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Study of Two Stream Instability in Hot Collisional Plasmas over Solar Atmosphere

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Abstract: ***The two stream instability arises when there is a relative velocity between the electrons and protons. Such situations are common in stellar atmospheres, often at the reconnection sites. We have derived a general dispersion relation for the two stream instability in hot collisional plasmas. The dispersion relation has been studied for photosphere plasma parameters. The growth rate is analyzed for various electron thermal velocities and found that it is maximum for finite range of frequencies and then become stable after a cut off wave vector. The maximum growth rate region shift toward the low frequency for higher thermal velocity of electrons. Also the angular frequency of maximum growth rate increases with increase in thermal velocity. The real plot shows that the wave propagation increases with increasing thermal velocity and become insensitive to low frequency region. As this a general dispersion relation and this could be for studying various plasmas such as magnetosphere and industrial purposes.***

Keywords: Streaming Instability, Hot Collisional Plasma, Solar atmosphere, thermal velocity, electrons, protons, magneto hydro dynamic theory

1. Introduction

Streaming instabilities arise when there is relative velocity between ions and electrons in a plasma. The simplest type of streaming instability is the two stream instability. This arises in an electron – proton plasma with electrons in relative motion with ions. This type of streaming instability can arise in stellar atmospheres since stellar plasmas are predominantly electron- proton type. A specific example for this is electron- precipitation related phenomenon in solar chromosphere [1]. The streaming instability can be ignited when a high energy beam of electrons created during the process of reconnection taking place at the site of a solar are, comes down toward chromosphere. But since the density of chromosphere plasma is high, these energetic electrons suffer collisions and transfer their energy to electrons and protons and to the small population of heavier ions also. These secondary electrons and ions are accelerated with different speeds [2],[3]. This creates plasma in which electrons and protons are at different velocity and always electrons drive fast to ions. The theory of two stream instability for hot plasmas is applicable here. Chromosphere foot point heating by energetic streams from magnetic reconnection is discussed in [4] and [5]. Similar streams can happen in earth magneto sphere, stellar atmospheres, helmet streamers, solar fares, cometary atmospheres etc. The electromagnetic wave propagation and instabilities in counter streaming astrophysical situations is discussed in [6, 7, 8]. The dispersion relation is a fourth power equation in the angular frequency [9], Nicholson [10], and Treumann & Baumjohann [11]. This can become imaginary depending upon the values of ω/ω_p and kv_0/ω_p where ω_p is the plasma frequency and v_0 the drift velocity of electrons relative to ions. For large value of kv_0/ω_p, no roots are imaginary and the system is stable. For sufficiently small values of kv_0/ω_p, it can be possible that two of the roots are imaginary and the one corresponding wave growth. The growth rate for sufficiently small kv_0 being proportional to $(m/M)^{1/3}$, where m is the mass of electron and M is the mass of proton. For studying more realistic astrophysical situations, we have to extend the cold plasma theory to the hot plasma case. We have derived a dispersion relation for two stream instability of hot collisional electron-proton plasma. As this dispersion relation is dimensionless in wave vector and angular frequency the result can be applied to any hot electron- proton plasma. It is found that the possibility for instability occurs even at sufficiently high kv_0/ω_p, compared to the cold plasma case. The growth rate is found to be sensitive to electron thermal velocities.

2. Theory

For the theoretical study of streaming instability in a hot plasma, we consider an electron- proton plasma, with ions assumed stationary and electrons moving with a velocity v_0 relative to ions. This is same as we assume the observer moving with a stream of ions. We consider hot plasma $KT = 0$. For simple analysis we consider the case of zero ambient magnetic field ($\mathbf{B}_0 = 0$). It can easily shown that the same results can be applied for electrostatic waves along magnetic field. The linearized equation of motion for protons and electrons are respectively:

$$Mn_0\frac{\partial \mathbf{v}_{i1}}{\partial t} = en_0\mathbf{E}_1 - \gamma_i KT_i \nabla n_{i1} - M n_0 \nu_{ie}[\mathbf{v}_{i1} - (\mathbf{v}_0 + \mathbf{v}_{e1})] \quad (1)$$

