# Crystal FCC Imposed Deformation: Active Systems in the New Representation Projective Plane for General Test «Purposes: Test Biaxial Extension».

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**Abstract:** ***In this work, we studied the behavior of a single crystal plane FCCP2 rigid plastic deformation imposed in the plane of symmetry (110).We are interested in testing mixed stress-strain. And we deduced the activity state systems are based on the study of the trajectory of the curve representing the general test in the new representation projective plane. We have introduced the dependence on speed, considering the viscoplastic Bingham. And we determined crystallographic perfectly glides stabilize deformation and stabilization of the crystal rotations.***

**Keywords:** crystal FCCP2, imposed deformation, the general test viscoplastic Bingham representation projective plane.

## 1. Introduction

The study of the single crystal with compulsory deformation gave rise to numerous research works, in particular the works of [1], [2]. This study bases essentially on the knowledge of the state of activity of the various systems cristallographic which are responsible for the deformation of materials. The main problem in the study of the behavior of the single crystal, in compulsory deformation, is the knowledge of the states of activity for the various systems. The works of [3]-[4] have surmounted this difficulty for single crystals plans by using the representation projective plan. The objective of this work is the study the state of activity of the various systems. In particular the study of the rotations cristallographic for essay biaxial extension, en base itself on the new representation projective plane, thus the evolution of the rotations will be obtained thus easily. To do it, the used basic model is the one of the cubic single crystals with centered faces stiff (FCC) [5] with isotropic strain hardening, by considering the viscoplastic law of Bingham.

## 2. The FCCP2 Plane Single Crystal

The f.c.c.P2 model is the plane single crystal corresponding to a plane stress and strain state in the (110) planes. In this case the crystallographic frame $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ is chosen

$\bar{x}_1 = [001]$ , $\bar{x}_2 = [1\,\bar{1}\,0]$ , $\bar{x}_3 = [110]$

The corresponding pseudo-slip systems are summarized in Table 1. This table is obtained by noting that, under plane stress and strain, two true systems disappear (because the corresponding resolved shear stress vanishes) and ten remaining true systems can be symmetrized into five plane pseudo-slip systems. For further details, the reader is referred to [6] - [7].

**Table 1:** f.c.c.P2 single crystal

| *Pseudo system (s)* | *Resolved shear* $\tau^s$ | *Pseudo-slip* $\bar{N}^s$ |
|---|---|---|
| *S = 1* | $\tau^1 = \frac{\bar{T}_{12}}{\sqrt{3}}$ | $\bar{N}^{(1)} = \frac{1}{2\sqrt{3}}\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}$ |
| *S = 2* | $\tau^2 = \frac{1}{2\sqrt{3}}(\bar{T}_{12} - \sqrt{2}\bar{T}_{11})$ | $\bar{N}^{(2)} = \frac{1}{2\sqrt{3}}\begin{bmatrix} -\sqrt{2} & 0 \\ 1 & 0 \end{bmatrix}$ |
| *S = 3* | $\tau^3 = \frac{1}{2\sqrt{3}}(\bar{T}_{12} + \sqrt{2}\bar{T}_{11})$ | $\bar{N}^{(3)} = \frac{1}{2\sqrt{3}}\begin{bmatrix} \sqrt{2} & 0 \\ 1 & 0 \end{bmatrix}$ |
| *S = 4* | $\tau^4 = \frac{1}{2\sqrt{3}}(\bar{T}_{12} + \sqrt{2}(\bar{T}_{11} - \bar{T}_{22}))$ | $\bar{N}^{(4)} = \frac{1}{2\sqrt{3}}\begin{bmatrix} \sqrt{2} & 2 \\ -1 & -\sqrt{2} \end{bmatrix}$ |
| *S = 5* | $\tau^5 = \frac{1}{2\sqrt{3}}(\bar{T}_{12} + \sqrt{2}(\bar{T}_{11} - \bar{T}_{22}))$ | $\bar{N}^{(5)} = \frac{1}{2\sqrt{3}}\begin{bmatrix} -\sqrt{2} & 2 \\ -1 & \sqrt{2} \end{bmatrix}$ |

### 2.1. Analyse cinématique plan

The plane single crystal which is defined by the kinematical equations:

$$F = RP \quad (1)$$

$$\dot{P}P^{-1} = \sum_s^N \dot{\alpha}^s \,\bar{N}^s \quad (2)$$

Where F, P and R respectively denote the deformation gradient, the plastic transformation and the lattice rotation tensors, while $\bar{N}^s$ is the plane pseudo-slip system, defined in the crystallographic (isoclinic) configuration, which represents the symmetric contribution of two systems symmetric to $\dot{P}P^{-1}$ ([4]–[5]). The velocity gradient L, strain rate $D = (L)^S$ and rotation rate

$$W = (L)^A \quad (3)$$

$$L = \dot{F}F^{-1} \quad (4)$$

$$\bar{L} = R^T L\, R \quad (5)$$

$$\bar{D} = R^T D\, R = (\bar{L})^s = \sum_s^N \dot{\alpha}^s \,(\bar{N}^s)^S \quad (6)$$

$$\bar{W} = R^T W\, R = R^T\dot{R} + \sum_s^N \dot{\alpha}^s \,(\bar{N}^s)^A \quad (7)$$

$$\bar{\omega}^p = \sum_s^N \dot{\alpha}^s \,(\bar{N}^s)^A \quad (8)$$

Where suffix $()^S$and $()^A$respectively denote the symmetric and skew-symmetric part of any tensor, and where a superimposed bar denotes tensors rotated in the crystallographic configuration.

