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An Analysis of Steady-State for Machine Repair Problem with a Single Service Station Subject to Breakdown

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Abstract: ***In this paper, we study the machine repair problem with a single service station subject to breakdown under the steady-state conditions. Assume that the system operating machine fails according to a Poisson distribution. The failure time and repair time all are assumed to be exponential distribution. The system is modeled as a finite state Markov chain and its steady-state distribution is obtained by Matrix-Geometry method.***

Keywords : Server breakdown, Matrix-Geometric Technique, Machine Repairs, Queue size

1. Introduction

In this paper we study the machine repair problem with subject to breakdown. It is assumed that each machine fails independently to others . We consider the machine repair problem with working in which the server works with different rates. The important contributions in the areas of machine repair problem are due to B.D. Sinazlian and K.H. Wank [6], P. Wartenhorst [8], M.F. Neuts and D.M. Lucantoni [4] and K.H. Wang [7] used Matrix-Geometry methods to drive various performance of machine repair problem with server breakdown. M.F. Neuts [5] first give the Matrix-Geometry solution of stochastic model. S.H. Choi, B. Kim and K. Sohraby [1] proposed matrix-Geometry solution of nested QBD chain. J. Yu, D. Yu and R. Tian [9] studied repairable queueing system with two heterogeneous servers by using the Matrix-Geometric method.

In this paper investigation we develop a queueing model with working server breakdown and busy period. Which require a sequence of state of repair before service is restored. The rest of the paper is organized as follows. In next section 2 we outline the underlying assumption and notations to develop mathematical model under study. The steady-state equation are constructed by taking appropriate transition rates. In section 3, Matrix-Geometry solution of the system is given. In section 4, various performance characteristic of the system are formulated explicitly in terms of steady-state probability. In the last section, we give the conclusion.

2. Model Description

We consider a machine repair mode with N operating machines which are maintained by a single repairman. Assume that the operating machine fails according to a Poisson process with parameter λ. When the system is working breakdown, Assume that the rate α whenever the server break down, it is immediately repaired at a repair. Repair time during a working period are according to a exponential distribution with rate μ_w. Repair time during a busy period are according to a exponential distribution with rate β. When the machine breakdown, it is immediately sent for repair. After repaired the machine goes immediately into service and the server is fixed it.

For mathematical formulation purpose, we define the following steady-state probability,

$P_0(n)$ = Probability that there are n failed the system when the server is working.

$P_1(n)$ = Probability that there are n failed the system when the server is busy.

The steady-state equation covering the model are constructed as follows,

$$\lambda_0 P_0(0) = \beta P_1(1) + \mu_w P_0 \tag{1}$$

$$[\lambda_n + \mu_w + \alpha] P_0(n) = \lambda_{N-n+1} P_0(N-n+1) + \mu_w P_0(n+1), \quad \text{if } 1 \le n \le N-1 \tag{2}$$

$$[\mu_w + \alpha] P_0(N) = \lambda_{N-1} P_0(N-1), \quad n \le N \tag{3}$$

$$[\lambda_1 + \beta] P_1(1) = \beta P_1(2) + \alpha P_0(1), n \le 1 \tag{4}$$

$$[\lambda_n + \beta] P_1(n) = \lambda_{N-1+1} P_1(N-n+1) + \beta P_1(n+1) + \alpha P_0(n), \quad 2 \le n \le N-1 \tag{5}$$

$$\beta P_1(n) = \lambda_{N-1} P_1(N-1) + \alpha P_0(N), \quad n = N \tag{6}$$

3. Matrix Geometry Solution

The theory of Matrix-Geometry approach was developed Neuts [5] to solve the stationary state probability for the vector state Markov process with repetitive structure. We develop the steady-state probability $P_{0,n}(1 \le n \le N), P_{1,n}(0 \le n \le k),\ P_{2,n}(0 \le n \le N-1)$. The corresponding transition rate matrix Q at this Markov chain has the block- tridiogonal form. Consider the generator matrix Q as shown below

$$Q = \begin{bmatrix} A_0 & C_0 & & & & & \\ B_1 & A_1 & C_1 & & & & \\ & B_2 & A_1 & C_2 & & & \\ & & B_2 & A_1 & C_3 & & \\ & & \ddots & \ddots & \ddots & & \\ & & & B_2 & A_{N-1} & C_{N-1} & \\ & & & & B_2 & A_N & \end{bmatrix} \quad (7)$$

