Bulk Viscous Higher Dimensional FRW Cosmological Model with Variable Gravitational Constant and Cosmological Constant in General Relativity

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Abstract: ***In this paper we study higher dimensional homogeneous and isotropic FRW viscous cosmological model with variable G and Λ. We obtain solutions of the field equations assuming that the cosmological term is proportional to H^2 (where H is a Hubble constant)and bulk viscosity proportional to H.From equation(19),we conclude that as $t \to \infty$ then $R \to \infty$.Implies due to bulk viscosity the model has no singularity and universe is accelerating if A=0.Recent observational data (Knop et al. [70]; Riess et al. [71,72]; Spergel et al. [73]; Tegmark et al. [74]; Perlmutter et al. [75] strongly suggest this acceleration occurs. Finally, the solutions presented in the paper are new and useful for a better understanding of the evolution of the universe in F-R-W space-time with variable G and Λ.***

Keywords: F-R-W Cosmological model, Hubble Constants, Gravitational Constant and Cosmological Constants

1. Introduction

Friedmann [1] was first to obtain a general relativistic cosmological solution of Einstein's equation describing, expanding, spatially homogeneous and isotropic universe. He first recognized that the expansion starts in a super dense state viz. Big Bang and related these models to the redshift measurements (Hubble, [2]). Robertson and Walker in a series of papers (Robertson [3, 4] walker [5, 6]) proved that the line-element depicting spatially homogeneous and isotropic world models.

It represents an incoherent matter (dust) distribution of closed universe with constant positive spatial curvature. The generalized Einstein’s theory of gravitation with time dependent G and Λ has been proposed by Lau [7]. This modification allows us to use Einstein's field equations form, which is unchanged since variation in Λ is accompanied by a variation of G. A number of authors investigated Friedman-Robertson-Walker (FRW) models and Bianchi models using this approach (Abdel-Rahman [8]; Berman &Som [9]; Sistero [10]; Kalligas et al. [11]; Abdussattar &Vishwakarma [12]; Vishwakarma [13,14]; Pradhan & Otarod [15]; Singh et al. [16]; Singh & Tiwari [17]; Borges & Carneiro [18] have considered that the cosmological term is proportional to the Hubble parameter in the FRW model and the Bianchi type-I model with variable G and Λ. Classification of the FRW universe with a cosmological constant and a perfect fluid in the equation of state have been studied by Ha et al. [19]. Recently, We have [17, 20, 21] considered whether or not the cosmological term is proportional to the Hubble parameter in the Bianchi type-I model and FRW model with varying G and Λ.

The Newtonian constant of gravitation G plays the role of a coupling constant between geometry and matter in the Einstein field equations. Variation of the gravitational constant was first suggested by Dirac [22]. Expanding universe models were studied by Lemaitre [23, 24] independently.Canuto et al. [25, 26] made numerous suggestions based on different arguments that G is indeed time dependent. Beesham [27] has studied the creation with variable G and pointed out the variation of form $G \sim t^{-1}$, originally proposed by Dirac [22].

The possibility of variable G and Λ in Einstein theory has also been studied by Dersarkissian [28]. Berman [29] and Sistero [10] have considered the Einstein field equations with perfect fluid and variable G for Rabertson-Walker line element. Kalligas et al. [11] have studied FRW models with variable Λ and G and discussed the possible connection with power law time dependence of G. Abdussattar and Vishwakarma [30] presented R-W models with variable Λ and G by admitting a contracted Ricci Collineation along the fluid flow vector. Recently some of us and others have studied cosmological models with variable G and Λ in a diversified field [31, 32, 33, 34, 35, 36, 16, 17, 37, 38, 20, 39]. Thus the implication of time varying Λ and G are important to study the early evolution of the universe.

