$$\times\int_a^b \frac{\xi e^{-\xi^2/4t}\phi_1(\xi, t)}{(\xi^2-z^2)^{1-m}}dz+\int_a^b \frac{\eta(z)(c^2-z^2)^{\nu-\sigma+m}}{(y^2-z^2)}dz$$

$$\times\int_z^b \frac{\xi e^{-\xi^2/4t}\phi_1(\xi, t)}{(\xi^2-z^2)^{1-m}}d\xi+\int_0^c \frac{\eta(z)(c^2-z^2)^{\nu-\sigma+m}}{(y^2-z^2)}dz$$

$$\left.\times\int_c^\infty \frac{\xi e^{-\xi^2/4t}\phi_2(\xi, t)}{(\xi^2-z^2)^{1-m}}d\xi,\right\} \quad c<x<\infty \qquad (2.45)$$

Substituting the values from (2.27) and (2.28) in above equation, we obtain

$$\eta(y)\overline{\phi_2(y)}=F_4(y, t)-\frac{\mathrm{Sin}(1-\nu+\sigma-m)\pi \mathrm{Sin}(1-m)\pi}{\pi^2(y^2-c^2)^{\nu-\sigma+m}}$$

$$\times\left\{\int_0^a \frac{\eta(z)(a^2-z^2)^m(c^2-z^2)^{\nu-\sigma+m}}{(y^2-z^2)(x^2-z^2)}dz\int_a^b \frac{2x\overline{\phi_1(x)}}{(x^2-a^2)^m}dx\right.$$

$$\left.+\int_0^c \frac{\eta(z)(c^2-z^2)^{\nu-\sigma+2m}}{(y^2-z^2)(x^2-z^2)}dz\int_c^\infty \frac{2x\overline{\phi_2(x)}}{(x^2-c^2)^m}dx\right\}$$

$$-\frac{\mathrm{Sin}(1-\nu+\sigma-m)\pi}{\pi(y^2-c^2)^{\nu-\sigma+m}}\int_a^b \frac{\eta(z)(c^2-z^2)^{\nu-\sigma+m}}{(y^2-z^2)}dz$$

$$\left.\times\int_z^b \frac{\xi e^{-\xi^2/4t}\phi_1(\xi, t)}{(\xi^2-z^2)^{1-m}}d\xi\right\}, \quad c<x<\infty \qquad (2.46)$$

Changing the order of integration of equation (2.46), we get

$$\eta(y)\overline{\phi_2(y)}=F_4(y, t)-\frac{\mathrm{Sin}(1-\nu+\sigma-m)\pi \mathrm{Sin}(1-m)\pi}{\pi^2(y^2-c^2)^{\nu-\sigma+m}}$$

$$\times\left\{\int_a^b \frac{2x\overline{\phi_1(x)}}{(x^2-a^2)^m}dx\int_0^a \frac{\eta(z)(a^2-z^2)^m(c^2-z^2)^{\nu-\sigma+m}}{(y^2-z^2)(x^2-z^2)}dz\right.$$

$$\left.+\int_c^\infty \frac{2x\overline{\phi_2(x)}}{(x^2-c^2)^m}dx\int_0^c \frac{\eta(z)(c^2-z^2)^{\nu-\sigma+2m}}{(y^2-z^2)(x^2-z^2)}dz\right\}$$

$$-\frac{\mathrm{Sin}(1-\nu+\sigma-m)\pi}{\pi(y^2-c^2)^{\nu-\sigma+m}}\int_a^b \frac{\xi e^{-\xi^2/4t}\phi_1(\xi, t)}{(\xi^2-z^2)^{1-m}}d\xi$$

$$\times\int_z^b \frac{\eta(z)(c^2-z^2)^{\nu-\sigma+m}}{(y^2-z^2)}dz, \quad c<x<\infty \qquad (2.47)$$