In this plane case, the lattice rotation R and $R^T\dot{R}$ are given by:

$$R = \begin{bmatrix} \cos\theta & \sin 2\theta & 0 \\ -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (9)$$

$$R^T\dot{R} = \dot{\theta}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (10)$$

Finally, for this model the kinematical relations (6) and (7) become

$$\begin{aligned} \bar{D}_{11} &= \frac{1}{\sqrt{6}}(\dot{\alpha}^3 + \dot{\alpha}^4 - \dot{\alpha}^5 - \dot{\alpha}^2) \\ \bar{D}_{22} &= \frac{1}{\sqrt{6}}(\dot{\alpha}^5 - \dot{\alpha}^4) \quad (11) \\ \bar{D}_{12} &= \frac{1}{4\sqrt{3}}(2\dot{\alpha}^1 + \dot{\alpha}^2 + \dot{\alpha}^3 + \dot{\alpha}^4 + \dot{\alpha}^5) \end{aligned}$$

$$\bar{\varpi}^p = \frac{1}{4\sqrt{3}}(-2\dot{\alpha}^1 - \dot{\alpha}^2 - \dot{\alpha}^3 + 3\dot{\alpha}^4 + 3\dot{\alpha}^5) \quad (12)$$

which gives $\dot{\alpha}^s$ in terms of $\bar{D}$ and two indeterminate quantities $\xi$ and $\bar{\varpi}^p$.

$$\begin{cases} \dot{\alpha}^1 = \frac{\sqrt{3}}{2}(3\bar{D}_{12} - \bar{\varpi}^p - \xi) \\ \dot{\alpha}^2 = \frac{\sqrt{3}}{2}\left(\xi - \sqrt{2}(\bar{D}_{11} + \bar{D}_{22})\right) \\ \dot{\alpha}^3 = \frac{\sqrt{3}}{2}\left(\xi + \sqrt{2}(\bar{D}_{11} + \bar{D}_{22})\right) \\ \dot{\alpha}^4 = \frac{\sqrt{3}}{2}(\bar{D}_{12} + \bar{\varpi}^p - \sqrt{2}\bar{D}_{22}) \\ \dot{\alpha}^5 = \frac{\sqrt{3}}{2}(\bar{D}_{12} + \bar{\varpi}^p + \sqrt{2}\bar{D}_{22}) \end{cases} \quad (13)$$

These kinematic equations (6) and (7), are completed by N slip laws relating, on each slip system, the slip rate $\dot{\alpha}^s$to the resolved shear stress $\tau^s$. Two cases will be considered in the following:

Schmid's slip law:

$$\begin{aligned} \dot{\alpha}^s &\geq 0 \text{ if } \tau^s = \tau^c \\ \dot{\alpha}^s &= 0 \text{ if } |\tau^s| \leq \tau^c \quad (14) \\ \dot{\alpha}^s &\leq 0 \text{ if } \tau^s = -\tau^c \end{aligned}$$

Viscoplastic law of Bingham type:

$$\tau^s = (\tau^c + \mu|\dot{\alpha}^s|)\mathrm{sgn}(\dot{\alpha}^s) \quad (15)$$

With the same critical shear stress $\tau^c$ for all systems.

## 3. Strain Rate Representation

The CFCP2 rigid plastic model case has been analysed in [7] for a rate-independent behaviour. An appropriate geometric representation of the strain rate is obtained by starting from the three-dimensional space (Y1, Y2, Y3) defined as: $Y_1 = \bar{D}_{11}\,; Y_2 = \bar{D}_{22}\,; Y_3 = \sqrt{2}\bar{D}_{12}$

### 3.1. Representation Projective Plane for General Test

Now we shall focus our attention on the general test which is defined as:

$$L = \begin{bmatrix} 1 & \Gamma & 0 \\ 0 & \rho & 0 \\ 0 & 0 & -(1+\rho) \end{bmatrix}\epsilon \quad (16)$$

Where $\epsilon$ is the stretching rate.

$$\text{Strain rate tensor: } D = \frac{1}{2}\begin{bmatrix} 1 & \Gamma & 0 \\ \Gamma & \rho & 0 \\ 0 & 0 & -(1+\rho) \end{bmatrix}\epsilon \ (17)$$

$$\text{rotation rate tensor:} W = \frac{1}{2}\begin{bmatrix} 0 & \Gamma & 0 \\ -\Gamma & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\epsilon \quad 18)$$

Using (6) and (9), strain rate tensor rotated in the crystallographic configuration:

$$\begin{aligned} \bar{D}_{11} &= \left(\frac{1+\rho}{2} + \frac{1-\rho}{2}\cos 2\theta - \frac{\Gamma}{2}\sin 2\theta\right)\epsilon \\ \bar{D}_{22} &= \left(\frac{1+\rho}{2} - \frac{1-\rho}{2}\cos 2\theta + \frac{\Gamma}{2}\sin 2\theta\right)\epsilon \quad (19) \\ \bar{D}_{12} &= \left(\frac{1-\rho}{2}\sin 2\theta + \frac{\Gamma}{2}\cos 2\theta\right)\epsilon \end{aligned}$$

These relations (19) can be rewritten under the following shape:

$$\begin{aligned} \bar{D}_{11} &= \left(\frac{1+\rho}{2} + \frac{\Delta}{2}\left(\frac{1-\rho}{\Delta}\cos 2\theta - \frac{\Gamma}{\Delta}\sin 2\theta\right)\right)\epsilon \\ \bar{D}_{22} &= \left(\frac{1+\rho}{2} - \frac{\Delta}{2}\left(\frac{1-\rho}{\Delta}\cos 2\theta - \frac{\Gamma}{\Delta}\sin 2\theta\right)\right)\epsilon \quad (20) \\ \bar{D}_{12} &= \frac{\Delta}{2}\left(\frac{1-\rho}{\Delta}\sin 2\theta + \frac{\Gamma}{\Delta}\cos 2\theta\right)\epsilon \end{aligned}$$

With: $\Delta = \sqrt{(1-\rho)^2 + \Gamma^2}$ and $\cos 2\alpha = \frac{1-\rho}{\Delta}$ et $\sin 2\alpha = \frac{\Gamma}{\Delta}$

$\alpha$ is orientation principal reference of deformation relative reference in eulerien.

