

# Gauss-PSO Parameter Identification Algorithm for Single-Phase Induction Motors

Duy C. Huynh

Ho Chi Minh City University of Technology, Ho Chi Minh City, Vietnam

**Abstract:** This paper proposes a new parameter identification approach for a single-phase induction motor (SPIM) whose parameters are usually obtained using several traditional techniques such as the DC, no-load, load and locked-rotor tests. It can be realized that the traditional techniques are complicated and require a higher cost with extra equipment. The proposal is based on using a Gauss particle swarm optimization (Gauss-PSO) algorithm. The Gauss-PSO algorithm modifies the algorithm parameters to improve the performance of the standard PSO algorithm. The algorithms use the experimental measurements of the currents and active powers in the SPIM main and auxiliary windings as the inputs to the parameter estimator. The experimental results obtained compare the identified SPIM parameters with the SPIM parameters achieved using the traditional tests. There is also a comparison of the solution quality between the standard PSO and Gauss-PSO algorithms. The results show that the Gauss-PSO algorithm is better than the standard PSO algorithm for parameter identification of the SPIM.

**Keywords:** parameter identification, induction motors, particle swarm optimization algorithms.

## 1. Introduction

Single-phase induction motors (SPIM) are very popular in the fractional horsepower range, especially in home appliances. This is due to the widespread availability of single-phase power supplies and the advantages of the SPIM such as the simple electrical and mechanical structures, ruggedness, high reliability and low cost. In most high performance SPIM applications, accurate knowledge of the SPIM parameters is necessary. These parameters are usually provided by the manufacturer. However, manufacturers usually do not supply all the parameter information. Thus, research about how to obtain the parameters of the SPIM is an important topic. In [1]-[3], the parameters of the SPIM are achieved through tests such as the DC, no-load and locked-rotor tests.

This paper proposes a new approach for parameter identification of the SPIM using stochastic optimization techniques. A Gauss particle swarm optimization (Gauss-PSO) algorithm is an advanced variant of the standard PSO algorithm which is one of the relatively new stochastic optimization techniques. It can be realized that the PSO algorithm does not have genetic operators such as selection, crossover between particles and mutation. This means that it is simpler and easier to implement than other evolutionary algorithms such as a genetic algorithm (GA) and evolution programming (EP) [4], as it only has a few parameters to adjust. Obviously, the PSO algorithm is suitable for this parameter identification application. The magnitudes of the complex currents and active powers in the SPIM main and auxiliary windings are the inputs of the estimator which can be easily measured.

The experimental results of the identified parameters using the Gauss-PSO algorithm is compared with the parameters obtained from the standard PSO algorithm and the load tests.

The remainder of this paper is organized as follows. The mathematical model for parameter identification of the SPIM is described in Section 2. A new proposal using the Gauss-PSO algorithm for parameter identification of a SPIM is presented in Section 3. The experimental results then follow to confirm the validity of the proposed application in Section 4. Finally, the advantages of the new application are summarized through comparison with related existing approaches.

## 2. Parameter Identification of a Single-Phase Induction Motor

### 2.1. Single-Phase Induction Motor Model

The equivalent circuit model of the SPIM using the double-revolving-field theory under steady-state conditions is given in Figure 1 [5]. The main winding voltages of the forward and backward-rotating fields from Figure 1 are:

$$\dot{V}_{mf} = \left( \frac{R_{1m} + jX_{1m}}{2} + \frac{R_{1a} + jX_{1a}}{2a^2} + Z_f \right) \dot{I}_{mf} - \frac{1}{2} \left( \frac{R_{1a} + jX_{1a}}{a^2} - R_{1m} - jX_{1m} \right) \dot{I}_{mb} \quad (1)$$

$$\dot{V}_{mb} = \left( \frac{R_{1m} + jX_{1m}}{2} + \frac{R_{1a} + jX_{1a}}{2a^2} + Z_b \right) \dot{I}_{mb} - \frac{1}{2} \left( \frac{R_{1a} + jX_{1a}}{a^2} - R_{1m} - jX_{1m} \right) \dot{I}_{mf} \quad (2)$$