Again using the value from equation (2.27) in equation (2.47), we get

$$\eta(y)\overline{\phi_2(y)}=F_4(y, t)-\frac{\mathrm{Sin}(1-\nu+\sigma-m)\pi \mathrm{Sin}(1-m)\pi}{\pi^2(y^2-c^2)^{\nu-\sigma+m}}$$

$$\times\left\{\int_a^b \frac{2x\overline{\phi_1(x)}}{(x^2-a^2)^m}dx\int_0^a \frac{\eta(z)(a^2-z^2)^m(c^2-z^2)^{\nu-\sigma+m}}{(y^2-z^2)(x^2-z^2)}dz\right.$$

$$+\left\{\int_a^b \frac{2x\overline{\phi_1(x)}}{(x^2-a^2)^m}dx\int_z^b \frac{\eta(z)(a^2-z^2)(c^2-z^2)^{\nu-\sigma+m}}{(y^2-z^2)(x^2-z^2)}dz\right.$$

$$\left.+\int_c^\infty \frac{2x\overline{\phi_2(x)}}{(x^2-c^2)^m}dx\int_0^c \frac{\eta(z)(c^2-z^2)^{\nu-\sigma+2m}}{(y^2-z^2)(x^2-z^2)}dz\right\} \quad c<x<\infty \qquad (2.48)$$

Equation (2.39) can now be written as

$$\eta(y)\overline{\phi_2(y)}+\int_c^\infty P(x, y)\overline{\phi_2(x)}dx=F_4(y, t)-\int_a^b Q(x, y)\overline{\phi_1(x)}dx,$$

$c<x<\infty$ (2.49)

Where

$$P(x, y)=\frac{\mathrm{Sin}(1-\nu+\sigma-m)\pi \mathrm{Sin}(1-m)\pi}{\pi^2(y^2-c^2)^{\nu-\sigma+m}}.\frac{2x}{(x^2-c^2)^m}$$

$$\int_0^c \frac{\eta(z)(c^2-z^2)^{\nu-\sigma+2m}}{(y^2-z^2)(x^2-z^2)}dz \qquad (2.50)$$

And

$$\left\{\int_0^a \frac{\eta(z)(a^2-z^2)^m(c^2-z^2)^{\nu-\sigma+m}}{(y^2-z^2)(x^2-z^2)}dz\right.$$

$$\left.+\int_z^b \frac{\eta(z)(a^2-z^2)^m(c^2-z^2)^{\nu-\sigma+m}}{(y^2-z^2)(x^2-z^2)}dz\right\} \qquad (2.51)$$

Equations (2.31) and (2.39) are Fredholm integral equations of the second kind which determine $\overline{\phi_1(x)}$ and $\overline{\phi_2(x)}$. Values of $\overline{\phi_1(\xi, t)}$ and $\overline{\phi_2(\xi, t)}$ can be determined with the help of equations (2.25) and (2.26) respectively. Finally, the coefficients A_n can be computed from equation(2.3), which satisfy the quadruple series equations involving heat polynomials, of the first kind.

$$\sum_{n=0}^{\infty}\frac{B_n}{\Gamma\left(\mu+\frac{1}{2}+n+p\right)}P_{n+p,\sigma}(x,-t)=\begin{cases}\psi_1(x, t), & 0\le x<a \quad (2.52)\\ \psi_2(x, t), & b<x<a \quad (2.53)\end{cases}$$

where $\psi_1(x, t)$ and $\psi_2(x, t)$ are unknown functions. Using equation (1.1) in equations (2.52), (2.53), (1.1) in equations (2.52), (2.53), (1.6) and (1.8), we obtain

$$B_n=\frac{\Gamma\left(\sigma+\frac{1}{2}\right)\Gamma\left(\mu+\frac{1}{2}+n+p\right)}{2^{4(n+p)}(n+p)!\Gamma\left(\sigma+\frac{1}{2}+n+p\right)}\left[\int_0^a \psi_1(x, t)+\int_a^b g_2(x, t)\right.$$

$$\left.+\int_b^c \psi_2(x, t)+\int_c^\infty g_4(x, t)\right]W_{n+p,\sigma}(x, t)d\Omega(x) \qquad (2.54)$$

