Optimal Solution for Degeneracy Fuzzy Transportation Problem Using Zero Termination and Robust Ranking Methods

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Abstract: In this paper, we propose a new method for the Fuzzy optimal solution to the Degeneracy transportation problem with Fuzzy parameters. We develop Fuzzy version of Zero Termination and FOMDI algorithms for finding Fuzzy basic feasible and fuzzy optimal solution of fuzzy transportation problems with change into crisp form using Robust ranking technique. The proposed method is easy to understand and to apply for finding Fuzzy optimal solution of degenerate Fuzzy transportation problem occurring in real world situation. To illustrate the proposed method, numerical examples are provided and the results obtained are discussed.

Keywords: fuzzy transportation problem, Trapezoidal fuzzy numbers, Robust ranking, Zero Termination method, FOMDI method

1. Introduction

The transportation problem is one of the earliest applications of linear programming problems. The occurrence of randomness and imprecision in the real world is inevitable owing to some unexpected situations. In a typical problem a product is transported from m sources to n designations and their capacities are a1, a2, ..., an and b1, ..., bn, respectively. In addition, there is a penalty cij associated with transporting unit of product from source i to destination j. This penalty may be on cost or delivery time or safety of delivery etc. The transportation problem, in which the transportation costs, supply and demand quantities are represented in terms of fuzzy numbers, is called a fuzzy transportation problem. The objective of the fuzzy transportation problem is to determine the transportation schedule that minimizes the total fuzzy transportation cost while satisfying the availability and requirement limits. In order to solve a transportation problem, the decision parameters such as availability, requirement and the unit transportation cost of the model must be tried at crisp values. But in real life applications, supply, demand and unit transportation cost may be uncertain due to several factors. These imprecise data may be represented by fuzzy numbers. The idea of fuzzy set was introduced by Zadeh [10] in 1965. Bellmann and Zadeh [2] proposed the concept of decision making in fuzzy environment. After this many authors have suggested various efficient methods for solving transportation problems. Chanas et. al [4], Pandian et.al [8], Liu and Kao [5] etc proposed different methods for the solution of Fuzzy transportation problems. Chanas and Kuchta [4] proposed the concept of the optimal solution for the Transportation with fuzzy coefficient expressed as Fuzzy numbers. A concept of optimal solution of the transportation with Fuzzy cost coefficient, Fuzzy sets and systems, Mohanaseslvi and Ganesan [6] proposed the initial fuzzy feasible solution to a fuzzy transportation problem. Arshamkhan’s Algorithm to solve a Fuzzy Transportation problem. Nagoor Gani and Abdul Razak [7] obtained a fuzzy solution for a two stage cost minimizing.

In this paper we propose a simple method, for the solution of fuzzy transportation problems with converting to crisp form. The rest of this paper is organized as: In section 2, we recall the basic concepts of Fuzzy numbers and related results. In section 3, we define degenerate Fuzzy transportation problem and prove the related theorems. In Section 4, we propose a new algorithm to find the initial fuzzy feasible solution for the given fuzzy transportation problem and obtained the fuzzy optimal solution, applying the zero termination method. In Section 5, we briefly mention the method of solving a degenerate fuzzy transportation problem using zero termination method on Trapezoidal fuzzy number. Numerical example is illustrated.

2. Preliminaries

2.1 Fuzzy Set

A fuzzy set is characterized by a membership function mapping element of a domain, space, or the universe of discourse X to the unit interval [0, 1].

2.2 Fuzzy Number

A real fuzzy number ā is a fuzzy subset of the real number R with Membership function µā satisfying the following conditions,

(i) µā is continuous from R to [0,1]
(ii) µā is strictly increasing &continuous on [a1, a2]
(iii) µā is strictly decreasing & continuous on [a3, a4]
2.3 Trapezoidal fuzzy number

A fuzzy number \(A=(a_1, a_2, a_3, a_4)\) is trapezoidal if

\[
\mu_A(x) = \begin{cases} 
\frac{x-a_2}{a_2-a_1}, & a_1 \leq x \leq a_2 \\
1, & a_2 \leq x \leq a_3 \\
\frac{x-a_3}{a_4-a_3}, & a_3 \leq x \leq a_4 
\end{cases}
\]

2.4 Robust Ranking Technique

Robust ranking technique which satisfy compensation, linearity, and additive properties and provides results which are consist human intuition. If \(\tilde{a}\) is a fuzzy number then the Robust Ranking is defined by

\[
R(\tilde{a}) = \int_0^1 0.5 \left( (a_L^\alpha a_U^\alpha) \, da \right) \text{where (} a_L^\alpha a_U^\alpha \text{) is the } \alpha \text{ level cut of the fuzzy number } \tilde{a} \text{. In this paper we use this method for ranking the objective values. The Robust ranking index } R(\tilde{a}) \text{ gives the representative value of fuzzy number } \tilde{a}.
\]