$$mn_0\frac{\partial \mathbf{v}_{e1}}{\partial t} = en_0\mathbf{E}_1 - \gamma_e KT_e \nabla n_{e1} - mn_0 \nu_{ei}[(\mathbf{v}_0 + \mathbf{v}_{e1}) - \mathbf{v}_{i1}] \quad (2)$$

where the last terms in the equations represents the collisional terms ν_{ie} and ν_{ei}, the proton and electron collisional frequencies respectively. We consider electrostatic waves of the form,

$$E_1 = E\exp[i(kx - \omega t)]\,\hat{x} \quad (3)$$

Where $\hat{x}$ is in the direction of v_0 and k.
Under these conditions Eq.1 give

$$-i\omega M n_0 v_{i1}\hat{x} = e n_0 E\hat{x} - \gamma_i K T_i i k n_{i1}\hat{x} - M n_0 \nu_{ie}[\mathbf{v}_{i1} - (\mathbf{v}_0 + \mathbf{v}_{e1})]\hat{x}$$

i.e.,

$$\mathbf{v}_{i1} = \frac{ie}{M\omega}E\hat{x} + \frac{\gamma_i K T_i}{M\omega}k\left(\frac{n_{i1}}{n_0}\right)\hat{x} - \frac{i\nu_{ie}}{\omega}[\mathbf{v}_{i1} - (\mathbf{v}_0 + \mathbf{v}_{e1})]\hat{x} \tag{4}$$

Similarly Eq.2 gives

$$\mathbf{v}_{e1} = \frac{-ie}{m}\frac{E}{(\omega - k v_0)}\hat{x} + \frac{\gamma_e K T_e}{m(\omega - k v_0)}k\left(\frac{n_{e1}}{n_0}\right)\hat{x} - \frac{i\nu_{ei}}{\omega - k v_0}[(\mathbf{v}_0 + \mathbf{v}_{e1}) - \mathbf{v}_{i1}]\hat{x} \tag{5}$$

The same results hold good for parallel electrostatic plasma oscillations (i.e., wave propagation parallel to $\mathbf{B}_0$). These can be simplified into a form

$$v_{i1} = A + B v_{e1} \tag{6}$$

And

$$v_{e1} = C + D v_{i1} \tag{7}$$

where,

$$A = \frac{1}{1 + \frac{i\nu_{ie}}{\omega}}\left[\frac{ieE}{M\omega} + \frac{\gamma_i K T_i}{M\omega}k\frac{n_{i1}}{n_0}\right] + \frac{i\nu_{ie}v_0}{\omega\left(1 + \frac{i\nu_{ie}}{\omega}\right)}$$

$$B = \frac{i\nu_{ie}}{\omega\left(1 + \frac{i\nu_{ie}}{\omega}\right)}$$

$$C = \frac{1}{1 + \frac{i\nu_{ei}}{\omega - k v_0}}\left[\frac{-ieE}{m(\omega - k v_0)} + \frac{\gamma_e K T_e}{m(\omega - k v_0)}k\frac{n_{e1}}{n_0}\right] + \frac{i\nu_{ei}v_0}{(\omega - k v_0)\left(1 + \frac{i\nu_{ei}}{\omega - k v_0}\right)}$$

$$D = \frac{i\nu_{ei}}{(\omega - k v_0)\left[1 + \frac{i\nu_{ei}}{\omega - k v_0}\right]}$$

Thus we obtain,

$$v_{i1} = \frac{A + BC}{1 - BD} \tag{8}$$

and

$$v_{e1} = \frac{C + AD}{1 - BD} \tag{9}$$

The ion continuity equation for our case is

$$\frac{\partial n_{i1}}{\partial t} + n_0(\nabla.\mathbf{v}_{i1}) = 0 \tag{10}$$