Substituting this expression for B_n in equations (1.5) and (1.7), we get

$$\sum_{n=0}^{\infty}\frac{t^{-n}e^n\Gamma\left(\sigma+\frac{1}{2}\right)\Gamma\left(\mu+\frac{1}{2}+n+p\right)P_{n+p,\nu}(x,-t)}{2^{4(n+p)}(n+p)!\Gamma\left(\sigma+\frac{1}{2}+n+p\right)\Gamma\left(\nu+\frac{1}{2}+n+p\right)}$$

$$\left[\int_0^a \psi_1(x,\ t)+\int_a^b g_2(x,\ t)+\int_b^c \psi_2(x,\ t)+\int_c^\infty g_4(x,\ t)\right] W_{n+p,\sigma}(\xi,\ t)d\xi(\xi)$$

$$=\begin{cases} g_1(x,\ t), & 0\le x<a \quad (2.55)\\ g_3(x,\ t), & b<x<c \quad (2.56)\end{cases}$$

Using the results (1.2) in above equations and changing the order of integration and summation, we get

$$\left[\int_0^a \xi^{2\sigma}\psi_1(\xi,\ t)d\xi+\int_a^b \xi^{2\sigma}g_2(\xi,\ t)d\xi+\int_b^c \xi^{2\sigma}\psi_2(\xi,\ t)d\xi+\int_c^\infty \xi^{2\sigma}g_4(\xi,\ t)d\xi\right]$$

$$2^{\frac{1}{2}-\sigma}\sum_{n=0}^{\infty}\frac{\left(\frac{\ell}{t}\right)^n\Gamma\left(\mu+\frac{1}{2}+n+p\right)P_{n+p,\ \nu}(x,-t)}{2^{4(n+p)}(n+p)!\Gamma\left(\sigma+\frac{1}{2}+n+p\right)\Gamma\left(\nu+\frac{1}{2}+n+p\right)}\times W_{n+p,\ \sigma}(\xi,\ t)$$

$$=\begin{cases} g_1(x,\ t), & 0\le x<a \quad (2.57)\\ g_3(x,\ t), & b<x<c \quad (2.58)\end{cases}$$

Using summation result (1.4) in equations (2.57) and (2.58), we have

$$\int_0^a \xi^{2\sigma}\psi_1(\xi,\ t)S(x,\ \xi,\ t)d\xi+\int_a^b \xi^{2\sigma}g_2(\xi,\ t)S(x,\ \xi,\ t)d\xi$$

$$+\int_b^c \xi^{2\sigma}\psi_2(\xi,\ t)S(x,\ \xi,\ t)d\xi+\int_c^\infty \xi^{2\sigma}g_4(\xi,\ t)S(x,\ \xi,\ t)d\xi$$

$$=\begin{cases} g_1(x,\ t), & 0\le x<a \quad (2.59)\\ g_3(x,\ t), & b<x<c \quad (2.60)\end{cases}$$

$$\int_0^a \xi^{2\sigma}\psi_1(\xi,\ t)S(x,\ \xi,\ t)d\xi+\int_b^c \xi^{2\sigma}\psi_2(\xi,\ t)S(x,\ \xi,\ t)d\xi$$

$$=\begin{cases} g_1(x,\ t), & 0\le x<a \quad (2.61)\\ g_3(x,\ t), & b<x<c \quad (2.62)\end{cases}$$

where

$$G_1(x,\ t)=g_1(x,\ t)-\int_a^b \xi^{2\sigma}g_2(\xi,\ t)S(x,\ \xi,\ t)d\xi$$

$$-\int_c^\infty \xi^{2\sigma}g_4(\xi,\ t)S(x,\ \xi,\ t)d\xi \quad (2.63)$$

and

$$G_2(x,\ t)=g_3(x,\ t)-\int_a^b \xi^{2\sigma}g_2(\xi,\ t)S(x,\ \xi,\ t)d\xi$$

$$-\int_c^\infty \xi^{2\sigma}g_4(\xi,\ t)S(x,\ \xi,\ t)d\xi \quad (2.64)$$

Starting from equation (2.61), using notation given by (1.6), we get

$$\int_0^a \xi^{2\sigma}\psi_1(\xi,\ t)\left\{\frac{\xi^{-2\sigma+1}e^{-\xi^2/4t}a^*}{x^{-1+2\nu}\Gamma(m)\Gamma(\nu-\sigma+m)}S_\omega(\xi,\ x,\ y)\right\}d\xi$$

