}(p'_{qx}) > 0, \mathcal{M}(q_x) > k$) then return true else return false.

(iii) If $ij : E/a^r \to a^s$ is a spiking rule of σ_i and passing of spikes to σ_d is influenced by astrocyte of threshold k and $s = k$, then add new places p'_{qx}, q'_x and transitions t_{ij} and t'_{qxd}, t''_{qx_2}, t''_{qx_3} and a sink transition t_{c_2} to T for $1 \leq i \leq m$, $1 \leq x \leq l$ with following connectivity
$W(p_i, t_{ij}) = r, W(t_{ij}, p'_{qx}) = (i, s), W(t_{ij}, q_x) = (i, s)$,
$W(p_{is}, t_{ij}) = W(t_{ij}, p_{is}) = 1$ with guard function
$G(t_{ij})$: if ($\mathcal{M}(p_i) \in \Psi(L(E))$ and $\mathcal{M}(p'_{qx}) = 0$) then return true
else return false.
q_x non–deterministically chooses either of the transitions t''_{qx_2} or t''_{qx_3}. Then add synchronizing place q_{xs} with $\mathcal{M}(q_{xs}) = 1$ and with
a)If ($G(t''_{qx_2}) : (\mathcal{M}(p'_{qx}) > 0, \mathcal{M}(q_x) = 2k$)

$W(q_x, t''_{qx_2}) = (i,s), W(t''_{qx_2}, q'_x) = (i,s),$
$W(q_{xs}, t''_{qx_2}) = W(t''_{qx_2}, q_{xs}) = 1$
If ($G(t'_{qxd})$: ($\mathcal{M}(p'_{qx}) > 0, \mathcal{M}(q'_x) > 0$)
$W(p'_{qx}, t'_{qxd}) = \mathcal{M}(p'_{qx}), W(q'_x, t'_{qxd}) = \mathcal{M}(q'_x),$
$W(t'_{qxd}, p_d) = s.$
else return false
b)If $(G(t''_{qx_3}) : (\mathcal{M}(p'_{qx}) > 0, \mathcal{M}(q_x) = 2k)$
$W(q_x, t''_{qx_3}) = s, W(t''_{qx_3}, p'_{qx}) = (i,s),$
$W(q_{xs}, t''_{qx_3}) = W(t''_{qx_3}, q_{xs}) = 1$
the place q'_x remains empty. Therefore this inhibits the firing of transition t'_{qxd} and inhibits the passing of tokens. Then tokens are accumulated in p'_{qx}, which should be removed for the next firing of any transition t_{ij}. Then for removing accumulated tokens from p'_{qx}, we add a sink transition t_{c_2} with $W(p'_{qx}, t_{c_2}) = \mathcal{M}(p'_{qx})$, with guard function
$G(t_{c_2})$: if $(\mathcal{M}(p'_{qx}) > 0, \mathcal{M}(q'_x) = 0)$ then return true else return false.

3. $\zeta : 2^T/\{\phi\} \to V$ where $\zeta(u) = 1$ if $t \in U$ such that $W(t, p_0) = 1$ otherwise $\zeta(u) = 0$.

To prove the equivalence of ESNPA system and the corresponding Petri net we show that the languages generated by both the systems is same. To capture a very tight correspondence between the ESNPA system Π and the corresponding Petri net NL_Π, we introduce a straight forward bijection between the configurations of Π and the sub markings of NL_Π based on the correspondence between places and neurons.