Linearizing this and noting that $\nabla.\mathbf{v}_0$ and ∇n_0 vanishes, and using the value of v_{i1}, we get for protons

$$n_{i1} = \frac{k}{\omega}n_0 v_{i1} \tag{11}$$

Simplifying this with substitutions $\psi_i = \left(1 + \frac{i\nu_{ie}}{\omega}\right)$, $\phi_i = \frac{i\nu_{ie}}{\omega}$, $\psi_e = \left(1 + \frac{i\nu_{ei}}{\omega - k v_0}\right)$, $\phi_e = \frac{i\nu_{ei}}{\omega - k v_0}$ we get

$$A_1 n_{i1} = A_2(i e n_0)E + A_3 n_{e1} + A_4 \tag{12}$$

where

$$A_1 = \left[1 - \frac{k^2}{\omega^2}\frac{1}{\psi_i}\left(\frac{\psi_i\psi_e}{\psi_i\psi_e - \phi_i\phi_e}\right)\frac{\gamma_i K T_i}{M}\right]$$

$$A_2 = \frac{k}{\omega}\frac{1}{\psi_i}\left(\frac{\psi_i\psi_e}{\psi_i\psi_e - \phi_i\phi_e}\right)\left[\frac{1}{M\omega} - \frac{\phi_i}{\psi_e m(\omega - k v_0)}\right]$$

$$A_3 = \frac{k^2}{\omega(\omega - k v_0)}\frac{1}{\psi_i}\left(\frac{\psi_i\psi_e}{\psi_i\psi_e - \phi_i\phi_e}\right)\frac{\phi_i}{\psi_e}\frac{\gamma_e K T_e}{m}$$

$$A_4 = \frac{k}{\omega}\left(\frac{\psi_i\psi_e}{\psi_i\psi_e - \phi_i\phi_e}\right)\left(\frac{\phi_i}{\psi_i} - \frac{\phi_e}{\psi_e}\right)n_0 v_0$$

Similarly the electron continuity equation is,

$$\frac{\partial n_{e1}}{\partial t} + n_0(\nabla.\mathbf{v}_{e1}) + (\mathbf{v}_o.\nabla)n_{e1} = 0 \tag{13}$$

Linearizing this we get

$$n_{e1} = \frac{k n_0}{\omega - k v_0}v_{e1} \tag{14}$$

Simplifying this we get,

$$B_1 n_{e1} = B_2(i e n_0)E + B_3 n_{i1} + B_4 \tag{15}$$

where

$$B_1 = \left[1 - \frac{k^2}{(\omega - k v_0)^2}\frac{1}{\psi_e}\left(\frac{\psi_i\psi_e}{\psi_i\psi_e - \phi_i\phi_e}\right)\frac{\gamma_e K T_e}{m}\right]$$

$$B_2 = \frac{k}{\omega - k v_0}\frac{1}{\psi_e}\left(\frac{\psi_i\psi_e}{\psi_i\psi_e - \phi_i\phi_e}\right)\left[\frac{-1}{m(\omega - k v_0)} + \frac{\phi_e}{\psi_i M\omega}\right]$$

$$B_3 = \frac{k^2}{\omega(\omega - k v_0)}\frac{1}{\psi_e}\left(\frac{\psi_i\psi_e}{\psi_i\psi_e - \phi_i\phi_e}\right)\frac{\phi_e}{\psi_i}\frac{\gamma_i K T_i}{M}$$

$$B_4 = \frac{k}{\omega - k v_0}\left(\frac{\psi_i \psi_e}{\psi_i \psi_e - \phi_i \phi_e}\right)\left(\frac{\phi_i}{\psi_i} - \frac{\phi_e}{\psi_e}\right) n_0 v_0$$

Therefore the proton and electron perturbed number densities become

$$n_{i1} = P_1(ien_0)E + P_2 n_{e1} + P_3 \quad (16)$$

and

$$n_{e1} = Q_1(ien_0)E + Q_2 n_{i1} + Q_3 \quad (17)$$

where $P_1 = \frac{A_2}{A_1}$, $P_2 = \frac{A_3}{A_1}$, $P_3 = \frac{A_4}{A_1}$, $Q_1 = \frac{B_2}{B_1}$, $Q_2 = \frac{B_3}{B_1}$, $Q_3 = \frac{B_4}{B_1}$ respectively.