$$+\int_b^c \xi^{2\sigma}\psi_2(\xi,\ t)\left\{\frac{\xi^{1-2\sigma}e^{-\xi^2/4t}a^*}{x^{2\nu-1}\Gamma(m)\Gamma(\nu-\sigma+m)}S_\omega(\xi,\ x,\ y)\right\}d\xi$$

$$=G_1(x,\ t),\qquad 0\le x<a \quad (2.65)$$

$$\frac{a^*}{\Gamma(m)\Gamma(\nu-\sigma+m)}\left[\int_0^x \xi e^{-\xi^2/4t}\psi_1(\xi,\ t)S_\xi(\xi,\ x,\ y)d\xi\right.$$

$$\left.+\int_x^a \xi e^{-\xi^2/4t}\psi_1(\xi,\ t)S_x(\xi,\ x,\ y)d\xi+\int_b^c \xi e^{-\xi^2/4t}\psi_2(\xi,\ t)S_x(\xi,\ x,\ y)d\xi\right]$$

$$=x^{2\nu-1}G_1(x,\ t),\qquad 0\le x<a \quad (2.66)$$

Now putting the value of summation in terms of integral from (1.5) in equation (2.66), we obtain

$$\int_0^x \xi e^{-\xi^2/4t}\psi_1(\xi,\ t)\int_0^\xi \eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}d\xi dy$$

$$+\int_x^a \xi e^{-\xi^2/4t}\psi_1(\xi,\ t)\int_0^x \eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}d\xi dy$$

$$+\int_b^c \xi e^{-\xi^2/4t}\psi_2(\xi,\ t)\int_0^x \eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}d\xi dy$$

$$=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{1-2\nu}}G_1(x,\ t),\ 0\le x<a \quad (2.67)$$

Inverting the order of integration of equation (2.67), we get

$$\int_0^x \frac{\eta(y)}{\left(x^2-y^2\right)^{1-\nu+\sigma-m}}dy\int_y^a \frac{\xi e^{-\xi^2/4t}\psi_1(\xi,\ t)}{\left(x^2-y^2\right)^{1-m}}d\xi$$

$$+\int_0^x \frac{\eta(y)}{\left(x^2-y^2\right)^{1-\nu+\sigma-m}}dy\int_b^c \frac{\xi e^{-\xi^2/4t}\psi_2(\xi,\ t)}{\left(x^2-y^2\right)^{1-m}}d\xi$$

$$=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{1-2\nu}}G_1(x,\ t),\qquad 0\le x<a \quad (2.68)$$

Assuming,

$$\overline{\psi_1(y)}=\int_y^a \frac{\xi e^{-\xi^2/4t}\psi_1(\xi,\ t)}{\left(x^2-y^2\right)^{1-m}}d\xi \quad (2.69)$$

Now equation (2.68) can be written as

$$\int_0^x \frac{\eta(y)}{\left(x^2-y^2\right)^{1-\nu+\sigma-m}}dy\left\{\overline{\psi_1(y)}+\int_b^c \frac{\xi e^{-\xi^2/4t}\psi_2(\xi,\ t)}{\left(x^2-y^2\right)^{1-m}}d\xi\right\}$$

$$=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{1-2\nu}}G_1(x,\ t),\ 0\le x<a \quad (2.70)$$

With the help of equations (1.8), we can solve the above equation as

$$\eta(y)\left\{\overline{\psi_1(y)}+\int_b^c \frac{\xi e^{-\xi^2/4t}\psi_2(\xi,\ t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi\right\}$$

$$=\frac{\mathrm{Sin}(1-\nu+\sigma-m)\pi}{\pi}-\frac{d}{dy}\int_0^y \frac{2xdx}{(y^2-x^2)^{\nu-\sigma+m}}$$

$$=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{1-2\nu}}G_1(x,\ t),\quad 0\le x<a \quad (2.71)$$

