Segmentation and Catheter Detection in Angiographic Images

R. Anu Prabha¹, M. Savitha², M. Dinesh³

¹Cheran College of Engineering, K. Paramathi, Karur

²Cheran College of Engineering, K.Paramathi, Karur

³Cheran College of Engineering, K. Paramathi, Karur

Abstract: Segmentation of coronary arteries in angiography images is a fundamental tool to evaluate arterial diseases and choose proper coronary treatment. The accurate segmentation of coronary arteries has become an important topic for the registration of different modalities, which allows physicians rapid access to different medical imaging information from Computed Tomography (CT) scans or Magnetic Resonance Imaging (MRI). An accurate fully automatic algorithm based on Graph-cuts for vessel centerline extraction, caliber estimation, and catheter detection is proposed. Vesselness, geodesic paths, and a new multiscale edgeness map are combined to customize the Graph-cuts approach to the segmentation of tubular structures, by means of a global optimization of the Graph-cuts energy function. A novel supervised learning methodology that integrates local and contextual information is proposed for automatic catheter detection.

Keywords: Coronary artery, Computed Tomography, Graph cut algorithm, Adaboost classifier, Catheter detection.

1. Introduction

Coronary Artery Disease (CAD) is a complex disease that causes reduced or absent blood flow in one or more of the arteries that encircle and supply the heart. The disease may be focal or diffuse. Apart from rare congenital anomalies (birth defects), CAD is usually a degenerative disease, uncommon as a clinical problem before the age of 30 years and common by the age of 60 years. One in four people will have a heart attack. The first recognized symptom may be death. The term coronary is derived from crown, referring to the way these arteries sit on the heart. When the heart has inadequate blood supply, pressure may be felt in the chest that moves to the left arm; one may feel weak, sweaty, or short of breath or nauseated; palpitations (ie, change in heart rhythm) may occur; or there may be a sensation of pressure or tightness just in the chest, neck, or arms.

Coronary artery disease (CAD) has become one of the mostly occurring diseases in the world and has increasing trend in its incidence in future. It is the cause of 20-30% of deaths in most industrialised countries. According to WHO (World Health Organisation) report, CAD has become a modern epidemic. It is estimated that by 2010 India is expected to have 60% of world's heart disease and that in India problems of heart are increasing in younger generation. According to recent research at least 20 million people suffer from heart attacks. In India it is also reported that CAD tends to occur earlier in life in Indians than other ethnic groups. CAD has multi-factorial aetiology with many of the risk factors influenced by lifestyle.CAD is the result of accumulation of plaques within the coronary arteries. These arteries supply the myocardium (the muscle of heart) with oxygen-rich blood. The plaque is made up of fat, cholesterol, calcium and other substances. This condition is called Atherosclerosis. When the plaque is growing it narrows the lumen of the coronary arteries. Consequently the blood flow to the heart muscle decreases. This causes a discomfort or a pain. The pain may be felt in chest, neck, jaws, abdomen, arms and shoulder also called Angina. During Angina the amount of the oxygenated blood flow decreases. But as the disease progresses the lumen of coronary arteries goes on narrowing due to the increased size of plaque. Hence the amount of blood supplied to this tissue becomes inadequate to supply the needs of the tissue. This condition is called myocardial ischemia and the tissues do not work at its fullest capacity. When the lumen of coronary artery has near-complete blockage, severely restricting the flow of oxygenated blood, the tissue in the areas of myocardium dies leading to myocardial infarction (particularly known as heart attack) which also accounts for sudden death. Although CAD has now become much familiar disease, but death rate is high due to the lack of awareness among the common people. Generally the patients neglect the initial symptoms and they only consult medical experts when those symptoms become severe or critical. But at that time the treatment becomes complicated and sometimes due to the acuteness and severity of the disease, the patients die before getting proper medication. Another situation is that there is not enough facility to diagnosis in the countryside like India and treat the patients soon.

In order to circumvent such a burning problem one of the possible solutions is to make the people aware of their respective CAD risks in advance and take preventive measures accordingly. It is only possible when an early detection of CAD occurs. According to the medical experts an early detection at the stage of angina may prevent the death due to CAD if proper medication is given thereafter. Here lies the importance of developing a CAD screening system.

Quantitative Coronary angiography has become an invaluable tool for the interventional cardiologist, providing objective and reproducible measurements of coronary artery

dimensions, which can be used to study progression or regression of coronary atherosclerosis, as well as the immediate and long term effects of percutaneous interventions.Coronary lesions can be assessed qualitatively and quantitatively, in terms of severity of the lesion itself. The qualitative evaluation is based on the visual estimation of the lesion and it depends very much on the operator's experience. The quantitative evaluation, based on QCA, allows instead obtaining numeric parameters that are much more independent from the operator acquiring the images or the one performing the analyses.

2. Related Work

Quantitative Coronary Angiography (QCA) tools are used by clinicians on daily basis to evaluate the degree of coronary lesions and proceed with the proper intervention. Automatic enhancement and segmentation of vessel structures has become a basic tool to assist clinicians for a more accurate, fast, and objective patient data analysis. Vessel segmentation in angiography sequences is challenging to be highly reliable and fully automatic method.

Graph-cuts (GC) theory is to model vessel structures and obtain a globally optimal segmentation of the coronary tree in angiography images, achieving accurate detection of both the centerline and the vessel borders. One of the critical issues for GC is the design of proper energy terms to assure optimal analysis of local image structures and global segmentation solution. In particular, the original GC definition suffers from the shrinking bias drawback since it tends to produce small contours corresponding to the minimal cut. Hence, with the original GC energy formulation, it is not well suited to segment tubular structures and thin objects like blood vessels.