From the above equations we obtain,

$$n_{i1} = \frac{P_1 + P_2 Q_1}{1 - P_2 Q_2} ien_0 E + \frac{P_2 Q_3 + P_3}{1 - P_2 Q_2} \quad (18)$$

and

$$n_{e1} = \frac{Q_1 + P_1 Q_2}{1 - P_2 Q_2} ien_0 E + \frac{Q_2 P_3 + Q_3}{1 - P_2 Q_2} \quad (19)$$

The plasma waves leading to two stream instability are high frequency plasma oscillations. To deal with these types of waves it is well known that we should use Poisson's equations

$$\varepsilon_0 \nabla.\mathbf{E} = \rho \quad (20)$$

i.e.,

$$\varepsilon_0 \nabla.\mathbf{E}_1 = e(n_{i1} - n_{e1}) \quad (21)$$

Since we assume that the electric field in plasma is due to the wave.

Substituting the value of n_{i1} and n_{e1} in Eq.21 and simplifying we obtain the following dispersion relation

$$ik\varepsilon_0 E = \left[\frac{P_1(1-Q_2) - Q_1(1-P_2)}{1 - P_2 Q_2}\right] i e^2 n_0 E + e\left(\frac{P_2 Q_3 - Q_2 P_3 + P_3 - Q_3}{1 - P_2 Q_2}\right) \quad (22)$$

This reduces to the dispersion relation of the two stream instability if $\left(\frac{P_2 Q_3 - Q_2 P_3 + P_3 - Q_3}{1 - P_2 Q_2}\right) = 0$.

Thus we obtain the condition of two stream instability as,

$$1 = \frac{m}{k}\left[\frac{P_1(1-Q_2) - Q_1(1-P_2)}{1 - P_2 Q_2}\right]\omega_p^2 \quad (23)$$

where ω_p being the electron plasma frequency. The condition $\left(\frac{P_2 Q_3 - Q_2 P_3 + P_3 - Q_3}{1 - P_2 Q_2}\right) = 0$ leads to the equation

$$\left(\frac{\phi_i}{\psi_i} - \frac{\phi_e}{\psi_e}\right) H = 0 \quad (24)$$

where

$$H = \frac{\psi_i \psi_e}{\psi_i \psi_e - \phi_i \phi_e}\frac{1}{\psi_i \psi_e}\left[\frac{k^3}{(\omega - k v_0)^2 \omega}\phi_i \frac{\gamma_e K T_e}{m} - \frac{k^3}{\omega^2(\omega - k v_0)}\phi_e \frac{\gamma_i K T_i}{M}\right]$$
$$+ \frac{k}{\omega}\left[1 - \frac{k^2}{(\omega - k v_0)^2}\frac{1}{\psi_e}\left(\frac{\psi_i \psi_e}{\psi_i \psi_e - \phi_i \phi_e}\right)\frac{\gamma_e K T_e}{m}\right]$$
$$- \frac{k}{\omega - k v_0}\left[1 - \frac{k^2}{\omega^2}\frac{1}{\psi_i}\left(\frac{\psi_i \psi_e}{\psi_i \psi_e - \phi_i \phi_e}\right)\frac{\gamma_i K T_i}{M}\right]$$

Two cases can arise: Either $\left(\frac{\phi_i}{\psi_i} - \frac{\phi_e}{\psi_e}\right) = 0$ or $H = 0$.