Then:

$$\begin{aligned} \bar{D}_{11} &= \left(\frac{1+\rho}{2} + \frac{\Delta}{2}(\cos 2\alpha\cos 2\theta - \sin 2\alpha\sin 2\theta)\right)\epsilon \\ \bar{D}_{22} &= \left(\frac{1+\rho}{2} - \frac{\Delta}{2}(\cos 2\alpha\cos 2\theta - \sin 2\alpha\sin 2\theta)\right)\epsilon \quad (21) \\ \bar{D}_{12} &= \frac{\Delta}{2}(\cos 2\alpha\sin 2\theta + \sin 2\alpha\cos 2\theta)\epsilon \end{aligned}$$

Then:

$$\begin{aligned} \bar{D}_{11} &= \left(\frac{1+\rho}{2} + \frac{\Delta}{2}\cos\left(2(\alpha+\theta)\right)\right)\epsilon \\ \bar{D}_{22} &= \left(\frac{1+\rho}{2} - \frac{\Delta}{2}\cos\left(2(\alpha+\theta)\right)\right)\epsilon \quad (22) \\ \bar{D}_{12} &= \frac{\Delta}{2}\sin\left(2(\alpha+\theta)\right)\epsilon \end{aligned}$$

The strain rate $\bar{D}$ in the three-dimensional space (Y1, Y2, Y3) is defined as:

$$\begin{aligned} Y_1 &= \left[\frac{1+\rho}{2} + \frac{\Delta}{2}\cos\left(2(\alpha+\theta)\right)\right]\epsilon \\ Y_2 &= \left[\frac{1+\rho}{2} - \frac{\Delta}{2}\cos\left(2(\alpha+\theta)\right)\right]\epsilon \quad (23) \\ Y_3 &= \pm\sqrt{2}\left[\frac{\Delta}{2}\sin\left(2(\alpha+\theta)\right)\right]\epsilon \end{aligned}$$

With $Y_1 = \bar{D}_{11}\,; Y_2 = \bar{D}_{22}\,; Y_3 = \sqrt{2}\bar{D}_{12}$

According to (23), general test is given in the projective geometric representation$Y_3 = \pm 1$, described above, by the following equations:

$$\begin{aligned} Y_1 &= \pm\frac{\sqrt{2}}{2}\left[A(\rho,\Gamma)\frac{1}{\sin 2\beta} + \frac{1}{\tan 2\beta}\right] \\ Y_2 &= \pm\frac{\sqrt{2}}{2}\left[A(\rho,\Gamma)\frac{1}{\sin 2\beta} - \frac{1}{\tan 2\beta}\right] \end{aligned} \quad (24)$$

With: $\beta = \alpha + \theta\,; \tan 2\alpha = \Gamma\,;\ \Delta = \sqrt{(1-\rho)^2 + \Gamma^2}$ and $A(\rho,\Gamma) = \frac{1+\rho}{\Delta}$

For each value of $A(\rho,\Gamma)$, we can thus determine the state of activity of the various systems (figure 1 and 2), active systems in the new representation projective plane for general test, thus the evolution of the rotations will be obtained thus easily This applies to any plane kinematics imposed.

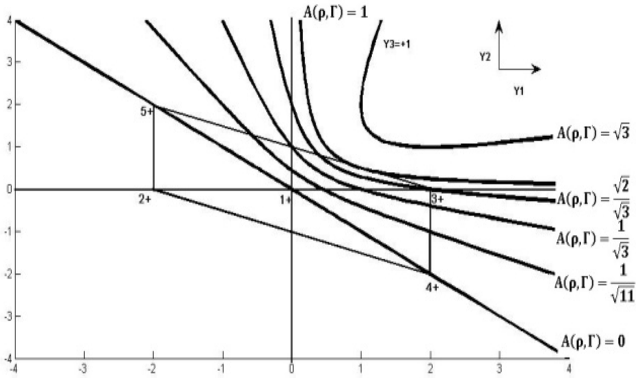


**Figure 1:** Diagram projective plan for general essay $\mathbf{Y_3 = +1}$

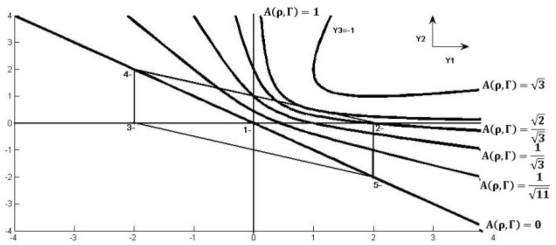


**Figure 2:** Diagram projective plan for general essay $\mathbf{Y_3 = -1}$

It would thus be necessary for every value of $A(\rho,\Gamma)$to make a precise analysis the active systems following the value of angle of rotation$\beta$ and also study the rotation which will be determined for the concerned system. To do it, we are going to handle the case biaxial extension$\Gamma = 0$.