3. Degenerate Fuzzy Transportation Problem

Consider Degeneracy fuzzy transportation with m sources and n destinations with trapezoidal fuzzy numbers. Let \(\tilde{a}_i\) (\(\tilde{a}_i \geq 0\)) be the fuzzy availability at source \(i\) and \(\tilde{b}_j\) (\(\tilde{b}_j \geq 0\)) be the fuzzy requirement at destination \(j\). Let \(c_{ij}\) be the fuzzy unit transportation cost from source \(i\) to destination \(j\). Let \(x_{ij}\) denote the number of fuzzy units to be transported from source \(i\) to destination \(j\). Then the problem is to determine a feasible way of transporting the available amount at each source to satisfy the demand at each destination so that the total transportation cost is minimized. The mathematical formulation of the fuzzy transportation problem whose parameters are trapezoidal fuzzy numbers under the case that the total supply is equivalent to the total demand is given by the following Table

Mathematical formation of Fuzzy Transportation Table

| | \(c_{11}\) | \ldots | \(c_{1n}\) | \(\tilde{a}_1\) |
| | \vdots | \vdots | \vdots | \vdots |
| Demand | \(b_1\) | \ldots | \(b_n\) |

Mathematically, a fuzzy transportation problem can be stated as follows:

Minimize \(z=\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij} \otimes \tilde{x}_{ij}\)

Subject to \(\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i\), for \(i=1,2,\ldots,m\)

\(\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j\), for \(j=1,2,\ldots,n\)

where

\(m = \) total number of sources
\(n = \) total number of destinations
\(\tilde{a}_i = \) fuzzy availability of the product at \(i\)th source
\(\tilde{b}_j = \) fuzzy demand of the product at \(j\)th destination
\(\tilde{C}_{ij} = \) fuzzy cost of transporting one unit of the product from \(i\)th source to \(j\)th destination
\(\tilde{x}_{ij} = \) fuzzy quantity of the product that should be transported from \(i\)th source to \(j\)th destination

The above fuzzy transportation problem is said to be balanced \(\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j\). Otherwise it is called unbalanced

3.1 Definitions: Fuzzy feasible solution of the Transportation Problem

Now we give some important definitions relevant for developing the feasible solution for the transportation problem using the trapezoidal fuzzy numbers. They are briefly enumerated as follows

a) Fuzzy feasible solution: Any set of non-negative allocations \(\tilde{x}_{ij}\) which satisfies (in the sense equivalent) the row and the column restrictions is known as fuzzy feasible solution.

b) Fuzzy basic feasible solution: A fuzzy feasible solution is a fuzzy basic feasible solution if the number of non-negative allocations is at most \((m + n - 1)\), where, \(m\) is the number of rows, \(n\) is the number of columns in the transportation table.

c) Fuzzy degenerate basic feasible solution: A fuzzy feasible solution to a fuzzy transportation problem with \(m\) sources and \(n\) destinations is said to be a fuzzy degenerate basic feasible solution if the number of positive allocations is \((m+n)-1\). If the number of allocations in a fuzzy basic solution is less than \((m+n-1)\), it is called fuzzy degenerate basic feasible solution.

d) Fuzzy non-degenerate basic feasible solution: Any fuzzy feasible solution to the transportation problem containing origins and \(n\) destinations is said to be fuzzy non-degenerate, if it contains exactly \((m+n-1)\) occupied it is called fuzzy non-degenerate basic feasible solution.

e) Fuzzy optimal Solution: A fuzzy feasible solution is said to be a fuzzy optimal solution if it minimizes the total fuzzy transportation cost.

3.2 Theorems

Finally, we present some important theorems often found useful while developing the feasible solution for the
transportation problem using the trapezoidal fuzzy numbers as well as testing the optimality of the obtained solution.

**Theorem 1:** the number of basics variables in a transportation problem is at most (m+n-1).

**Theorem 2:** The transportation problem always has a feasible solution.

**Theorem 3:** All the transportation problem are triangular (upper or lower) in nature

**Theorem 4:** The values of the basic variables in a basic feasible solution to the transportation problem are given by the expressions of the form

\[ X_{ij} = \pm \sum_{\text{some } p} A_p \mp \sum_{\text{some } q} B_q \]

where, in \( \pm \) and \( \mp \) the upper signs apply to some basic variables and the lower signs apply to the remaining basic variables.

