Algorithm of Power Allocation for Capacity Enhancement of MIMO OFDM

Kavita Devi¹, Rajneesh Talwar²
Jasdev Singh Sandhu Group of Institutes, Kauli, Patiala, Pb. India

Abstract: Channel Capacity is a measure of how much information that can be transmitted and received with a negligible probability of error. When the transmitter has perfect knowledge of the channel, water filling algorithm can optimize the transmitted signal power. The capacity can be enhanced by using the good channels i.e. those with the highest gain by applying an unequal power distribution.

Keywords: Channel Capacity, Power Distribution, Water Level, MIMO, SISO.

1. Introduction

Channel Capacity is a measure of the information that can be transmitted and received with a negligible probability of error. Claude Elwood Shannon [1] developed the following equation for theoretical channel capacity:

$$C_{\text{iso}} = B \log (1+\text{SNR})$$

It includes the transmission bandwidth $B$ and signal to noise ratio $\text{SNR}$. The Shannon capacity of MIMO system depends on the number of antenna. For MIMO the capacity is given by the following equation:

$$C_{\text{mimo}} = NB \log (1+\text{SNR})$$

Where $N$ is the minimum of $N_t$ (number of transmitting antennas) or $N_r$ (number of receiving antennas). When channel parameters are known at transmitter, capacity can be increased by adaptively assigning transmitted power under total transmit power constraint for maximal transported bits.

Problem of maximizing the mutual information between the input and the output of a channel composed of several subchannels (such as a frequency-selective channel, a time-varying channel, or a set of parallel subchannels arising from the use of multiple antennas at both sides of the link) with a global power constraint at the transmitter. This capacity-achieving solution has the visual interpretation of pouring water over a surface given by the inverse of the subchannel gains [2] as shown in the adjacent figure 1.

When the transmitter has perfect knowledge of the channel, this algorithm can optimize the transmitted signal power. The division of total power is in such a way that a greater portion goes to the sub channels with higher gain and less or even none to the channels with small gains. The sub channels with lower gain i.e., those with higher noise for which no power is allocated at all refer to those sub channels which are not used for transmitting any signal during the transmission [3]. The objective of this algorithm is to allocate power across the channel so as to maximize the total capacity.

This power allocation is subject to the constraint that the sum of the power poured into all sub-channel is equal to $P_T$, the total power available to the transmitter. The relative channel strengths and the amount of power to allocate to each channel is determined by knowledge of the channel matrix, $H$. Channel gain is $1/\lambda$, $\lambda$ is eigen value of channel matrix.

![Figure 1: Interpretation of pouring water over a surface given by the inverse of the subchannel gains](image-url)
The first step is to determine the parameter $\mu$. The parameter $\mu$, is a mathematical parameter, used to determine the power assigned to each of the sub channels of the composite MIMO channel. After determining $\mu$, the square of the inverse of eigen values are compared with $\mu$.

If the square of the inverse of $i^{th}$ eigen value is greater than $\mu$, i.e.

$$\frac{1}{\lambda_i^2} \geq \mu$$  \hspace{1cm} (1)

then that $i^{th}$ eigen channel is too weak to be used for the communication process. The last two sub channels in the above figure of a (7-by-7) MIMO channel are such eigen channels which are not used for transmitting any signal at that point of time. Such channels are said to be switched off and they are put away from the communication process which means that those particular sub channels are not allocated with any transmitting power.

Once the total available power, $P_T$ and the gains of the parallel sub channels are known, the optimum power allocated to the $i^{th}$ sub channel is

$$P_i = \mu - \frac{1}{\lambda_i^2}$$  \hspace{1cm} (2)

If this quantity is positive then the power $P_i$ is allocated to the $i^{th}$ sub channel otherwise, the sub channel is left unused.

And the power allocated to each of these eigen channels, $P_i$ is determined by the waterfilling rule such that the following equations are satisfied

$$\frac{1}{\lambda_1^2} + P_1 = \frac{1}{\lambda_2^2} + P_2 = \frac{1}{\lambda_3^2} + P_3 = \frac{1}{\lambda_4^2} + P_4 = \ldots \frac{1}{\lambda_m^2} + P_m = \mu$$  \hspace{1cm} (6)

$$P_T = P_1 + P_2 + P_3 + P_4 + \ldots + P_m$$  \hspace{1cm} (7)

$$P_T = \sum (\mu - \frac{1}{\lambda_i^2})$$  \hspace{1cm} (8)

4. The water filling parameter $\mu$ is determined next part.

$$\frac{1}{\lambda_1^2} + P_1 = \mu$$
$$\frac{1}{\lambda_2^2} + P_2 = \mu$$
$$\frac{1}{\lambda_3^2} + P_3 = \mu$$
$$\ldots$$
$$\frac{1}{\lambda_m^2} + P_m = \mu$$

Adding these equations we get

$$\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} + \frac{1}{\lambda_3^2} + \frac{1}{\lambda_4^2} + \ldots \frac{1}{\lambda_m^2} + P_1 + P_2 + P_3 + P_4 + \ldots + P_m = m \mu$$  \hspace{1cm} (9)

$$\sum(\frac{1}{\lambda_i^2} + P_i) = m\mu$$  \hspace{1cm} (10)

$$\mu = \frac{\sum(\frac{1}{\lambda_i^2} + P_i)}{m}$$  \hspace{1cm} (11)

Flowchart showing these above mentioned steps has been placed in Figure 3.
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References


Author Profile

Rajneesh Talwar is Principal, CGC College of Engineering, Chandigarh, India
Flowchart for water filling algorithm

1. Start
2. Assume Water level $\mu$, Keeping in view Total Power Constraint
3. Find Channel Matrix $H$, which varies with time due to fading etc.
   $y = Hx + n$
   $y$ = Receiver Vector, $x$ = Transmitter Vector, $n$ = Noise Vector
4. Calculate gain of subchannel from $H$
5. Calculate inverse of gain of subchannel i.e. $1/H_i^{-2}$
6. Assume Water level $\mu$,
   Keeping in view Total Power Constraint
7. Determine Water Level:
   \[
   \frac{1}{x_1^2} + P_1 = \mu
   \]
   \[
   \frac{1}{x_2^2} + P_2 = \mu
   \]
   \[
   \frac{1}{x_3^2} + P_3 = \mu
   \]
   \[
   \vdots
   \]
   \[
   \frac{1}{x_m^2} + P_m = \mu
   \]
   Adding above eqns
   \[
   \frac{1}{x_1^2} + \frac{1}{x_2^2} + \frac{1}{x_3^2} + \ldots + \frac{1}{x_m^2} + P_1 + P_2 + P_3 + \ldots + P_m = m \mu
   \]
   \[
   \sum (\frac{1}{x_i^2} + P_i) = m \mu
   \]
   $\mu = \left(\sum (\frac{1}{x_i^2} + P_i)\right)/m$
8. Allocate Power to subchannels $P_i = \mu - 1/H_i^{-2}$
9. Allocate to subchannels till
   $P_1 + P_2 + P_3 + \ldots + P_m = \text{Total}$
10. Do not allocate Power to that sub channel

**Figure 3: Flowchart**