The matrix can be decomposed in to sub matrices A_0, C_0, B_1, B_2, A_N and C_N are as follows (7)

$$A_0 = [-\lambda_0]$$

$$C_0 = [\lambda, 0]$$

$$B_1 = \begin{bmatrix} \mu_w \\ \beta \end{bmatrix} \quad (8)$$

$$B_2 = \begin{bmatrix} \mu_w & 0 \\ 0 & \beta \end{bmatrix}$$

$$A_N = \begin{bmatrix} -(\lambda_N + \mu_w + \alpha) & \alpha \\ 0 & -(\lambda_N + \beta) \end{bmatrix}, \quad 1 \le n \le N$$

$$C_N = \begin{bmatrix} \lambda_{N-n} & 0 \\ 0 & \lambda_{N-n} \end{bmatrix} = \lambda_{N-n}\, I, 1 \le n \le N-1$$

Let X be the vector of the steady-state probability with Q as co-efficient matrix, such that $X\,Q = 0$ and the normalizing condition is $X e = 1$, where e is the column vector of appropriate dimension with all element equal to 1.

The steady-state probability as $X = \{X_1, X_2, X_3, \ldots, X_{N-1}, X_N\}$. Where $X_0 = X_0(0)$ is positive real number and $X_n = \{X_0(n), X_1(n)\}, (1 \le n \le N)$ is a row vector of dimension 2.

By solving steady-state equation $X\,Q = 0$, the balance equation is given by

$$X_0 A_0 + X_1 B_1 = 0 \quad (8)$$

$$X_0 C_0 + X_1 A_1 = 0 \quad (9)$$

$$X_{n-1} C_{n-1} + X_n A_n + X_{n+1} B_2 = 0 \quad \text{for} \quad 2 \le n \le N-1 \quad (10)$$

$$X_{n-1} C_{n-1} + X_n A_n + X_{n+1} B_2 = 0 \quad \text{for} \quad 2 \le n \le N-1 \quad (10)$$

$$X_{N-1} C_{N-1} + X_N A_N = 0 \quad (11)$$

This after routine substitutions, we get

$$P_N = -X_{N-1} C_{N-1} A_N^{-1} \quad (12)$$

$$P_N = X_{N-1} R_N, \quad \text{where } R_N = -C_{N-1} A_N^{-1}$$

$$P_n = X_{n-1} R_n, \quad 2 \le n \le N-1 \quad (13)$$

$$P_1 = -X_0 C_0 [A_1 + X_2 B_2] \quad (14)$$

$$P_0 = [A_0 - C_0 (A_1 + X_2 B_2)] B_1 = 0 \quad (15)$$

Where $R_n = -\lambda_{N-1+1} (A_n + R_{n+1} B_2)^{-1}, \quad 2 \le n \le N$

we have $X_N = -\lambda B_N^{-1}$

Finally we are interested to calculated the vector $X = [X_0, X_1, X_2, \ldots, X_{N-1}, X_N]$ for this purpose. The balance equations for the boundary states given by equations $[12,15]$ can be written uniquely determined by the following normalizing condition

$$X_0(0) + \sum_{n=1}^{k} X_n e = 1. \quad (16)$$

Where e is the column matrix of suitable dimension having all elements 1, this gives a unique solution for $X = \{X_1, X_2, X_3, \ldots, X_{N-1}, X_N\}$.

4. Performance Measures

We define the system validity of the model and the system performance characteristic can be analyzed by computing the system performance characteristic, in terms of the steady-state probability explicit. Some of the system performance induces are as follows

• Probability that the system failed with working

$$P(W) = \sum_{i=1}^{N} n\, P_0(i) \quad (17)$$

• Probability that the failed with busy period

$$P(B) = \sum_{i=1}^{N} n\, P_1(i) \quad (18)$$

• Probability that the number of failed machine in the system

$$P(S) = \sum_{i=1}^{N} n\,[P_0(i) + P_1(i)] \quad (19)$$

• Probability that the number of operating machine in the system

$$P(O) = N - P(W) - P(B)$$
$$= N - \sum_{i=1}^{N} n\,[P_0(i) + P_1(i)] \quad (20)$$

• Average of Machine availability

$$A_{MA} = 1 - \frac{1}{N} \sum_{i=1}^{N} n\,[P_0(i) + P_1(i)] \quad (21)$$

• Average of Operating Utilization machine

$$A_{OU} = 1 - P_0(0) \quad (22)$$

5. Conclusion

This paper provide a comparative analysis of steady-state performance for a machine repair problem with a service station subject to breakdown. We first established the steady -state equation and applied matrix -geometric method to drive the steady-state probabilities various system performance measure, such as the probability number of failed machine, probability number of operation machine, machine availability and operative utilization .The model developed may be applied by the system Engineer, Management personal etc., in industrial organization which encounter complexity in the maintenance of the system which making decision regarding the setting up of the number of machines and the repairman for continuance of the system operation.

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