**Theorem 6:** A subset of the columns of the coefficient matrix of a transportation problem is linearly dependent, if and only if, the corresponding cells or a subset of them can be sequenced to the form a loop.

**Theorem 7:** If there will be a feasible solution having (m+n-1) independent positive allocations and if there be number \( \mu_i, \nu_j \) \((i = 1, \ldots, m; j = 1, \ldots, n)\)

satisfying \( c_{ij} \) for each occupied cell \((i,j)\) then the cell evaluation \( \Delta_{ij} \) corresponding to the unoccupied cell \((i,j)\) will be given by \( \Delta_{ij} = c_{ij} - (\mu_i + \nu_j) \).

4. Proposed Algorithms

4.1 Zero Termination Method

The procedure of Zero termination method is as follows:

**Step 1:** Construct the transportation table

**Step 2:** select the smallest unit transportation cost value for each row and subtract it from all costs in that row. In a similar way this process is repeated column wise.

**Step 3:** In the reduced cost matrix obtained from step 2, there will be at least one zero in each row and column. Then we find the termination value of all the zeros in the reduced cost matrix, using the following rule; the zero termination cost is 

\[ T = \text{Sum of the costs of all the cells adjacent to zero is divided by the Number of non-zero cells added in the sum} \]

**Step 4:** In the revised cost matrix with zero termination costs if it has a unique maximum \( T \), allocate maximum possible to that the cell. If it has more one, then the select the cell with the largest cost and allocate the maximum possible.

**Step 5:** After the allocating the columns and rows corresponding to exhaust demands and supplies are trimmed. The resultant matrix must possess at least one zero in each row and column, else repeat step (2)

**Step 6:** Repeat step (3) to step (5) until the optimal solution is obtained

4.2 Test for Optimality (U-V Distribution Method)

The various steps of this method are as follows:

**Step 1:** Assign a zero trapezoidal fuzzy number to any row or column having maximum number of allocations. If the maximum number of allocations is more than one, choose any one arbitrarily

**Step 2:** For each basic cell, find out a set of numbers \( \tilde{u}_i \) and \( \tilde{v}_j \) satisfying \( \tilde{u}_i + \tilde{v}_j = \tilde{c}_{ij} \)

**Step 3:** For each non basic cell, find out the net evaluation \( \tilde{u}_i + \tilde{v}_j = \tilde{c}_{ij} \)

Case (1). If \( \tilde{u}_i + \tilde{v}_j = \tilde{c}_{ij} \) for all \( i, j \), then the solution is optimal and a unique solution exists.

Case( 2). If \( \tilde{u}_i + \tilde{v}_j = \tilde{c}_{ij} \) for all \( i, j \), then the solution is optimal, but an alternate solution exists.

Case( 3). If \( \tilde{u}_i + \tilde{v}_j = \tilde{c}_{ij} \) for at least one \( i, j \), then the solution is not optimal. In this case, we go to the following step.

**Step 4:** Select the non basic cell having the largest positive value of \( \tilde{u}_i + \tilde{v}_j = \tilde{c}_{ij} \) to enter the basis. Let the cell \((r, s)\) enter the basis. Allocate an unknown quantity, say \( 0 \), to the cell \((r, s)\). From this cell \((r, s)\), draw a closed path horizontally and vertically to the nearest basic cell with the restriction that the corner of the closed path must not lie in any non basic cell. Assign signs + and – alternately to the cells of the loop, starting with a + sign for the entering cell. Then \( \min \) of the allocations made in the cells having a negative sign. Add this value of to all cells having + sign and subtract the same from the cells having a – sign. Then the allocation of one basic cell reduces to zero. This yields a better solution by making one (or more) basic cell as non basic cell and one non basic cell as basic cell.

**Step 5:** For the new set of fuzzy basic feasible solution obtained in Step 4, repeat the procedure until a fuzzy optimal solution is obtained.

5. Numerical Examples

Consider the following fuzzy interval transportation problem of minimal cost representation; the optimal cost is obtained by the fuzzy intervals using a new algorithm. The problem is a balanced fuzzy transportation problem.