The first case leads to three modes:

$$\omega_1 = i\,\nu_{ie} \quad (25)$$

$$\omega_2 = \frac{\nu_{ie}}{\nu_{ie} - \nu_{ei}} k v_0 \quad (26)$$

$$\omega_3 = i\,\nu_{ei} + k v_0 \quad (27)$$

Case II (H=0) lead to a dispersion relation

$$v_0 \omega^2 + \left(\frac{\gamma_i K T_i}{M} - \frac{\gamma_e K T_e}{m} - v_0^2\right) k\omega + k^2 \frac{\gamma_i K T_i}{M} v_0 + i\left[v_0(\nu_{ie} + \nu_{ei})\omega - \nu_{ie} v_0^2 k\right] = 0 \quad (28)$$

which results in a growth / damping rate given by

$$\gamma = \frac{-\operatorname{Im} D}{\frac{\partial \operatorname{Re}(D)}{\partial \omega_r}} = \frac{-[v_0(\nu_{ie} + \nu_{ei})\omega - \nu_{ie} v_0^2 k]}{2 v_0 \omega + \left(\frac{\gamma_e K T_e}{m} - \frac{\gamma_i K T_i}{M} - v_0^2\right) k} \quad (29)$$

3. Results and Discussions

It would be convenient to make the following normalizations: $z = \frac{\nu_{ie}}{\omega_p}$, $s = \frac{\nu_{ei}}{\omega_p}$, $x = \frac{\omega}{\omega_p}$, $y = \frac{k v_0}{\omega_p}$, $a = \frac{\sqrt{\gamma_i K T_i / M}}{v_0}$, $b = \frac{\sqrt{\gamma_e K T_e / m}}{v_0}$. Then (Case I) Eqs.25, 26, 27 become respectively,

$$\frac{\omega_1}{\omega_p} = i z \quad (30)$$

$$\frac{\omega_2}{\omega_p} = \frac{z y}{z - s} \quad (31)$$

$$\frac{\omega_3}{\omega_p} = y + i\,s \qquad (32)$$

Eq.27 and 28 (Case II) become:

$$y x^2 + y^2 (b^2 - a^2 - 1) x + y^2 a^2 = 0 \qquad (33)$$

With growth/damping rate

$$\frac{\gamma}{\omega_p} = \frac{-[y(z+s) - z\,y^2]}{2yx + y^2(b^2 - a^2 - 1)} \qquad (34)$$

We can obtain the dispersion relation for the two stream instability from Eq.23 by the above procedure as:

$$x^4 - 2\,y x^3 + \left(y^2(1 - a^2 - b^2) - (1 + m/M)\right)x^2 + 2y\left(y^2 a^2 + m/M\right)x$$
$$+ y^2 a^2 \left(1 + y^2 b^2 - y^2\right) + (m/M)\left(y^2 b^2 - y^2\right) = 0 \qquad (35)$$

with growth/damping rate

$$\frac{\gamma}{\omega_p} = \frac{-(s+z)x^3 - y(s+z)x^2 + y^2(z - a^2 s - b^2 z)x + [2y(a^2 s + z) + (m/M) y\,s]}{4x^2 - 6y\,x^2 + 2[y^2(1 - b^2 - a^2) - (1 + (m/M))]x + 2y(y^2 a^2 + (m/M))} \qquad (36)$$

Waves corresponding to Eqs.30, 31, 32, 33 and 34 can trigger two stream instability of the form given by Eq.36. Eq.30 contains purely imaginary term only and hence cannot represent propagating wave. Since for most cases $s > z$, which result in negative frequencies for Eq.31 can also be discarded. But Eq.32 represents a wave with growth equal to electron collision frequency which would be important especially in high density regions. It is seen in conditions, corresponding to solar atmosphere, especially photosphere, Eq.33 can show only damping.

The Coulomb collisional frequency for electrons is given by

$$\nu_{ei}^{-1} = 2.8 \times 10^5 \frac{T_e^{3/2}}{n_e \ln \Lambda} \qquad (37)$$

and for protons,

$$\nu_{ie}^{-1} = 1.7 \times 10^7 \frac{T_i^{3/2}}{n_e \ln \Lambda} \qquad (38)$$

$\ln \Lambda$ is the Coulomb Logarithm [12] which has a value ≈ 20 in solar atmosphere. Coulomb collisions are very rare in upper solar corona, but as we go inward towards photosphere, the density increases and due to this the collisional frequencies increase. Typical photosphere temperature $6 \times 10^3\,K$, gives a value of normalized electron collision frequency (s) as0.86 and normalized proton collision frequency (z) as 0.014. The wave propagation and growth are analyzed for these values varying the value of normalized thermal velocity (b).