### 3.1.1. *Active systems case*$0 \le A(\rho,0) < \frac{1}{\sqrt{11}}$.

To deduct the state of activity of the systems, it is enough to study the trajectory of the representative curve of the essay biaxial extension in the new representation projective plan (figure 3 and 4).

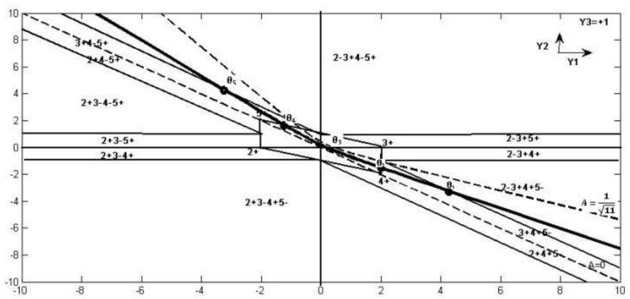


**Figure 3:** Diagram projective plan of biaxial extension $\mathbf{Y_3 = +1}$

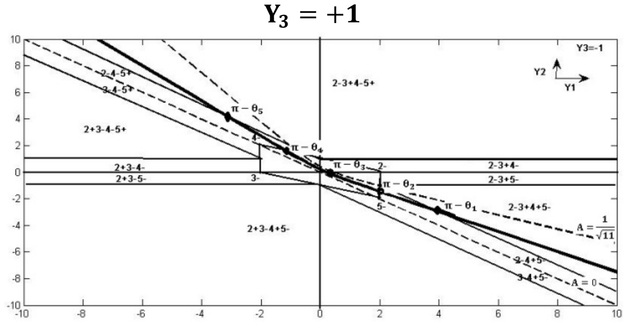


**Figure 4:** Diagram projective plan of biaxial extension $\mathbf{Y_3 = -1}$

With : $\tan 2\theta_1 = \frac{\sqrt{2}A}{\sqrt{1-2A^2}}$ ; $\tan 2\theta_2 = \frac{2\sqrt{2}A+\sqrt{9-A^2}}{-A+2\sqrt{2}\sqrt{9-A^2}}$ ;
$\tan 2\theta_3 = \frac{\sqrt{1-A^2}}{A}$ ; $\tan 2\theta_4 = \frac{2\sqrt{2}A+\sqrt{1-A^2}}{A-2\sqrt{2}\sqrt{1-A^2}}$ ; $\tan 2\theta_5 = \frac{-\sqrt{2}A}{\sqrt{1-2A^2}}$
Or : $A = A(\rho,0)$. Active systems are grouped in Table2.

**Table 2:** Active systems and Condition of sliding

| The interval of θ | Active systems | Condition of sliding |
|---|---|---|
| $0:\theta_1$ | $2^-3^+4^+5^-$ | $\lvert\tau^1\rvert \le 1; \dot\alpha^2 \le 0;\ \dot\alpha^3 \ge 0; \dot\alpha^4 \ge 0; \dot\alpha^5 \le$ |
| $\theta_1:\theta_2$ | $3^+4^+5^-$ | $\lvert\tau^1\rvert \le 1; \lvert\tau^2\rvert \le 1; \dot\alpha^3 \ge 0; \dot\alpha^4 \ge 0; \dot\alpha^5 \le 0$ |
| $\theta_2:\theta_3$ | $1^+3^+4^+$ | $\lvert\tau^2\rvert \le 1; \lvert\tau^5\rvert \le 1; \dot\alpha^1 \ge 0; \dot\alpha^3 \ge 0; \dot\alpha^5 \ge 0$ |
| $\theta_3:\theta_4$ | $1^+3^+5^+$ | $\lvert\tau^2\rvert \le 1; \lvert\tau^5\rvert \le 1; \dot\alpha^1 \ge 0; \dot\alpha^3 \ge 0; \dot\alpha^5 \ge 0$ |
| $\theta_4:\theta_5$ | $3^+4^-5^+$ | $\lvert\tau^1\rvert \le 1; \lvert\tau^2\rvert \le 1; \dot\alpha^3 \ge 0; \dot\alpha^4 \le 0; \dot\alpha^5 \ge 0$ |
| $\theta_5:\pi-\theta_5$ | $2^-3^+4^-5^+$ | $\lvert\tau^1\rvert \le 1; \dot\alpha^2 \le 0;\ \dot\alpha^3 \ge 0; \dot\alpha^4 \le 0; \dot\alpha^5 \ge$ |
| $\pi-\theta_5:\pi-\theta_4$ | $2^-4^-5^+$ | $\lvert\tau^1\rvert \le 1; \lvert\tau^3\rvert \le 1; \dot\alpha^2 \le 0; \dot\alpha^4 \le 0; \dot\alpha^5 \ge 0$ |
| $\pi-\theta_4:\pi-\theta_3$ | $1^-2^-4^-$ | $\lvert\tau^3\rvert \le 1; \lvert\tau^5\rvert \le 1; \dot\alpha^1 \le 0; \dot\alpha^2 \le 0; \dot\alpha^4 \le 0$ |
| $\pi-\theta_3:\pi-\theta_2$ | $1^-2^-5^-$ | $\lvert\tau^3\rvert \le 1; \lvert\tau^4\rvert \le 1; \dot\alpha^1 \le 0; \dot\alpha^2 \le 0; \dot\alpha^5 \le 0$ |
| $\pi-\theta_2:\pi-\theta_1$ | $2^-4^+5^-$ | $\lvert\tau^1\rvert \le 1; \lvert\tau^3\rvert \le 1; \dot\alpha^2 \le 0; \dot\alpha^4 \ge 0; \dot\alpha^5 \le 0$ |
| $\pi-\theta_1:\pi$ | $2^-3^+4^+5^-$ | $\lvert\tau^1\rvert \le 1; \dot\alpha^2 \le 0;\ \dot\alpha^3 \ge 0; \dot\alpha^4 \ge 0; \dot\alpha^5 \le$ |