Now equation (2.71) takes the form

$$\eta(y)\overline{\psi_1(y)}=G_3(y,\ t)-\eta(y)\int_b^c \frac{\xi e^{-\xi^2/4t}\psi_2(\xi,\ t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi,\ 0\le x<a \quad (2.72)$$

Where

$$G_3(y,\ t)=\frac{\mathrm{Sin}(1-\nu+\sigma-m)\pi}{\pi}\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*}$$

$$\times\frac{d}{dy}\int_0^y \frac{2x^{2\nu}G_1(x,\ t)}{\left(y^2-x^2\right)^{\nu-\sigma+m}}dx \quad (2.73)$$

Solving the integral equation (2.69) as

$$\xi e^{-\xi^2/4t}\psi_1(\xi,\ t)=-\frac{\mathrm{Sin}(1-m)\pi}{\pi}\frac{d}{d\xi}\int_\xi^a \frac{2y\overline{\psi_1(y)}}{(y^2-\xi^2)^m}dy \quad (2.74)$$

With the help of equation (2.74), we obtain

$$\int_0^a \frac{\xi e^{-\xi^2/4t}\psi_1(\xi,\ t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi=\frac{\operatorname{Sin}(1-m)\pi}{\pi\left(-y^2\right)^{-m}}\int_0^a \frac{2x\overline{\psi_1(x)}}{\left(x^2-y^2\right)\left(x^2\right)^m} \quad (2.75)$$

Again, let

$$\overline{\psi_2(y)}=\int_y^c \frac{\xi e^{-\xi^2/4t}\psi_2(\xi,\ t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi \quad (2.76)$$

Solving (2.76) similarly as (2.74) and (2.75) respectively, we get

$$\xi e^{-\xi^2/4t}\psi_2(\xi,\ t)=-\frac{\operatorname{Sin}(1-m)\pi}{\pi}\frac{d}{d\xi}\int_\xi^c \frac{2y\overline{\psi_2(y)}}{\left(y^2-\xi^2\right)^m}dy$$

(2.77)

$$\int_b^c \frac{\xi e^{-\xi^2/4t}\psi_2(\xi,\ t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi=\frac{\operatorname{Sin}(1-m)\pi}{\pi\left(b^2-y^2\right)^{-m}}\int_b^c \frac{2x\overline{\psi_2(x)}dx}{\left(x^2-b^2\right)^m\left(x^2-y^2\right)} \quad (2.78)$$

Now using the equation (2.79) in (2.72), we get

$$\eta(y)\overline{\psi_1(y)}=G_3(y,\ t)-\frac{\operatorname{Sin}(1-m)\pi}{\pi}\frac{\eta(y)}{\left(b^2-y^2\right)^{-m}}$$

$$x\int_b^c \frac{2x\overline{\psi_2(x)}dx}{\left(x^2-b^2\right)^m\left(x^2-y^2\right)} \quad (2.79)$$

Equcation (2.80) reduces to the following form

$$\eta(y)\overline{\psi_1(y)}=G_3(y,\ t)-\int_b^c R(x,\ y)\overline{\psi_2(y)}dx,\quad 0\le x<a \quad (2.80)$$

where

$$R(x,\ y)=\frac{\operatorname{Sin}(1-m)\pi}{\pi}\frac{\eta(y)}{\left(b^2-y^2\right)^{-m}}\frac{2x}{\left(x^2-b^2\right)^m\left(x^2-y^2\right)} \quad (2.81)$$

Again starting from equation (2.62) and using the notation (1.6), we have

$$\int_0^a \xi^{2\sigma}\psi_1(\xi,\ t)\left\{\frac{\xi^{-2\sigma+1}e^{-\xi^2/4t}a^*}{x^{2\nu-1}\Gamma(m)\Gamma(\nu-\sigma+m)}S_\omega(\xi,\ x,\ y)\right\}d\xi$$

$$+\int_b^c \xi^{2\sigma}\psi_2(\xi,\ t)\left\{\frac{\xi^{2\sigma+1}e^{-\xi^2/4t}a^*}{x^{2\nu-1}\Gamma(m)\Gamma(\nu-\sigma+m)}S_\omega(\xi,\ x,\ y)\right\}d\xi$$