Simpler forms of (1) and (2) are as follows:

$$\dot{V}_{mf} = Z_{11} \dot{I}_{mf} - Z_{12} \dot{I}_{mb} \quad (3)$$

$$\dot{V}_{mb} = -Z_{12} \dot{I}_{mf} + Z_{22} \dot{I}_{mb} \quad (4)$$

where

$$\dot{V}_{mf} = \frac{1}{2} \left( \dot{V}_m - j \frac{\dot{V}_a}{a} \right) \quad (5)$$

$$\dot{V}_{mb} = \frac{1}{2} \left( \dot{V}_m + j \frac{\dot{V}_a}{a} \right) \quad (6)$$

$\dot{V}_m$  and  $\dot{V}_a$  are the complex voltages of the main and auxiliary windings.

$\dot{V}_{mf}$ ,  $\dot{V}_{mb}$ ,  $\dot{I}_{mf}$ , and  $\dot{I}_{mb}$  are the forward and backward complex voltage and current components of the main winding.

$R_{1m}$ ,  $X_{1m}$ ,  $R_{1a}$ , and  $X_{1a}$  are the stator resistance and reactance of the main and auxiliary windings.

$R_2$  and  $X_2$  are the rotor resistance and reactance, referred to the stator.

$X_m$  is the magnetizing reactance, referred to the stator.

$a$  is the ratio between the number of turns of the auxiliary winding,  $N_a$ , and the number of turns of the main winding,  $N_m$ .

$s$  is the slip with respect to the forward-rotating field component.

By using (3) and (4), the forward- and backward-rotating field components of the main winding complex current are:

$$\dot{I}_{mf} = \frac{\dot{V}_{mf}Z_{22} + \dot{V}_{mb}Z_{12}}{Z_{11}Z_{22} - Z_{12}^2} \quad (7)$$

$$\dot{I}_{mb} = \frac{\dot{V}_{mf}Z_{12} + \dot{V}_{mb}Z_{11}}{Z_{11}Z_{22} - Z_{12}^2} \quad (8)$$

Then, the main and auxiliary winding complex currents are:

$$\dot{I}_m = \dot{I}_{mf} + \dot{I}_{mb} = |\dot{I}_m| \angle \theta_m \quad (9)$$

$$\dot{I}_a = \dot{I}_{af} + \dot{I}_{ab} = j \frac{\dot{I}_{mf}}{a} - j \frac{\dot{I}_{mb}}{a} = |\dot{I}_a| \angle \theta_a \quad (10)$$

where

$\dot{I}_m$ ,  $|\dot{I}_m|$ ,  $\dot{I}_a$ , and  $|\dot{I}_a|$  are the complex and magnitude components of the main and auxiliary winding currents.

$|\dot{I}_{mf}|$ ,  $|\dot{I}_{mb}|$ ,  $|\dot{I}_{af}|$ , and  $|\dot{I}_{ab}|$  are the magnitudes of the forward- and backward-rotating field components of the main and auxiliary winding currents.

$\theta_m$  and  $\theta_a$  are the phase angles of the main and auxiliary winding complex currents.

The active powers of the main and auxiliary windings are:

$$P_m = \text{Re}(\dot{V}_m \dot{I}_m) \quad (11)$$

$$P_a = \text{Re}(\dot{V}_a \dot{I}_a) \quad (12)$$

where

$P_m$  and  $P_a$  are the active powers of the main and auxiliary windings.

## 2.2. Parameter Identification

The parameter identification technique used in this paper is based on the output error method [6] which compares the response between the real system and an identified parameter model using the same inputs. The real system vectors are defined for parameter identification of the SPIM as follows:

$$y = \begin{bmatrix} |\dot{I}_m| & |\dot{I}_a| & P_m & P_a \end{bmatrix} \quad (13)$$

$$\theta = [X_{1m} \quad X_{1a} \quad R_2 \quad X_2 \quad X_m \quad a] \quad (14)$$

$$u = [\dot{V}_m \quad \dot{V}_a] \quad (15)$$

where

$|\dot{I}_m|$ ,  $|\dot{I}_a|$ ,  $P_m$ , and  $P_a$  are the variables sampled and recorded from the SPIM.