Kaluza [40] and Klein[41] disused the concept of gravitation and electromagnetism could be unified by well-known five dimensional theories in the single geometrical structure. Thiry [42] and Jordan[43] further generalized the concept of consideration of coefficient of fifth coordinate as constant. Later on Marcianos [44] theory suggested that strong evidence of higher dimensions may be experimental directions of time variation of fundamental constants. Various researchers investigated on theories consists the concept of higher dimensions (Banerjee et.al [45], Krori et.al[46], Chatterjee and Bhui [45], Singh et.al [47], Rehaman et.al[48]). The phenomenological Λ decay scenarios have been considered by a number of authors. Chen & Wu [49] considered $\Lambda \alpha a^{-2}$ (a is the scale factor of the Robertson-Walker metric). Hoyle et al. [50] considered $\Lambda \alpha a^{-3}$ while $\Lambda \alpha a^{-m}$ (a is a scale factor and m is a constant) was considered by Olson & Jordan [51]; Pavon [52]; Maia &Silva [53];Silveira&Waga [54, 55] andBloomfield Torres &Waga [56].
Various researchers Misner[57,58].Santos et.al[59],Coley and Tupper[60],Roy and Prakash[61],Goenner and Kowalewski[62],Padmanabhan and Chitre[63],Ram and Singh[64],Bali et.al[65,66,67,68] have studied effect of bulk viscosity on the evolution of universe at large.Gron[69] studied viscous inflationary universe models.

In this paper we study higher dimensional homogeneous and isotropic FRW viscous cosmological model with variable G and Λ .We obtain solutions of the field equations assuming that the cosmological term is proportional to H^2 (where H is a Hubble constant)and bulk viscosity proportional to H.

2. The Model and Field Equations

The higher dimensional Friedman-Robertson-Walker metric is given by

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{(1-kr^2)} + r^2(d\theta_1^2 + sin^2\theta_1 d\theta_2^2 + sin^2\theta_1 sin^2\theta_2 d\theta_3^2)\right] \quad ....(1)$$

where R(t) is the scale factor and k = 1, 0, or + 1 is the curvature parameter for an open, flat or closed universe, respectively.
The cosmic matter is represented by the energy-momentum tensor due to bulk viscous fluid is

$$T_{ij} = (\rho + \bar{p})\upsilon_i \upsilon_j \quad + \quad \bar{p} g_{ij} \quad ....(2)$$

Where $\rho\,\theta,\varepsilon$ are the energy density, scalar expansion and coefficient of bulk viscosity υ_i is the four-velocity vector of the particle .We have relations satisfying following conditions

$$v_i v_j = -1 \qquad ....(3)$$

$$\bar{p} = p - \varepsilon\theta \qquad ....(4)$$

$$P = w\rho, 0 \le w \le\le 1 \qquad ....(5)$$

Where p is equilibrium pressure,$\bar{p}$ is dissipative pressure.
The Einstein field equations with variable G and Λ both functions of "t" are given by Weinberg [17]

$$R_{ij} - \frac{1}{2}R_k^k g_{ij} - 8\pi G(t)T_{ij} + \Lambda(t)g_{ij}. \quad ....(6)$$

For the metric Equation (1) and energy-momentum tensor Equation (2) in a commoving system of coordinates, the field Equation (6) yields

$$\frac{6\ddot{R}}{R} = -8\pi G[\rho + 2(p - 3H\varepsilon)] + \Lambda \qquad ....(7)$$

$$\frac{6\dot{R}^2}{R^2} + \frac{6k}{R^2} = 8\pi G\left[\rho + \frac{\Lambda}{8\pi G}\right] ....(8)$$

Eliminating $\ddot{R}$ from equations (7) and (8) we get

$$\dot{\rho} + (4p + \rho)H - 12\varepsilon H^2 = -\left[\frac{\rho\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G}\right] ....(9)$$

The usual energy conservation equation $T^j_{i,j} = 0$ yields

$$\dot{\rho} + 4(\rho + \bar{p})H = 0 \qquad ....(10)$$

Using equation (10) in (9) we get

$$\frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}}{8\pi G} = 3\rho H \qquad ....(11)$$