$$=G_2(x,\ t),\qquad b<x<c \quad (2.82)$$

$$\frac{a^*}{\Gamma(m)\Gamma(\nu-\sigma+m)}\int_0^a \xi e^{-\xi^2/4t}\psi_1(\xi,\ t)S_\xi(\xi,\ x,\ y)d\xi$$

$$+\int_b^x \xi e^{-\xi^2/4t}\psi_2(\xi,\ t)S_\xi(\xi,\ x,\ y)d\xi$$

$$+\int_x^c \xi e^{-\xi^2/4t}\psi_2(\xi,\ t)S_x(\xi,\ x,\ y)d\xi$$

$$=x^{2\nu-1}G_2(x,\ t),\qquad b<x<c \quad (2.83)$$

Putting the value of summation in terms of integral from (1.5) in above equation, we get

$$\int_0^a \xi e^{-\xi^2/4t}\psi_1(\xi,\ t)d\xi\int_0^\xi \eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}dy$$

$$+\int_b^x \xi e^{-\xi^2/4t}\psi_2(\xi,\ t)d\xi\int_0^\xi \eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}dy$$

$$+\int_x^c \xi e^{-\xi^2/4t}\psi_2(\xi,\ t)d\xi\int_0^x \eta(y)\left(\xi^2-y^2\right)^{m-1}\left(x^2-y^2\right)^{\nu-\sigma+m-1}dy$$

$$=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{1-2\nu}}G_2(x,\ t), \quad (2.84)$$

Inverting the order of integration we get

$$\int_0^a \frac{\eta(y)}{\left(x^2-y^2\right)^{1-\nu+\sigma-m}}dy\int_y^a \frac{\xi e^{-\xi^2/4t}\psi_1(\xi,\ t)}{\left(x^2-y^2\right)^{1-m}}d\xi$$

$$+\int_0^b \frac{\eta(y)}{\left(x^2-y^2\right)^{1-\nu+\sigma-m}}dy\int_b^c \frac{\xi e^{-\xi^2/4t}\psi_2(\xi,\ t)}{\left(x^2-y^2\right)^{1-m}}d\xi$$

$$+\int_b^x \frac{\eta(y)}{\left(x^2-y^2\right)^{1-\nu+\sigma-m}}dy\int_y^c \frac{\xi e^{-\xi^2/4t}\psi_2(\xi,\ t)}{\left(x^2-y^2\right)^{1-m}}d\xi$$

$$=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{1-2\nu}}G_2(x,\ t),\qquad b<x<c \quad (2.85)$$

Using (2.69) and (2.76) the above equation becomes

$$\int_b^x \frac{\eta(y)\overline{\psi_2(y)}dy}{\left(x^2-y^2\right)^{1-\nu+\sigma-m}}=\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*x^{1-2\nu}}G_2(x,\ t)$$

$$-\int_0^a \frac{\eta(y)\overline{\psi_1(y)}}{\left(x^2-y^2\right)^{1-\nu+\sigma-m}}dy-\int_0^b \frac{\eta(y)}{\left(x^2-y^2\right)^{1-\nu+\sigma-m}}dy$$

$$\int_b^c \frac{\xi e^{-\xi^2/4t}\psi_2(\xi,\ t)}{\left(\xi^2-y^2\right)^{1-m}}d\xi\quad b<x<c \quad (2.86)$$

The equation (2.87) can be solved as

$$\eta(y)\overline{\psi_2(y)}=\frac{\operatorname{Sin}(1-\nu+\sigma-m)\pi}{\pi}\frac{d}{dy}\int_b^y \frac{2xdx}{(y^2-x^2)^{\nu-\sigma+m}}$$

$$\times\left\{\frac{\Gamma(m)\Gamma(\nu-\sigma-m)}{a^*x^{1-2\nu}}G_2(x,\ t)-\int_0^a \frac{\eta(z)\overline{\psi_1(z)}dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}}\right.$$

$$\left.-\int_0^b \frac{\eta(z)dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}}\int_b^c \frac{\xi e^{-\xi^2/4t}\psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi\right\},$$