Let $C =< \alpha_1, \alpha_2, \ldots, \alpha_m >$ be a configuration of ESNPA system Π. The corresponding configuration mapped sub marking $\phi(C)$ of NL_Π is defined as
$\phi(C) =< \beta_1, \beta_2, \ldots, \beta_m >$ where for $1 \le i \le m$.

$$\phi(C)\beta_i = \begin{cases} \mathcal{M}(p_i) & \text{if } \mathcal{M}(p'_{qx}) = 0 \\ \mathcal{M}(p'_{qx}) & \text{otherwise} \end{cases}$$

Similarly for any vector rule $v =< 1j_1, 2j_2, \ldots, mj_m >$ of Π enabled at configuration C, we define an enabled maximal step $\xi(v)$ of transitions of NL_Π such that

$$\xi(v) = \{t_{ij}/v(i) = ij \text{ with } j \ge 1, 1 \le i \le m\}$$

It is clear that ϕ is a bijection from the configurations of Π to the configuration mapped sub markings of NL_Π, and ξ is a bijection from vector rules of Π to enabled maximal steps of NL_Π. As there is a mapping between configuration and markings, $\phi(C)$ is the marking of net NL_Π corresponding to the configuration C of Π. There is a one-to one mapping between the rules in the ESNPA system and transitions in net. So there exists a maximal step $[\xi(v)\rangle$ enabled at the marking $\phi(C)$. The relationship between the dynamics of ESNPA system Π and that of the corresponding Petri net

$$C \overset{v}{\Rightarrow} C' \text{ if and only if } [\xi(v)\rangle_m M_1 [H\rangle_m \phi(C')$$

where M_1 is the intermediate configuration mapped sub marking of the Petri net between $\phi(C)$ and $\phi(C')$ and H is an intermediate step of transitions. In order to implement excitatory or inhibitory influence of astrocyte places q_x's on the transitions of t_{ij}'s, we introduce a maximal step
$H_1 = \{t''_{qx_1}/\mathcal{M}(p'_{qx}) > 0, \mathcal{M}(q_x) > 0, s < k\} \cup \{t'_{qxd}/\mathcal{M}(p'_{qx}) > 0, \mathcal{M}(q'_x) > 0\}$
$H_2 = \{t_{c_1}/s > k, \mathcal{M}(p'_{qx}) > 0, \mathcal{M}(q_x) > k\}$

$H_3 = \{t^{''}_{qx_2}/\mathcal{M}(p^{'}_{qx}) > 0, , \mathcal{M}(q_x) = 2k\} \cup \{t^{'}_{qxd}/\mathcal{M}(p^{'}_{qx}) > 0, \mathcal{M}(q^{'}_x) > 0\}$
$H_4 = \{t^{''}_{qx_3}/\mathcal{M}(p^{'}_{qx}) > 0, \mathcal{M}(q_x) = 2k\} \cup \{t_{c_2}/\mathcal{M}(p^{'}_{qx}) > 0, \mathcal{M}(q^{'}_x) = 0\}$
enabled at M_1.

After the execution of the step $[\xi(v)\rangle$, the Petri net reaches the marking M_1 where $\mathcal{M}(p^{'}_{qx})$ is non-empty and the tokens in $\mathcal{M}(p^{'}_{qx})$ gives the number of spikes transmitted through the synapse of neuron σ_i, which are influenced by inhibitory or excitatory role of astrocytes. After the execution of the step $H_j; j = 1, 2, 3, 4$, the system reaches the configuration $\phi(C')$. So here we map each vector rule of the ESNPA system with atmost two consecutive maximal steps. So the evolution of the Petri net is same as the evolution of the ESNPA system Π. $r = C_0 \overset{v_1}{\Rightarrow} C_1 \overset{v_2}{\Rightarrow} \ldots \overset{v_k}{\Rightarrow} C_k$ is an halting computation of Π if and only if $\Im(r) = \phi(C_0)[\xi(v_1)\rangle_m M_1$ $[H^1_j\rangle_m \phi(C_1)[\xi(v_2)\rangle_m M_2[H^2_j\rangle_m \ldots [\xi(v_k)\rangle_m M_k[H^k_j\rangle_m \phi(C_k)$ is the halting maximal step sequence of NL_Π. So the evolution of the Petri net NL_Π is same as the evolution of the ESNPA system. Hence evolution of ESNPA system is equivalent to that of the corresponding Petri net.