According to the equations (11) and (12) we can calculate $\dot\alpha^s$ and $\bar\omega^p_{12}$. (Table3)

**Table 3:** $\dot\alpha^s$ and $\bar\omega^p_{12}$

| Active Systems | slip rates $\dot\alpha^s$ | plastic rotation $\bar\omega^p_{12}$ |
|---|---|---|
| $2^-3^+4^+5^-$ | $\dot\alpha^2 = \frac{\sqrt{3}}{2}\left(2\bar D_{12} - \sqrt{2}(\bar D_{11} + \bar D_{22})\right)$<br>$\dot\alpha^3 = \frac{\sqrt{3}}{2}\left(2\bar D_{12} + \sqrt{2}(\bar D_{11} + \bar D_{22})\right)$<br>$\dot\alpha^4 = \frac{\sqrt{3}}{2}\left(2\bar D_{12} - \sqrt{2}\bar D_{22}\right)$<br>$\dot\alpha^5 = \frac{\sqrt{3}}{2}\left(2\bar D_{12} + \sqrt{2}\bar D_{22}\right)$ | $\bar\omega^p_{12} = \bar D_{12}$ |
| $3^+4^+5^-$ | $\dot\alpha^3 = \frac{\sqrt{3}}{2}\left(2\sqrt{2}(\bar D_{11} + \bar D_{22})\right)$<br>$\dot\alpha^4 = \frac{\sqrt{3}}{2}\left(4\bar D_{12} - \sqrt{2}(\bar D_{11} + \bar D_{22})\right)$<br>$\dot\alpha^5 = \frac{\sqrt{3}}{2}\left(4\bar D_{12} - \sqrt{2}\bar D_{11}\right)$ | $\bar D_{12} - \sqrt{2}(\bar D_{11} + \bar D_{22})$ |
| $1^+3^+4^+$ | $\dot\alpha^1 = \frac{\sqrt{3}}{2}\left(4\bar D_{12} - \sqrt{2}\bar D_{11}\right)$<br>$\dot\alpha^3 = \frac{\sqrt{3}}{2}\left(2\sqrt{2}(\bar D_{11} + \bar D_{22})\right)$<br>$\dot\alpha^4 = \frac{\sqrt{3}}{2}\left(-2\sqrt{2}\bar D_{11}\right)$ | $4\bar D_{12} - \sqrt{2}\bar D_{11}$ |

| | | |
|---|---|---|
| **$1^+3^+5^+$** | $\dot{\alpha}^1=\frac{\sqrt{3}}{2}\left(4\bar{D}_{12}-\sqrt{2}(\bar{D}_{11}+2\bar{D}_{22})\right)$ <br> $\dot{\alpha}^3=\frac{\sqrt{3}}{2}\left(2\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})\right)$ <br> $\dot{\alpha}^5=\frac{\sqrt{3}}{2}(2\sqrt{2}\bar{D}_{11})$ | $-\bar{D}_{12}+\sqrt{2}\bar{D}_{11}$ |
| **$3^+4^-5^+$** | $\dot{\alpha}^3=\frac{\sqrt{3}}{2}\left(2\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})\right)$ <br> $\dot{\alpha}^4=\frac{\sqrt{3}}{2}\left(4\bar{D}_{12}-\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})\right)$ <br> $\dot{\alpha}^5=\frac{\sqrt{3}}{2}(4\bar{D}_{12}-\sqrt{2}\bar{D}_{11})$ | $3\bar{D}_{12}-\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})$ |
| **$2^-3^+4^-5^+$** | $\dot{\alpha}^2=\frac{\sqrt{3}}{2}\left(2\bar{D}_{12}-\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})\right)$ <br> $\dot{\alpha}^3=\frac{\sqrt{3}}{2}\left(2\bar{D}_{12}+\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})\right)$ <br> $\dot{\alpha}^4=\frac{\sqrt{3}}{2}(2\bar{D}_{12}-\sqrt{2}\bar{D}_{22})$ <br> $\dot{\alpha}^5=\frac{\sqrt{3}}{2}(2\bar{D}_{12}+\sqrt{2}\bar{D}_{22})$ | $\bar{D}_{12}$ |
| **$2^-4^-5^+$** | $\dot{\alpha}^2=\frac{\sqrt{3}}{2}\left(-2\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})\right)$ <br> $\dot{\alpha}^4=\frac{\sqrt{3}}{2}(4\bar{D}_{12}+\sqrt{2}\bar{D}_{11})$ <br> $\dot{\alpha}^4=\frac{\sqrt{3}}{2}\left(4\bar{D}_{12}+\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})\right)$ | $3\bar{D}_{12}+\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})$ |
| **$1^-2^-4^-$** | $\dot{\alpha}^1=\frac{\sqrt{3}}{2}\left(4\bar{D}_{12}+\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})\right)$ <br> $\dot{\alpha}^2=\frac{\sqrt{3}}{2}\left(-2\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})\right)$ <br> $\dot{\alpha}^4=\frac{\sqrt{3}}{2}(-2\sqrt{2}\bar{D}_{22})$ | $-\bar{D}_{12}-\sqrt{2}\bar{D}_{22}$ |
| **$1^-2^-5^-$** | $\dot{\alpha}^1=\frac{\sqrt{3}}{2}(4\bar{D}_{12}+\sqrt{2}\bar{D}_{11})$ <br> $\dot{\alpha}^2=\frac{\sqrt{3}}{2}\left(-2\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})\right)$ <br> $\dot{\alpha}^5=\frac{\sqrt{3}}{2}(2\sqrt{2}\bar{D}_{22})$ | $-\bar{D}_{12}+\sqrt{2}\bar{D}_{22}$ |
| **$2^-4^+5^-$** | $\dot{\alpha}^2=\frac{\sqrt{3}}{2}\left(-2\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})\right)$ <br> $\dot{\alpha}^4=\frac{\sqrt{3}}{2}(4\bar{D}_{12}+\sqrt{2}\bar{D}_{11})$ <br> $\dot{\alpha}^4=\frac{\sqrt{3}}{2}\left(4\bar{D}_{12}+\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})\right)$ | $3\bar{D}_{12}+\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})$ |
| **$2^-3^+4^+5^-$** | $\dot{\alpha}^2=\frac{\sqrt{3}}{2}\left(2\bar{D}_{12}-\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})\right)$ <br> $\dot{\alpha}^3=\frac{\sqrt{3}}{2}\left(2\bar{D}_{12}+\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})\right)$ <br> $\dot{\alpha}^4=\frac{\sqrt{3}}{2}(2\bar{D}_{12}-\sqrt{2}\bar{D}_{22})$ <br> $\dot{\alpha}^5=\frac{\sqrt{3}}{2}(2\bar{D}_{12}+\sqrt{2}\bar{D}_{22})$ | $\bar{D}_{12}$ |