$$b<x<c \quad (2.87)$$

$$\eta(y)\overline{\psi_2(y)}G_4(y,\ t)-\frac{\operatorname{Sin}(1-\nu+\sigma-m)\pi}{\pi}\frac{d}{dy}\int_b^y \frac{2xdx}{(y^2-x^2)^{\nu-\sigma+m}}$$

$$\times\left\{\int_0^a \frac{\eta(z)\overline{\psi_1(z)}}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}}dz+\int_0^b \frac{\eta(z)dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}}\int_b^c \frac{\xi e^{-\xi^2/4t}\psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi\right\},$$

$$b<x<c \quad (2.88)$$

Where

$$G_4(y,\ t)=\frac{\operatorname{Sin}(1-\nu+\sigma-m)\pi}{\pi}\frac{\Gamma(m)\Gamma(\nu-\sigma+m)}{a^*}$$

$$\times\frac{d}{dy}\int_b^y \frac{2x^{2\nu}G_2(x,\ t)}{\left(y^2-x^2\right)^{\nu-\sigma+m}}dx \quad (2.89)$$

$$\eta(y)\overline{\psi_2(y)}=G_4(y,\ t)-\frac{\operatorname{Sin}(1-\nu+\sigma-m)\pi}{\pi}\frac{d}{dy}\int_b^y \frac{2xdx}{(y^2-x^2)^{\nu-\sigma+m}}$$

$$\times\left[\int_0^a \frac{dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}}\left\{G_3(z,\ t)-\eta(z)\int_b^c \frac{\xi e^{-\xi^2/4t}\psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi\right\}\right.$$

$$+\int_0^b \frac{\eta(z)dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}}\int_b^c \frac{\xi e^{-\xi^2/4t}\psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi\Bigg],\quad b<x<c \qquad (2.90)$$

Breaking the last term of the above equation into two parts, we get

$$\eta(y)\overline{\psi_2(y)}=G_4(y,\ t)-\frac{\text{Sin}(1-\nu+\sigma-m)\pi}{\pi}\frac{d}{dy}\int_b^y\frac{2xdx}{(y^2-x^2)^{\nu-\sigma+m}}$$

$$\times\left[\int_0^a\frac{G_3(z,\ t)dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}}+\int_a^b\frac{\eta(z)dz}{\left(x^2-z^2\right)^{1-\nu+\sigma-m}}\times\int_b^c\frac{\xi e^{-\xi^2/4t}\psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi\right],$$

$b<x<c$ (2.91)

Changing the order of integration, the equation (2.92) becomes

$$\eta(y)\overline{\psi_2(y)}=G_4(y,\ t)-\frac{\text{Sin}(1-\nu+\sigma-m)\pi}{\pi}\Bigg\{\int_0^a G_3(z,\ t)dz$$

$$\times\frac{d}{dy}\int_b^y\frac{2xdx}{(y^2-x^2)^{\nu-\sigma+m}(x^2-z^2)^{1-\nu+\sigma-m}}+\int_a^b\eta(z)dz$$

$$\times\frac{d}{dy}\int_b^y\frac{2xdx}{(y^2-x^2)^{\nu-\sigma+m}(x^2-z^2)^{1-\nu+\sigma-m}}\times\int_b^c\frac{\xi e^{-\xi^2/4t}\psi_2(\xi,\ t)}{\left(\xi^2-z^2\right)^{1-m}}d\xi\Bigg\}$$

$b<x<c$ (2.92)

We know that

$$\frac{d}{dy}\int_b^y\frac{2xdx}{(y^2-x^2)^{\nu-\sigma+m}(x^2-z^2)^{1-\nu+\sigma-m}}=\frac{(b^2-z^2)^{\nu-\sigma+m}}{(y^2-b^2)^{\nu-\sigma+m}(y^2-z^2)} \qquad (2.93)$$