Let $C_{i-1} \overset{v_i}{\Rightarrow} C_i$ is the i^{th} step of γ and if $bin(v_i) = 1$. By the definition of the bin, $bin(v_i) = 1$ if and only if $v_i(i_0)$ is a spiking rule with $rhs(v_i(i_0)) = 1$. From the construction of Petri net and the definition of $\xi(v_i)$ and H^i_j we observe that the step $\xi(v_i)$ contains a transitions t with $W(t, p_0) = 1$ which implies that $\zeta(\xi(v_i)) = 1$ otherwise $\zeta(\xi(v_i)) = 0$. Since H^i_j contains transitions with no outgoing arcs to p_0, by the definition of ζ, $\zeta(H^i_j) = \lambda$. The output generated the Petri net after firing of step $\xi(v_i)$ and H_i is 1. $bin(v_i) = 1$ if and only if $\zeta(\xi(v_i))\zeta(H^i_j) = 1\lambda = 1$ and $bin(v_i) = 0$ if and only if $\zeta(\xi(v_i))\zeta(H^i_j) = 0\lambda = 0$. Therefore $w = bin(\gamma) \in \{0, 1\}^*$ if and only if $w = \zeta(\Im(\gamma))$. Therefore $L(\Pi) = L^m(NL_\Pi)$.

2.2 The Properties of ESNPA system derived from Petri nets

The behavioural properties of Petri nets depends on the initial state or marking of the Petri nets while structural properties do not depend on the initial marking of a Petri net, but depend on the net structure of the Petri nets. Petri nets have behavioral properties such as reachability, boundedness, liveness etc. We can introduce these properties for ESNPA system. It may provide insights to the system.

1. Reachability: Reachability property helps to find out whether the modelled system can reach a specific state as a result of a functional behaviour. In a Petri net, a marking M_n is said to be reachable from a marking M_0 if there exists a sequence of firings that transform M_0 to M_n. This type of analysis in given ESNPA system can be used to determine whether certain outcomes are possible from initial configuration or certain configurations are reachable when specific rules are excited or inhibited.

2. T–Invariants: T–invariants indicate the presence of cycles that are in a state of continuous operation. In ESNPA system, we can identify the sequence of vector rules that have to fire from a configuration and return to that configuration.

3. Boundedness and Safeness: This property helps to identify the existence of overflows in a modelled system. A place p is said to be k–bounded if the number of tokens in p is always less than or equal to k. It is safe if it is 1-bounded. ESNPA system is k–bounded or simply bounded as the number of spikes in each neuron for a reachable configuration does not exceed a finite number k.

4. Terminating: The sequence of transitions between configurations of a given ESNPA system is finite. That is the computation of the ESNPA system always halts.

5. Deadlock–free: As Petri net, each configuration enables next rule in ESNPA system.

6. Liveness: It is dead lock free and there is a sequence of vector rules.

Simulation is one of techniques for the analysis of Petri nets. For complex Petri nets, simulation using the execution algorithm to run net, is another way to check the properties. This allows the temporal performance of a system. Some of the Petri net simulation tools are PnetLab, CPN tools, PIPE etc. Another Petri net analysis known as reachability analysis can be conducted through the construction of a reachability tree. Given a Petri net, from its initial marking, we can obtain as many new markings as the number of enabled transitions. Repeating these procedure over the results constitute a reachability graph of markings.