### 3.1.2. System activity for every value of $A(\rho,0)$ (Table 4,5,6 , 7and 8)

**Table 4:** Lattice rotation rate in terms of the system activity $\frac{1}{\sqrt{11}}\le A(\rho,0)<\frac{1}{\sqrt{3}}$.

| The interval of θ | Zone | Rotation rate$\dot{\theta}$ |
|---|---|---|
| $0:\theta_1$ | $2^-3^+4^+5^-$ | $-\bar{D}_{12}$ |
| $\theta_1:\theta_2$ | $2^-3^+4^+$ | $\bar{D}_{12}+\sqrt{2}\bar{D}_{22}$ |
| $\theta_2:\theta_3$ | $1^+3^+4^+$ | $\bar{D}_{12}+\sqrt{2}\bar{D}_{22}$ |
| $\theta_3:\theta_4$ | $1^+3^+5^+$ | $\bar{D}_{12}-\sqrt{2}\bar{D}_{11}$ |
| $\theta_4:\theta_5$ | $3^+4^-5^+$ | $-3\bar{D}_{12}+\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})$ |
| $\theta_5:\pi-\theta_5$ | $2^-3^+4^-5^+$ | $-\bar{D}_{12}$ |
| $\pi-\theta_5:\pi-\theta_4$ | $2^-4^-5^+$ | $-3\bar{D}_{12}-\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})$ |
| $\pi-\theta_4:\pi-\theta_3$ | $1^-2^-4^-$ | $\bar{D}_{12}+\sqrt{2}\bar{D}_{22}$ |
| $\pi-\theta_3:\pi-\theta_2$ | $1^-2^-5^-$ | $\bar{D}_{12}-\sqrt{2}\bar{D}_{22}$ |
| $\pi-\theta_2:\pi-\theta_1$ | $2^-3^+5^-$ | $\bar{D}_{12}-\sqrt{2}\bar{D}_{11}$ |
| $\pi-\theta_1:\pi$ | $2^-3^+4^+5^-$ | $-\bar{D}_{12}$ |

Or: $\tan 2\theta_1=\frac{-\sqrt{2}A+\sqrt{3-A^2}}{A+\sqrt{6-2A^2}}$ ; $\tan 2\theta_2=\frac{2\sqrt{2}A+\sqrt{9-A^2}}{-A+2\sqrt{2}\sqrt{9-A^2}}$ ;
$\tan 2\theta_3=\frac{\sqrt{1-A^2}}{A}$ ; $\tan 2\theta_4=\frac{2\sqrt{2}A+\sqrt{1-A^2}}{A-2\sqrt{2}\sqrt{1-A^2}}$ ;
$\tan 2\theta_5=\frac{\sqrt{2}A}{\sqrt{1-2A^2}}$