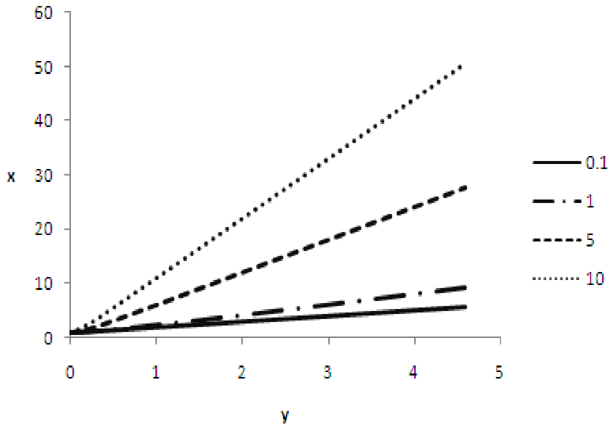

Figure 1: Plot of normalized angular frequency versus normalized wave vector. The graphs are drawn for photosphere temperatures $6 \times 10^3\ K$. Normalized proton collision frequency $z = 0.014$ and electron collision frequency $s = 0.86$. The graphs for normalized thermal velocities $b = 0.1, 1, 5, 10$ are drawn here. The above graphs show that the angular frequency increases with increase in thermal velocity. As $y \to 0$, they converge to a single frequency.

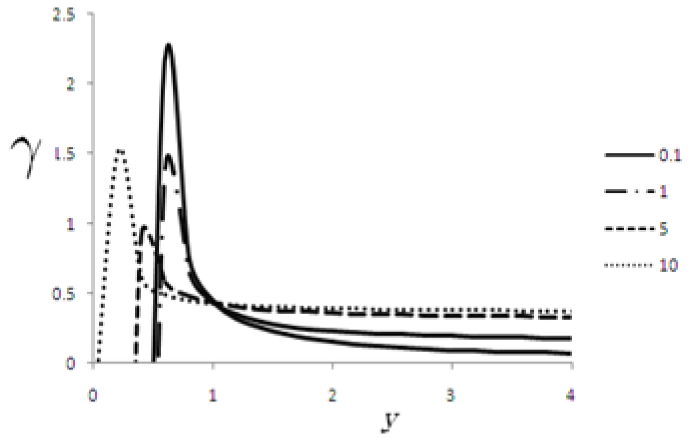

Figure 2: Plot of normalized growth rate versus normalized wave vector. The graphs are drawn for photosphere temperatures $6 \times 10^3\ K$. Normalized proton collision frequency $z = 0.014$ and electron collision frequency $s = 0.86$.

The graphs for normalized thermal velocities $b = 0.1, 1, 5, 10$ are drawn here. The growth rate is clearly sensitive to electron thermal speeds. It decreases with increase in the value of b and maximum growth being shifted to short wave vectors for higher values of b.

4. Conclusions

We have derived a general dispersion relation for two stream instability in hot collisional electron- proton plasma. The dispersion relation has been studied for photosphere parameters. The growth rate is analyzed for solar photosphere plasma, for various electron thermal velocities. It is found that the growth rate is maximum for finite range of frequencies and then become stable after a cut off wave vector. The maximum growth rate region shift towards the low frequency region for higher thermal velocity of electron. Also the angular frequency of maximum growth rate increases with increase in thermal velocity. The real plot shows that the wave propagation increases with increasing thermal velocity and become insensitive to low frequency region. Since this a general dispersion relation and this study could be extended for various plasmas such as magnetosphere, ionosphere and laboratory plasmas.

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