3 Example

In this section, an example is given to illustrate the simulation of a ESNPA system Π by a Petri net with guard. Consider the ESNPA system

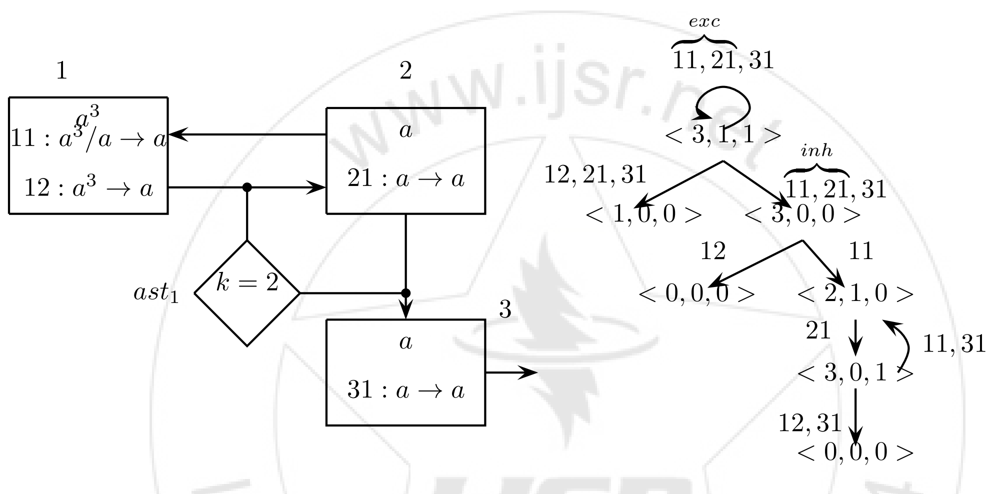

Figure 1: Extended SN P system with astrocyte Π (a) and its evolution (b)

The structure of Π is shown in figure, which consists of three neurons and one astrocyte. Neurons are denoted by rectangles with number of initial spikes and spiking rules inside. Arrows between these rectangles represent synapses and an arrow exits from the output neuron 3, pointing to the environment. The rule $11 : a^3/a \to a$ fires only if σ_1 has three spikes; one is consumed. the other remains available for the next step. The rule $12 : a^3 \to a$ also fires only if σ_1 has three spikes; all are consumed. So in σ_1, there is a non–determinism between its two rules. Each neuron σ_2 and σ_3 has only one rule. An astrocyte is denoted by a rhombic box with 'arms' touching the synapses; each arm indicates that the astrocyte controls the spike traffic of the corresponding touched synapse in excitatory or inhibitory way. The equation $k = 2$ inside the rhombic box denotes the astrocyte has the threshold 2.

The initial configuration of the system is <3,1,1>. It works as follows. All neurons can fire in the first step, with neuron σ_1 choosing non–deterministically between its two rules. The spike from σ_2 reaches σ_1. Output neuron σ_3 sends its spike to the environment. If σ_1 uses its first rule, then the two spikes on the synapses $(1, 2), (2, 3)$, which are influenced by the astrocyte ast_1 of threshold 2. So ast_1 chooses inhibitory or excitatory influence non–deterministically. If ast_1 chooses excitatory role, it allows to pass the spikes from σ_1 and σ_2 and reaches the same initial configuration <3,1,1>. If ast_1 chooses inhibitory role, it inhibits the passing of spikes from σ_1 and σ_2 and reaches the configuration <3,0,0>. If σ_1 chooses the second rule, then 4 spikes on the synapses $(1, 2)(2, 3)$, which are inhibited by the astrocyte ast_1 of threshold 2 and reaches the configuration <1,0,0> at which computation halts.

At the configuration <3,0,0>, σ_1 enables for second step. If σ_1 choose $11 : a^3/a \to a$ rule, the one spike on synapse $(1, 2)$ is excited by ast_1 and reaches the configuration <2,1,0>. If σ_1 choose $12 : a^3 \to a$ rule, the three spikes on synapse $(1, 2)$ is inhibited by ast_1 and reaches halting configuration <0,0,0>. Proceeding like this we get its evolution as in figure 1.