$X_{1m}$ ,  $X_{1a}$ ,  $R_2$ ,  $X_2$ ,  $X_m$ , and  $a$  are the actual parameters of the SPIM.

$\dot{V}_m$  and  $\dot{V}_a$  are the input variables in both the real system and the identified parameter model.

The identified system vectors are defined as follows:

$$\hat{y} = \begin{bmatrix} |\hat{I}_m| & |\hat{I}_a| & \hat{P}_m & \hat{P}_a \end{bmatrix} \quad (16)$$

$$\hat{\theta} = [\hat{X}_{1m} \quad \hat{X}_{1a} \quad \hat{R}_2 \quad \hat{X}_2 \quad \hat{X}_m \quad \hat{a}] \quad (17)$$

where

$|\hat{I}_m|$ ,  $|\hat{I}_a|$ ,  $\hat{P}_m$ , and  $\hat{P}_a$  are the variables which are calculated using (9)-(12).

$\hat{X}_{1m}$ ,  $\hat{X}_{1a}$ ,  $\hat{R}_2$ ,  $\hat{X}_2$ ,  $\hat{X}_m$ , and  $\hat{a}$  are the identified parameters of the SPIM.

Eventually, the fitness function for parameter identification of the SPIM is then defined as follows:

$$F(\hat{\theta}) = \frac{1}{N} \sum_{i=1}^N \left\{ \left( |\dot{I}_m| - |\hat{I}_m| \right)^2 + \left( |\dot{I}_a| - |\hat{I}_a| \right)^2 + \left( P_m - \hat{P}_m \right)^2 + \left( P_a - \hat{P}_a \right)^2 \right\} \quad (18)$$

where  $N$  is the series of measurement samples.

The fitness function  $F(\hat{\theta})$  depends on  $\hat{\theta}$  and obtains its minimum, when  $\hat{\theta} = \theta$ . In this case, the parameter identification problem is considered as the following optimization problem:

$$\text{Min}_{\hat{\theta}} F(\hat{\theta}) \quad (19)$$

In this application, it is assumed that the stator resistances of the main and auxiliary windings,  $R_{1m}$  and  $R_{1a}$ , are obtained by using the DC test.

## 3. Gauss Particle Swarm Optimization Parameter Identification Algorithm

It can be realized that the fitness function (18) of the parameter identification problem is non-linear and has many local optima. In order to determine the global optimum for parameter identification, stochastic optimization techniques, known as the standard PSO and Gauss-PSO algorithms are proposed to solve this problem.

The PSO algorithm is a population-based stochastic optimization method which was developed by Eberhart and Kennedy in 1995 [7]. The algorithm was inspired by the social behaviors of bird flocks, colonies of insects, schools of fishes and herds of animals. The algorithm starts by initializing a population of random solutions called particles and searches for optima by updating generations through the following velocity and position update equations.

However, for local optima problems, the particles sometimes become trapped in undesired states during the evolution process which leads to the loss of the exploration abilities.

Because of this disadvantage, premature convergence can happen in the standard PSO algorithm which affects the performance of the evolution process. This is one of the major drawbacks of the standard PSO algorithm. In order to improve the evolution process performance and solution quality for parameter identification of the SPIM, the Gauss-PSO algorithm is proposed for this application

The Gauss-PSO algorithm is a combination between the standard PSO algorithm and Gauss map which was presented in [8]-[12], where the Gauss map is described as a stochastic and unpredictable process in a deterministic non-linear system.

A Gauss map is given by [8]:

$$X_{k+1} = \begin{cases} 0, & X_k = 0 \\ \text{Frac}\left(\frac{1}{X_k}\right) = \frac{1}{X_k} \bmod(1), & X_k \in (0, 1) \end{cases} \quad (20)$$

where the fractional part  $\text{Frac}\left(\frac{1}{X_k}\right)$  of a number  $\frac{1}{X_k}$  is usually expressed using the modulo function, written as  $\frac{1}{X_k} \bmod(1)$  [8].