Using the result (2.79) and (2.94) to the equation (2.93), we get

$$\eta(y)\overline{\psi_2(y)}=G_4(y,\ t)-\frac{\text{Sin}(1-\nu+\sigma-m)\pi}{\pi}\Bigg\{\int_0^a G_3(z,\ t)dz$$

$$\times\frac{(b^2-z^2)^{\nu-\sigma+m}}{(y^2-b^2)^{\nu-\sigma+m}(y^2-z^2)}+\int_a^b\eta(z)dz\times\frac{(b^2-z^2)^{\nu-\sigma+m}}{(y^2-b^2)^{\nu-\sigma+m}(y^2-z^2)}$$

$$\frac{\text{Sin}(1-m)\pi}{\pi(b^2-z^2)^{-m}}\times\int_b^c\frac{2x\overline{\psi_2(x)}}{(x^2-b^2)^m(x^2-z^2)}d\Bigg\}x\ b<x<c \qquad (2.94)$$

$$\eta(y)\overline{\psi_2(y)}=G_4(y,\ t)-\frac{\text{Sin}(1-\nu+\sigma-m)\pi}{\pi\left(y^2-b^2\right)^{\nu-\sigma+m}}\int_0^a\frac{G_3(z,\ t)(b^2-z^2)^{\nu-\sigma+m}}{(y^2-z^2)}dz$$

$$-\frac{\text{Sin}(1-\nu+\sigma-m)\pi\text{Sin}(1-m)\pi}{\pi^2\left(y^2-b^2\right)^{\nu-\sigma+m}}+\int_a^b\frac{\eta(z)(b^2-z^2)^{\nu-\sigma+2m}}{(x^2-z^2)(y^2-z^2)}dz$$

$$\times\int_b^c\frac{2x\overline{\psi_2(x)}}{(x^2-b^2)^m}d\ ,x\quad b<x<c \qquad (2.95)$$

Now changing the order of integration of the last term of the equation (2.96), we get

$$\eta(y)\overline{\psi_2(y)}=G_4(y,\ t)-\frac{\text{Sin}(1-\nu+\sigma-m)\pi}{\pi\left(y^2-b^2\right)^{\nu-\sigma+m}}\int_0^a\frac{G_3(z,\ t)(b^2-z^2)^{\nu-\sigma+m}}{(y^2-z^2)}dz$$

$$-\frac{\text{Sin}(1-\nu+\sigma-m)\pi\text{Sin}(1-m)\pi}{\pi^2\left(y^2-b^2\right)^{\nu-\sigma+m}}\times\int_b^c\frac{2x\overline{\psi_2(x)}}{(x^2-b^2)^m}dx$$

(2.96) Now equation (2.97) can be rewritten as

$$\eta(y)\overline{\psi_2(y)}+\int_b^c S(x,\ y)\overline{\psi_2(x)}dx=G_4(y,\ t)-\frac{\text{Sin}(1-\nu+\sigma-m)}{\pi\left(y^2-b^2\right)^{\nu-\sigma+m}}$$

$$\int_0^a\frac{G_3(z,\ t)(b^2-z^2)^{\nu-\sigma+m}}{(y^2-z^2)}d\ ,z\,b<x<c \qquad (2.97)$$

Where S(x, y) is the symmetric kernel

$$S(x,\ y)=\frac{\text{Sin}(1-\nu+\sigma-m)\pi\text{Sin}(1-m)\pi}{\pi^2\left(y^2-b^2\right)^{\nu-\sigma+m}}.\frac{2x}{(x^2-b^2)^m}$$

$$\times\int_a^b\frac{\eta(z)(b^2-z^2)^{\nu-\sigma+2m}}{(x^2-z^2)(y^2-z^2)}dz, \qquad (2.98)$$

Equations (2.98) and (2.81) are Fredholm integral equations of the second kind determine $\overline{\psi_2(y)}$ and $\overline{\psi_1(y)}$. $\Psi_1(\xi,\ t)$ and $\Psi_2(\xi,\ t)$ can be then computed from equations (2.74) and (2.78) respectively. Finally, the coefficients B_n can be calculated with the help of equation (2.54) which satisfy the equations from (1.5) to (1.8).

Particular Case

If we let $c\to\infty$ in equation (1.1) to (1.8), they reduce to the corresponding triple series equation involving heat polynomials and this solution can be shown to agree with that obtained earlier for triple series equations. Similarly, we can obtain the corresponding dual series equations involving heat polynomials.

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