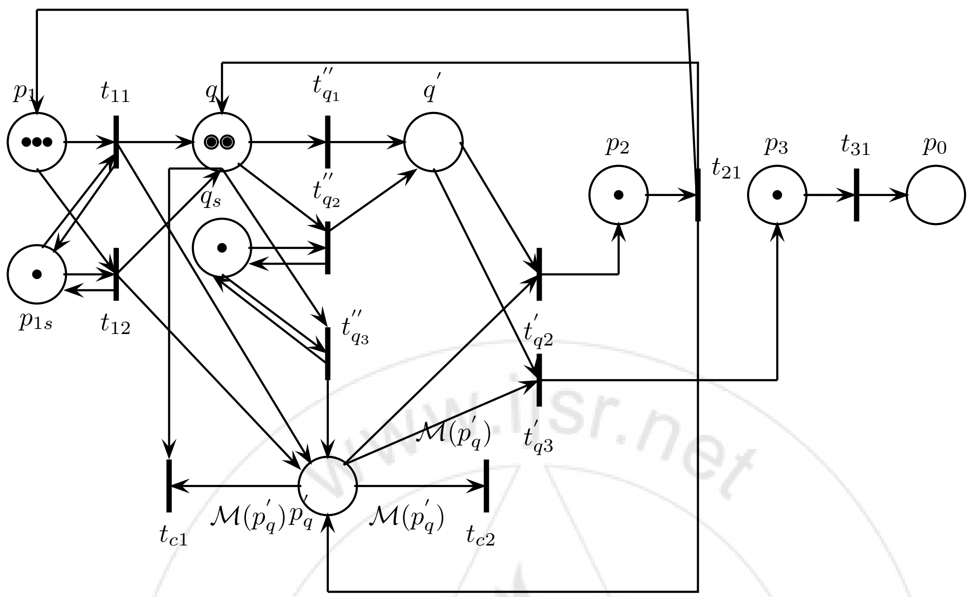

Figure 2: Corresponding Petri net of Π

Consider the number 0 if the output neuron does not send a spike and the number 1 if the output neuron sends a spike to the environment. In this way we can identify that the language generated by the system Π is, $L^3(\Pi) = L(1^+ + 1^+0(01)^*)$.

We translate the given example to Petri net $\mathcal{NL}_\Pi = (P, T, F, W, G, \mathcal{M}_0)$ as in figure 2. p_1, p_2, p_3, q and p_0 are places corresponding to neurons $\sigma_1, \sigma_2, \sigma_3, ast_1$ and environment of Π respectively. Only the place p_1 requires the synchronizing place p_{1s} since σ_1 has two rules. The places are initially marked with 3,1 and 1 tokens respectively which is same as the initial numbers of spikes in their corresponding neurons in Π. The place q is marked with 2 two element tokens which is same as the threshold of ast_1. For each rule ij in Π, a transition t_{ij} is introduced with an incoming arc from place p_i and outgoing arcs to all places p_k such that $(i, k) \in syn$. Here the passing of spikes on synapses (1,2) and (2,3) influenced by the ast_1. So we add new places p'_q, q', q_s and transitions $t''_{q_1}, t''_{q_2}, t''_{q_3}, t'_{q2}, t'_{q3}$and two sink transitions t_{c_1} and t_{c_2}.

In the step 1, the transitions t_{11}, t_{21}, t_{31} fire non–deterministically (corresponding to the rules 11,21,31 of Π). The place p_0 receives a token since spiking translation t_{31} fired in the step. A one element token, from p_2 directly reaches p_1. Each token from p_1 and p_2 are reached q and p'_q as two coloured tokens $(1, 1)$ and $(2, 1)$ respectively. Then the present submarking causes the non-deterministic choice of the firing of the transition t''_{q_2} or t''_{q_3} at the next step. If t''_{q_2} fires, the system reaches the submarking with tokens $(1, 1)$ and $(2, 1)$ in p'_q, and q', which enables the transitions t'_{q2} and t'_{q3}. The firing of the transitions t'_{q2} and t'_{q3} causes the passing of token $(1, 1)$ as a one colour token to p_2 and token $(2, 1)$ as a one colour token to p_3 respectively and the systems reaches the same submarking as initial submarking. Similarly we can observe the remaining steps of the execution as in the figure 3.