Additionally,  $\frac{1}{X_k} \bmod(1)$  is described as follows [8].

$$\frac{1}{X_k} \bmod(1) = \frac{1}{X_k} - \left\lfloor \frac{1}{X_k} \right\rfloor \quad (21)$$

where

$\lfloor z \rfloor$  is the largest integer less than  $z$  and acts as a shift on the continued fractional representation of numbers.

$X_k$  is the  $k$ th chaotic number,  $X_k \in (0, 1)$  with the following initial condition:  $X_1$  is a random number in the interval of  $(0, 1)$ .

The sequences are generated by using the Gauss map. These sequences have the characteristics of randomness, ergodicity and regularity, so that no state is repeated. The Gauss map sequences are recently considered as sources of random sequences which can be adopted instead of normally generated random sequences.

For the standard PSO algorithm, one of its main disadvantages is premature convergence, especially in local optima problems. Thus, in order to overcome this, the algorithm parameter sequences with a randomness-based choice are substituted by the Gauss map. In this case, the Gauss map sequences are obviously an appropriate tool to support the standard PSO algorithm so that it avoids getting stuck in a local optimum during the search process and overcomes the premature convergence phenomenon present in the standard PSO algorithm.

For the Gauss-PSO algorithm, the position and velocity update equation is written as follows:

$$x_i(k+1) = x_i(k) + v_i(k+1) \quad (22)$$

$$v_i(k+1) = w_k v_i(k) + c_1 r_k^1 (pbest_i(k) - x_i(k)) + c_2 r_k^2 (gbest(k) - x_i(k)) \quad (23)$$

$$1 \leq i \leq M \text{ and } 1 \leq k \leq n$$

where

$w_k$ ,  $r_k^1$  and  $r_k^2$  are the Gauss maps.

In this case, the acceleration coefficients,  $c_1$  and  $c_2$  are set to 2.

In this parameter identification application, there are six parameters,  $X_{1m}$ ,  $X_{1a}$ ,  $R_2$ ,  $X_2$ ,  $X_m$  and  $a$  which need to be identified. Each particle is treated as a point in a 6-dimensional space. The magnitudes of the complex currents in the main and auxiliary windings,  $|\hat{I}_m|$  and  $|\hat{I}_a|$ , as well as

the active powers in the main and auxiliary windings,  $\hat{P}_m$  and  $\hat{P}_a$ , in the identified parameter model are then

calculated using (9)-(12), whereas the magnitudes of the complex currents in the main and auxiliary windings,  $|\hat{I}_m|$

and  $|\hat{I}_a|$ , as well as the active powers in the main and

auxiliary windings,  $P_m$  and  $P_a$ , are sampled and recorded

from the real SPIM. The fitness function (18) is then used

together with  $|\hat{I}_m|$ ,  $|\hat{I}_a|$ ,  $\hat{P}_m$ ,  $\hat{P}_a$ ,  $|\hat{I}_m|$ ,  $|\hat{I}_a|$ ,  $P_m$  and  $P_a$

to search for the best position for the  $i$ th particle and the best

position of the swarm. This search process is repeated until

the user-defined end criterion is satisfied. In this parameter

identification problem, the end criterion is that the  $k$ th

current iteration number reaches the maximum iteration

number,  $n$ th.

The solution of the parameter identification is eventually:

$$\left[ \hat{X}_{1mi} \dots \hat{a}_i \right] = \left[ gbest_{X_{1m}}(n) \dots gbest_a(n) \right] \quad (24)$$

In this application, the inertia weight,  $w$ , is set to 0.9; the two independent random sequences,  $r_1$  and  $r_2$ , are uniformly distributed in  $U(0, 1)$ .