**Table 5:** Lattice rotation rate in terms of the system activity$\frac{1}{\sqrt{3}}\le A<\frac{\sqrt{2}}{\sqrt{3}}$

| The interval of θ | Zone | Rotation rate$\dot{\theta}$ |
|---|---|---|
| $0:\theta_1$ | $2^-3^+4^+5^-$ | $-\bar{D}_{12}$ |
| $\theta_1:\theta_2$ | $2^-3^+4^+$ | $\bar{D}_{12}+\sqrt{2}\bar{D}_{22}$ |
| $\theta_2:\theta_3$ | $1^+3^+4^+$ | $\bar{D}_{12}+\sqrt{2}\bar{D}_{22}$ |
| $\theta_3:\theta_4$ | $1^+3^+5^+$ | $\bar{D}_{12}-\sqrt{2}\bar{D}_{11}$ |
| $\theta_4:\theta_5$ | $3^+4^-5^+$ | $-3\bar{D}_{12}+\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})$ |
| $\theta_5:\pi-\theta_5$ | $2^-3^+4^-5^+$ | $-\bar{D}_{12}$ |
| $\pi-\theta_5:\pi-\theta_4$ | $2^-4^-5^+$ | $-3\bar{D}_{12}-\sqrt{2}(\bar{D}_{11}+\bar{D}_{22})$ |
| $\pi-\theta_4:\pi-\theta_3$ | $1^-2^-4^-$ | $\bar{D}_{12}+\sqrt{2}\bar{D}_{22}$ |
| $\pi-\theta_3:\pi-\theta_2$ | $1^-2^-5^-$ | $\bar{D}_{12}-\sqrt{2}\bar{D}_{22}$ |
| $\pi-\theta_2:\pi-\theta_1$ | $2^-3^+5^-$ | $\bar{D}_{12}-\sqrt{2}\bar{D}_{11}$ |
| $\pi-\theta_1:\pi$ | $2^-3^+4^+5^-$ | $-\bar{D}_{12}$ |

Or:$\tan 2\theta_1=\frac{-\sqrt{2}A+\sqrt{3-A^2}}{A+\sqrt{6-2A^2}}$ ; $\tan 2\theta_2=\frac{2\sqrt{2}A+\sqrt{9-A^2}}{-A+2\sqrt{2}\sqrt{9-A^2}}$
;
$\tan 2\theta_3=\frac{\sqrt{1-A^2}}{A}$ ; $\tan 2\theta_4=\frac{2\sqrt{2}A+\sqrt{1-A^2}}{A-2\sqrt{2}\sqrt{1-A^2}}$ ;
$\tan 2\theta_5=\frac{\sqrt{2}A+\sqrt{3-A^2}}{A-\sqrt{6-2A^2}}$

**Table 6:** Lattice rotation rate in terms of the system activity$\frac{\sqrt{2}}{\sqrt{3}}\le A<1$

| L'intervalle de θ | Systèmes de glissement | vitesse de Rotation$\dot{\theta}$ |
|---|---|---|
| $0:\theta_1$ | $2^-3^+4^+5^-$ | $-\bar{D}_{12}$ |
| $\theta_1:\theta_2$ | $2^-3^+4^+$ | $\bar{D}_{12}+\sqrt{2}\bar{D}_{22}$ |
| $\theta_2:\theta_3$ | $2^-3^+5^+$ | $\bar{D}_{12}-\sqrt{2}\bar{D}_{22}$ |
| $\theta_3:\theta_4$ | $1^+3^+5^+$ | $\bar{D}_{12}-\sqrt{2}\bar{D}_{11}$ |
| $\theta_4:\theta_5$ | $2^-3^+5^+$ | $\bar{D}_{12}-\sqrt{2}\bar{D}_{22}$ |
| $\theta_5:\pi-\theta_5$ | $2^-3^+4^-5^+$ | $-\bar{D}_{12}$ |
| $\pi-\theta_5:\pi-\theta_4$ | $2^-3^+4^-$ | $\bar{D}_{12}+\sqrt{2}\bar{D}_{22}$ |
| $\pi-\theta_4:\pi-\theta_3$ | $1^-2^-4^-$ | $\bar{D}_{12}+\sqrt{2}\bar{D}_{22}$ |
| $\pi-\theta_3:\pi-\theta_2$ | $2^-3^+4^-$ | $\bar{D}_{12}+\sqrt{2}\bar{D}_{22}$ |
| $\pi-\theta_2:\pi-\theta_1$ | $2^-3^+5^-$ | $\bar{D}_{12}-\sqrt{2}\bar{D}_{22}$ |
| $\pi-\theta_1:\pi$ | $2^-3^+4^+5^-$ | $-\bar{D}_{12}$ |

Or :$\tan 2\theta_1=\frac{-\sqrt{2}A+\sqrt{3-A^2}}{A+\sqrt{6-2A^2}}$ ; $\tan 2\theta_2=\frac{\sqrt{1-A^2}}{A}$ ;
$\tan 2\theta_3=\frac{2\sqrt{2}A-\sqrt{1-A^2}}{A+2\sqrt{2}\sqrt{1-A^2}}$ ; $\tan 2\theta_4=\frac{2\sqrt{2}A+\sqrt{1-A^2}}{A-2\sqrt{2}\sqrt{1-A^2}}$ ;
$\tan 2\theta_5=\frac{\sqrt{2}A+\sqrt{3-A^2}}{A-\sqrt{6-2A^2}}$

**Table 7:** Lattice rotation rate in terms of the system activity$1 \leq A < \sqrt{3}$

| The interval of θ | Zone | Rotation rate$\dot{\theta}$ |
|---|---|---|
| $0:\theta_1$ | $2^-3^+4^-5^+$ | $-\bar{D}_{12}$ |
| $\theta_1:\theta_2$ | $2^-3^+5^+$ | $\bar{D}_{12}-\sqrt{2}\bar{D}_{22}$ |
| $\theta_2:\pi-\theta_2$ | $2^-3^+4^-5^+$ | $-\bar{D}_{12}$ |
| $\pi-\theta_2:\pi-\theta_1$ | $2^-3^+4^-$ | $\bar{D}_{12}+\sqrt{2}\bar{D}_{22}$ |
| $\pi-\theta_1:\pi$ | $2^-3^+4^-5^+$ | $-\bar{D}_{12}$ |