<table>
<thead>
<tr>
<th>Table 1: Fuzzy Transportation problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_1 )</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>( F_1 )</td>
</tr>
<tr>
<td>( F_2 )</td>
</tr>
<tr>
<td>( F_3 )</td>
</tr>
<tr>
<td>( F_4 )</td>
</tr>
<tr>
<td>( Dem )</td>
</tr>
</tbody>
</table>

Now, the total fuzzy supply, \( \tilde{S} = (50,83,117,150) \) and the total fuzzy demand \( \tilde{D} = (50,83,117,150) \). Since \( Mag(\tilde{S}) = Mag(\tilde{D}) \), the given problem is a balanced.
using the Step 2 to the Step 3 of the fuzzy zero termination method, we have the following reduced fuzzy transportation table

<table>
<thead>
<tr>
<th>(5)</th>
<th>(15)</th>
<th>(20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(30)</td>
<td>(5)</td>
<td></td>
</tr>
</tbody>
</table>

required number of allocations m+n-1=8, but actual number of allocations=7<8 therefore given problem is Degeneracy. We shall allocate a very small positive value ε to one of the cell (1,2),(2,3),(3,4) and (4,5) each has same minimum transportation cost of Rs.6.5 out of unoccupied cells) allocations either of cells (3,4) and (4,5) results in closed loops hence no allocations will be made in these cells. Thus ε = (-0.05, 0, 0, 0.05) can be allocated to either cell (1,2) or (2,3) let us choose allocate it to cell(2,3) so that the number of allocated cells becomes 8

<table>
<thead>
<tr>
<th>W1</th>
<th>W2</th>
<th>W3</th>
<th>W4</th>
<th>W5</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>7.75</td>
<td>6.5</td>
<td>4.5</td>
<td>5.75</td>
<td>9.5</td>
</tr>
<tr>
<td>F2</td>
<td>8.5</td>
<td>5.75</td>
<td>6.5</td>
<td>7.75</td>
<td>8.5</td>
</tr>
<tr>
<td>F3</td>
<td>6.5</td>
<td>8.5</td>
<td>9.5</td>
<td>6.5</td>
<td>5.75</td>
</tr>
<tr>
<td>F4</td>
<td>5.75</td>
<td>7.75</td>
<td>7.75</td>
<td>8.5</td>
<td>6.5</td>
</tr>
<tr>
<td>Demand</td>
<td>30</td>
<td>30</td>
<td>15</td>
<td>20</td>
<td>5</td>
</tr>
</tbody>
</table>

Test for Optimality: Initial basic feasible solution is $W e = C e$. Since 1st row has maximum number of allocations, we take $V_1=0$. Now we compute $U_i$ and $V_j$ for all the basic cells $U_1+V_j=7.75$, $U_1+V_3=4.5$, $U_1+V_5=5.75$, $U_2+V_5=5.75$, $U_2+V_3=0$, $U_3+V_1=6.5$, $U_3+V_3=5.75$, $U_3+V_5=6.5$ then we have $U_1=7.75$, $U_2=9.25$, $U_3=6.5$, $U_4=5.75$, $V_1=0$, $V_2=3.25$, $V_3=3.25$, $V_4=2$, $V_5=-0.75$ Now we compute the net evaluation for all the non basic cells.

<table>
<thead>
<tr>
<th>W1</th>
<th>W2</th>
<th>W3</th>
<th>W4</th>
<th>W5</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>1.5</td>
<td>2.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2</td>
<td>1.25</td>
<td>1.25</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>F3</td>
<td>4.75</td>
<td>6.25</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>F4</td>
<td>4.75</td>
<td>5.25</td>
<td>4.75</td>
<td>1.5</td>
</tr>
</tbody>
</table>

$C_{ij}(U_i+V_j)$ for empty cells

Here, cell value in (2,1) is negative, so the initial basic feasible solution is not optimal and Allocated cells form a closed loop as below:

<table>
<thead>
<tr>
<th>15(+)</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

$\tilde{x}_{11}=(2,4,6,8)$ and $\tilde{x}_{11}=(5,8,12,15)$ with the fuzzy objective value $z = (250, 58, 188, 575)$ and the crisp value of the optimum fuzzy transportation cost for the problem, $z$ is Rs. 577.5
6. Conclusion

We have thus obtained an optimal solution for a fuzzy Degeneracy transportation problem using trapezoidal fuzzy numbers. Here, fuzzy parameters to change into crisp form using Robust ranking technique and the initial basic feasible solution obtained by we propose a new algorithm zero termination method. Here after, we have to propose the method of FOMD method to be finding out the optimal solution for the total fuzzy transportation minimum cost. The numerical examples are solved using the proposed algorithms and obtained results are better than the existing results. The same approach of solving the fuzzy problems may also be utilized in future studies of operational research.

References

[10] Zadeh, L, A Fuzzy sets, Information Control, 8 (1965), 338-353