#### 4. Experimental Results

The practical experiments are implemented with a capacitor-start SPIM. The stator resistances of the main and auxiliary windings are obtained using the DC test,  $R_{1m} = 0.382$  p.u and  $R_{1a} = 0.248$  p.u. The data acquisition process for the inputs of the estimator is based on four tests. The first two tests were made with a capacitor of 40  $\mu$ F and voltages of 80 V and 100 V. The remaining two tests were made with a capacitor of 20  $\mu$ F and voltages of 80 V and 100 V. The input values of the capacitors and voltages are the same for the real system and the identified parameter model. Then, the outputs of the real system, including the currents and active powers of the main and auxiliary windings, are sampled and recorded. The measurement data processing and parameter identification are performed in MATLAB. In all algorithms, the particle number of a generation is 70 and the maximum iteration number is set to 300. Each algorithm is run 50 times.

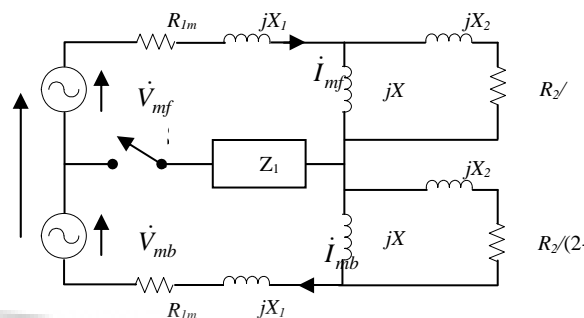
There are several differences which exist between the standard PSO and Gauss-PSO algorithms such as the initialization of the particles' positions and velocities, the inertia weight and the two independent random sequences in the velocity update equation of the Gauss-PSO algorithm. These enhance both the convergence speed and value of the Gauss-PSO algorithm. The convergence value of the standard PSO algorithm is 0.011402 whereas the convergence value of the Gauss-PSO algorithm is 0.00178 in Table 1. The standard PSO and Gauss-PSO algorithms are converged by the 39th and 17th iteration steps in Table 1 respectively. This means that both the convergence speed and value of the Gauss-PSO algorithm are better than the standard PSO algorithm. All features in the Gauss-PSO algorithm have improved the performance as well as avoiding premature convergence in the standard PSO algorithm. Table 2 shows the experimental results of the identified parameters when using the standard PSO and Gauss-PSO algorithms whereas Table 3 shows the error percentages of the identified parameters by using the two algorithms. Additionally, it can be realized that the errors produced by the Gauss-PSO algorithm is always less than 5% and less than the errors achieved when using the standard PSO algorithm. This shows that the Gauss-PSO algorithm is better than the standard PSO algorithm for parameter identification of the SPIM.

**5. Conclusion**

In this paper, the Gauss-PSO algorithm has been proposed for parameter identification of the SPIM. The experimental results of the identified parameters obtained were compared with the SPIM parameters achieved using the load tests. The experimental results of the identified parameters obtained were compared with the SPIM parameters achieved using the load tests. Additionally, the results of the identified parameters using the standard PSO and Gauss-PSO algorithms were also compared. The results confirm the benefits of the Gauss-PSO algorithm. The errors produced by the new algorithm are always less than 5% and less than the errors obtained when using the standard PSO algorithm for parameter identification of the SPIM.

**6. Future Works**

It can be realized that this proposal has been developed assuming steady-state operation of the single-phase IM. Therefore, it would be useful to further extend the research for transient conditions. Furthermore, it is assumed that no measurement noise is available in the parameter identification application. Thus, it would be also useful to examine this effect in future research.