Or: $\tan 2\theta_1 = \frac{\sqrt{2}A-\sqrt{3-A^2}}{A+\sqrt{6-2A^2}}$ ; $\tan 2\theta_2 = \frac{\sqrt{2}A+\sqrt{3-A^2}}{A-\sqrt{6-2A^2}}$

**Table 8:** Lattice rotation rate in terms of the system activity$A \geq \sqrt{3}$

| The interval of θ | Zone | Rotation rate$\dot{\theta}$ |
|---|---|---|
| $0:\pi$ | $2^-3^+4^-5^+$ | $-\bar{D}_{12}$ |

### 3.1.3. *Analysis of rotation* $\dot{\boldsymbol{\theta}}$

Using the Tables (5 ,6 and 7 ) , we obtain easily the value of $\dot{\boldsymbol{\theta}} = \frac{\mathbf{d\theta}}{\mathbf{d\varepsilon}}$in each zone. The results are plotted in Figure (4), (5) and (6). This shows that the rotation stabilizes at three orientation limits: $\boldsymbol{\theta_l}$ , 0 et $\pi - \boldsymbol{\theta_l}$ , with $\tan\theta_l = 2\sqrt{2}$.

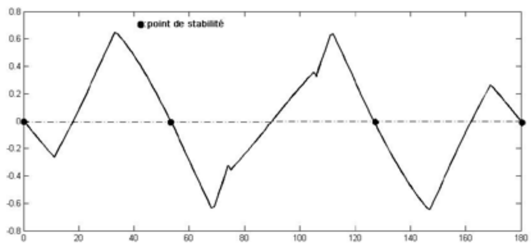


**Figure 5:** curve$\dot{\theta} = f(\theta)$ or $\frac{1}{\sqrt{11}} \leq A < \frac{1}{\sqrt{3}}$

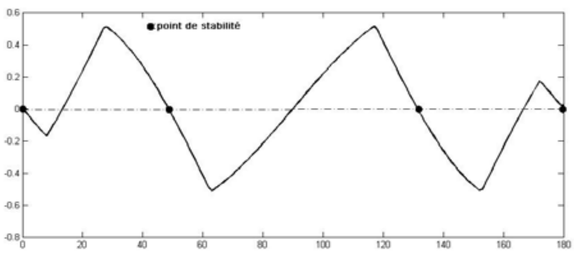


**Figure 6:** curve $\dot{\theta} = f(\theta)$ or $\frac{1}{\sqrt{3}} \leq A < \frac{\sqrt{2}}{\sqrt{3}}$

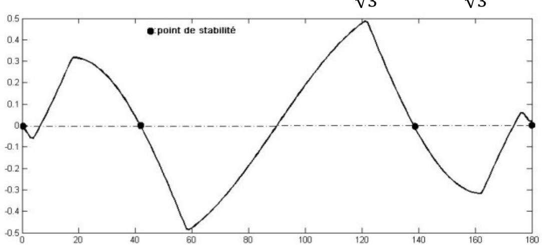


**Figure 7:** curve $\dot{\theta} = f(\theta)$ or $\frac{\sqrt{2}}{\sqrt{3}} \leq A < 1$

The corresponding rate $\dot{\theta} = \frac{d\theta}{d\varepsilon}$is plotted in Figure (8). In this case the rotation stabilizes at two orientation limits $\theta_l$ and $\pi - \theta_l$ , with $\tan\theta_l = 2\sqrt{2}$.

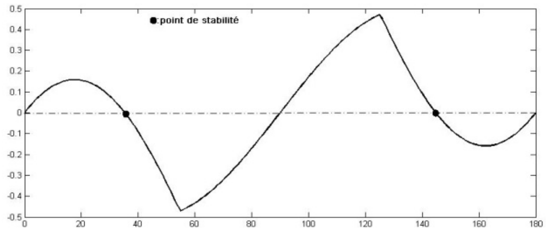


**Figure 8:** curve $\dot{\theta} = f(\theta)$or $1 \leq A < \sqrt{3}$

Finally, to complete the illustrative example, we analyse the case of strain test with $A \geq \sqrt{3}$ (Figure 9) In this case, when θ goes from 0 to $\pi$, solely the regime $2^-3^+4^-5^+$ (Table 8) is potentially active and corresponds to one orientation limit θ = 0.

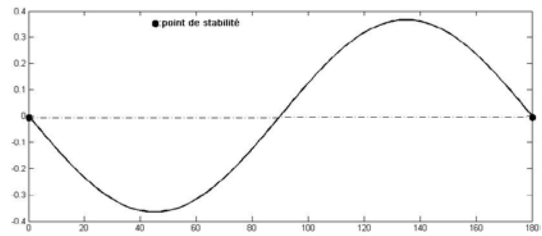


**Figure 9:** curve $\dot{\theta} = f(\theta)$ or $A \geq \sqrt{3}$

## 4. Conclusion

In this paper, our analysis is focused on the determination the slip systems activity of a plane single crystal FCCP2, in the new representation projective plane for general test. Based on the plane single crystal model and Bingham slip law, as illustration, the biaxial extension is studied and the complete analytical solution of single crystal behavior is obtained. In particular, the analytical description of the plastic spin is introduced. The lattice orientation and the straining path have an important influence on the plastic spin evolution. Different situations may be encountered, according to the strain path and the initial lattice orientation value. But in any case, this lattice rotation always stabilizes at a limit value resulting in a stabilized behavior for the crystal. All of these calculations, which are performed in the framework of simple plane model FCCP2, remain valid in the three-dimensional case. The plane single crystal model represents a reasonable compromise between the mathematical simplicity and the physical relevance for the analysis of some basic problems in the mechanics of single crystal.

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