We can observe from figure 2 that t_{31} is the only transition having an arc to place p_0, which corresponds to the environment of the system Π. By using the labeling function ζ defined in the previous section, if we label the steps having t_{31} as 1 and other steps as 0, we get the step languages generated by the Petri net as $L^3(\mathcal{NL}_\Pi) = L(1^+ + 1^+0(01)^*) = L^3(\Pi)$.

Extended Spiking Neural P systems with Astrocytes can be simulated using any Petri net tool that supports parallel execution of transitions with guard functions and coloured tokens.

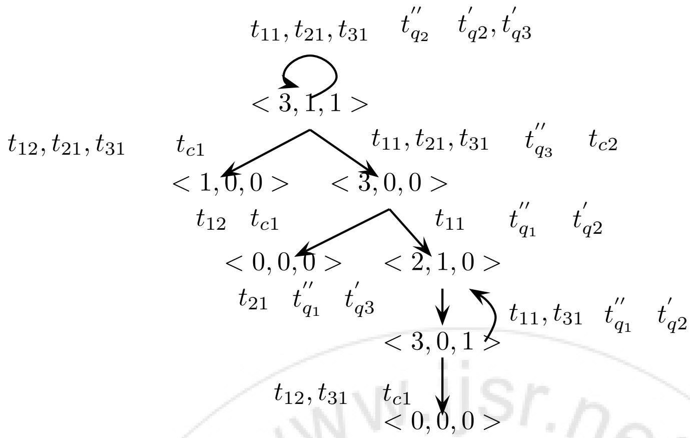

Figure 3: Reachability graph of $\mathcal{NL}_{\Pi}$ with submarkings of first three places

Throughout our study, a tool called PnetLab is used to simulate and check certain properties of these systems, for which synchronizing place is not required.

For the purpose of analysis of ESNPA systems we can also investigate the reachability graph of its Petri net as this is isomorphic to the evolution of ESNPA systems. Since the reachability graph combine step sequences and reachable states, they are useful for the analysis and verification of behavioural properties.

4 Conclusion

This paper provides a systematic procedure to translate Extended Spiking Neural P systems with Astrocytes into Petri nets that can be simulated using parallel execution of transitions and guard functions. This enables us to verify and analyze ESNPA systems, by using properties of Petri nets.

References

[1] A. Binder, R. Freund, M. Oswald, L. Vock, *"Extended Spiking Neural P systems with Excitatory and Inhibitory Astrocytes"*, Proceedings of 5th BWMC, Sevilla, 63-72, 2007.

[2] Gheorghe Păun, *Membrane Computing An Introduction*, Springer–Verlag, Berlin, 2002.

[3] M.Ionescu, Gh. Paun, T.Yokomori: *"Spiking Neural P systems"*, Fundamenta Informaticae, **71**, (2-3) 2006, 279-308.

[4] V. P Metta, Kamala Krithivasan, *"Modelling and Simulation of SN P systems using Petri nets"*. PhD Thesis, Thapar University Patiala, 2012.

[5] L. Pan, Jun Wang, H.J. Hoogeboom, *"Asynchronous Extended Spiking Neural P systems With Astrocytes"*, Proceedings of the 12th international conference on Membrane computing, CMC'11, Springer–Verlag, Berlin, 243-256, 2012.

[6] G. Rozenberg, A. Salomaa (Eds), *Handbook of Formal Languages*, Springer–Verlag, Berlin, 1997.

[7] Tadao Murata, *"Petri Nets: Properties, Analysis and Applications"*, Proceedings of the IEEE, vol. **77**, (4), 541-580, 1989.