In terms of Hubble parameter H, we rewrite equation (7) and (8)

$$\dot{H} + H^2 = -\frac{4\pi G}{3}[\rho + 2(p - 3H\varepsilon)] + \frac{\Lambda}{6} ....(12)$$

$$H^2 = \frac{4\pi G}{3}\left[\rho + \frac{\Lambda}{8\pi G}\right] - \frac{k}{R^2} \qquad ....(13)$$

3. Solution of the Field Equations

The system of field equations (12) and (13).We obtain solutions of the field equations assuming

$$\Lambda = aH^2 \qquad ....(14)$$

$$\varepsilon = \varepsilon_0 H \qquad ....(15)$$

Where a andε_0 are constants.
Put value of equation (5) in (12) we get

$$\dot{H} + H^2 = -\frac{4\pi G}{3}[\rho(1 + 2w)] + 8\pi G\varepsilon H + \frac{\Lambda}{6} ....(16)$$

Eliminate ρ from equations (13)and(16)we get

$$\dot{H} + 2H^2(1 + w) + \frac{k}{R^2}(1 + 2w) = 8\pi G\varepsilon H + \frac{2\Lambda(1+w)}{6} ....(17)$$

We assume universe is flat i.e.k=0 in equation (17)

$$\dot{H} + 2H^2(1 + w) = 8\pi G\varepsilon H + \frac{2\Lambda(1+w)}{6} ....(18)$$

Put equation (15) and (14) in (18) we get

$$R = (bt - Bt_0)^{\frac{1}{A}} \qquad ....(19)$$

Where b,B,A are all constants.
$A = 2(1 + w) - 8\pi G\varepsilon_0 - \frac{a(1+w)}{3}$ and BA=b.
The coefficient of bulk viscosity,cosmologicalconstant,density,gravitationalconstant,sherescalar,Shear scalar and deceleratio parameter are

$$\varepsilon = \varepsilon_0 H = \frac{\varepsilon_0}{(At - t_0)} \qquad ....(20)$$

$$\Lambda = aH^2 = \frac{a}{(At - t_0)^2} \qquad ....(21)$$

$$\rho = \frac{1}{(At-t_0)^{\frac{4(1+w)}{A}}} \quad ....(22)$$

$$G = \frac{6-a}{8\pi} \times \frac{1}{(At-t_0)^{\frac{-4(1+w)}{A}+2}} \quad ....(23)$$

$$q = -1 + A \quad ....(24)$$

$$\theta = \frac{3}{(At-t_0)} \quad ....(25)$$

$$\sigma = \frac{k}{\sqrt{3} \times (bt-Bt_0)^{\frac{3}{A}}} \quad ....(26)$$

4. Conclusion

In this paper, we study higher dimensional homogeneous and isotropic FRW viscous cosmological model with variable G and Λ .We obtain solutions of the field equations assuming that the cosmological term is proportional to H^2 (where H is a Hubble constant)and bulk viscosity proportional to H.From equation(19),we conclude that as $t \rightarrow \infty$ *then* $R \rightarrow \infty$.Implies due to bulk viscosity the model has no singularity and universe is accelerating if A=0.Recent observational data (Knop et al. [70]; Riess et al. [71,72]; Spergel et al. [73]; Tegmark et al. [74]; Perlmutter et al. [75] strongly suggest this acceleration occurs. The cosmological constant $\Lambda(t)\alpha 1/t^2$ which follow from the model of Kalligas et al. [76]; Berman &Som [77]; Berman [78]; Berman et al. [79] and Bertolami [80].

This form of Λ is physically reasonable as observations suggest that Λ is very small in the present universe. Finally, the solutions presented in the paper are new and useful for a better understanding of the evolution of the universe in F-R-W space-time with variable G and Λ. Thus in earlier universe more general situation is given by viscous cosmological fluid. Various researchers discussed viscous cosmological model with variable gravitational constant and cosmological constant(Singh[47],)whose work has been studied and extended in five dimensions.

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