**Figure 1:** Equivalent circuit model of a SPIM

**Table 1:** Convergence value and converged iteration step number of the standard PSO and Gauss-PSO algorithms for parameter identification of the SPIM

Index	Standard PSO	Gauss-PSO
Convergence value	0.011402	0.00178
Iteration step number	39	17

**Table 2:** Identified parameter values for parameter identification of the SPIM

Parameter (p.u)	Load tests	Standard PSO	Gauss-PSO
X <sub>1m</sub>	0.439	0.425	0.436
X <sub>1a</sub>	0.230	0.223	0.229
R <sub>2</sub>	0.070	0.086	0.068
X <sub>2</sub>	0.279	0.285	0.276
X <sub>m</sub>	1.848	1.855	1.862
A	0.409	0.399	0.417

**Table 3:** Percentage errors of the identified parameters for parameter identification of the SPIM

Error percentage (%)	Standard PSO	Gauss-PSO
Error percentage of X <sub>1m</sub> (%)	3.19	0.68
Error percentage of X <sub>1a</sub> (%)	3.04	0.43
Error percentage of R <sub>2</sub> (%)	22.86	2.86
Error percentage of X <sub>2</sub> (%)	2.15	1.08
Error percentage of X <sub>m</sub> (%)	0.38	0.76
Error percentage of a (%)	2.44	1.96

**References**

- [1] F. W. Suhr, "Towards an accurate evaluation of single-phase induction motor constants," Trans. American Institute of Electrical Engineers on Power Apparatus and Systems, Vol. 71, Issue 1, pp. 221-227, 1952.
- [2] C. V. D. Merwe and F. S. V. D. Merwe, "A study of methods to measure the parameters of single-phase induction motors," IEEE Trans. Energy Conversion, Vol. 10, Issue 2, pp. 248-253, 1995.
- [3] O. Ojo, O. Omozusi, M. Omoigui, and A. A. Jimoh, "Parameter estimation of single-phase induction machines," 36th IEEE Industry Applications Conf., IAS 2001, USA, pp. 2280-2287, 2001.
- [4] R. K. Ursem and P. Vadstrup, "Parameter identification of induction motors using stochastic optimization algorithms," J. Applied Soft Computing, Vol. 4, Issue 1, pp. 49-64, 2004.

- [5] G. McPherson and R. D. Laramore, An Introduction to Electrical Machines and Transformers, John Wiley & Son, 1990.
- [6] A. B. Proco and A. Keyhani, "Induction motor parameter identification from operating data electric drive applications," Proc. 18th Digital Avionics Systems Conf., pp. 1–6, 1999.
- [7] J. Kennedy and R. Eberhart, "Particle swarm optimization," Proc. IEEE Int. Conf. Neural Networks, Perth, Australia, pp. 1942–1948, 1995.
- [8] B. Alatas, E. Akin, and A. B. Ozer, "Chaos embedded particle swarm optimization algorithms," J. Chaos, Solitons & Fractals, Vol. 40, Issue 4, pp. 1715–1734, 2009.
- [9] B. Liu, L. Wang, Y. H. Jin, F. Tang, and D. X. Huang, "Improved particle swarm optimization combined with chaos," J. Chaos, Solitons & Fractals, Vol. 25, Issue 5, pp. 1261–1271, 2005.
- [10] H. J. Meng, P. Zheng, R. Y. Wu, X. J. Hao, and Z. Xie, "A hybrid particle swarm algorithm with embedded chaotic search," Proc. IEEE Conf. Cybernetics and Intelligent Systems, Singapore, pp. 367–371, 2004.
- [11] Y. Feng, G. F. Teng, A. X. Wang, and Y. M. Yao, "Chaotic inertia weight in particle swarm optimization," Proc. 2nd Int. Conf. Innovative Computing, Information and Control, ICICIC '07, pp. 475–478, 2007.
- [12] Y. Feng, Y. M. Yao, and A. X. Wang, "Comparing with chaotic inertia weights in particle swarm optimization," Proc. 6th Int. Conf. Machine Learning and Cybernetics, Hong Kong, pp. 329–333, 2007.

### Author Profile



**Duy C. Huynh** received the B.Sc. and M.Sc. degrees in electrical and electronic engineering from Ho Chi Minh City University of Technology, Ho Chi Minh City, Vietnam, in 2001 and 2005, respectively and Ph.D. degree from Heriot-Watt University, Edinburgh, U.K., in 2010. In 2001, he became a Lecturer at Ho Chi Minh City University of Technology. His research interests include the areas of energy efficient control and parameter estimation methods of induction machines.