A novel GC energy functional tailored to the vessel segmentation problem. The novel energy formulation takes into account 1) the local vessel appearance, using a vesselness measure; 2) the local connectivity to other vessel regions, using geodesic paths; and 3) a measure of edgeness based on a new multiscale version of the adaptive Canny detector, which allows an accurate vessel boundary detection. In order to discriminate the catheter guide from vessels, define a set of appearance features and propose a supervised learning approach based on the multiscale stacked sequential learning.

Several methods that exploit photometric and structural properties of tubular structures have been proposed. An excellent review of basic geometrical features for tubular like structures can be found. Nonetheless, in the case of vessel segmentation in angiography sequences, the problem is still challenging; highly reliable, fully automatic methods are not established yet. Moreover, the accurate vessel calibre estimation is still a hot topic far from being solved, as demonstrated by the scale selection method proposed. An extensive overview of different methods for vessel extraction can be found. Recently, an interesting approach to vessel segmentation has been proposed, which fuses local features with local directional information; unfortunately, the authors do not provide a quantitative evaluation of their method. A method for scale selection that improves the caliber estimation is proposed earlier. Nonetheless, most works are based on local image analysis to extract vessels or employ an a priori model to help vessel extraction. In contrast, the GC technique is an optimal segmentation tool that combines local and contextual image information analysis by modeling relations between neighboring pixel. The goodness of the GC solution depends on the suitability of the energy terms and their reliable computation. It is worth to mention that GC suffers from the "shrinking bias" problem; since the energy function definition makes it proportional to the length of the boundary of the result, GC is biased to segment small, isotropic regions. This problem can be overcome in different ways. Some earlier papers addressed this problem by incorporating flux information in the GC framework. A geodesic distance term to the GC energy function, is computed from some strokes manually defined by the user as an initialization step of the segmentation. Differently than previous methods, this paper tailor the GC energy function in such a way that long and thin structures can be easily and automatically segmented and none of the previous methods incorporates local appearance, geodesic paths, and an edgeness measure in a compact, unified framework.

GC formulation defines the cost function E(L), which describes soft constraints imposed on boundary and region properties of L as

$$E(L) = U(L) + \lambda B(L)$$
(1)

the unary term is denoted as

$$U(L) = \sum U_i (L_i)$$
 (2)

and the boundary term as

$$B(L) = \sum B\{i,j\} \Omega(Li, Lj)$$
(3)

{i,j}**∈**N

i∈P

where the characteristic function $\Omega(\text{Li}, \text{Lj})$ is 0 if Li = Ljand 1, otherwise. The unary term U(L) is defined assuming that individual penalties for assigning pixel i to "fore" and "back" [i.e., Ui(fore) and Ui(back)] are given by foreground and background models. In this case, the foreground is the vessel denoted by "vess". The term B(L) comprises the boundary properties of segmentation L. Any $B\{i,j\} \ge 0$ should be interpreted as a penalty for a discontinuity between i and j. Finally, the coefficient $\lambda \in \mathbb{R}^+$ specifies the relative importance of the boundary term against the unary term. The GC algorithm imposes hard constraints on the segmentation result by means of the definition of seed points where labels are predefined and cannot be modified. The notations $V \subset P$, $B \subset P$ refer to the subsets of vessel and background seeds, respectively. Boykov and Funka- Lea show how to efficiently compute the global minimum of E(L) among all segmentations L satisfying the hard constraints $\forall i \in V$, Li = "vess", $\forall i \in B$, Li = "back" using a minimum cut algorithm on a graph defined by nodes and edges, which are image pixels and pixel relations, respectively. Let us describe the details of the graph created to segment an image. A graph G = (Y, E) is created with nodes Y corresponding to pixels $i \in P$ of the image. There are two additional nodes: the foreground terminal (source S)

and the background terminal (sink T); therefore, $Y = P \cup \{S, T\}$. The set of edges E consists of two types of undirected edges: n-links (neighborhood links) and t-links (terminal links). Each pixel i has two t-links {i, S} and {i, T} connecting it to each terminal. Each pair of neighboring pixels {i, j} in N is connected by an n-link. Without introducing any ambiguity, an n-link connecting a pair of neighbors i and j will be denoted by {i, j}, giving

 $E = N \text{ } \acute{U} \{\{i, S\}, \{i, T\}\} (4)$ $i \in P$

Graph G is completely defined when assigning weights to the edges, where $K = 1 + \max_{i \in P} (\sum_{\{i,j\} \in N} B\{i,j\})$..Estimate the vesselness V at every pixel *i* as V (i) = max $s \in \{s(1), ..., s(Q)\} V(i, s)$. By merely its appearance, the catheter is not easily distinguishable from arteries. In order to discriminate between vessels and catheter, we propose a classification method based on suitable catheter features, where every CL path is considered as a 1-D object in the 2-D image plane. The algorithm is based on the multiscale stacked sequential learning, which is divided into two steps: first, a pointwise classification method is performed using an Adaboost classifier with decision stumps, and second, contextual information is extracted and used as input for another classifier, in order to refine the previous results using contextual information.

3. Proposed Method



Figure 1: Proposed method Flow Diagram

Graph cut algorithm is used for centerline ,diameter and boundary of vessel estimation and pointwise classification using adaboost classifier is used for catheter detection. GC energy functional tailored to the vessel segmentation problem. The novel energy formulation takes into account 1)the local vessel appearance, using a vesselness measure

- 2) the local connectivity to other vessel regions, using geodesic paths
- 3)a measure of edgeness based on a new multiscale version of the adaptive Canny detector ,which allows an accurate vessel boundary detection.

3.1 Preprocessing

The rough images obtained from real time cameras are not used for further processing. So, preprocessing methods should be employed. The preprocessing methods used here Gaussian filtering (image smoothening). Gaussian filtering is a pre-processing level in order to enhance structures in the images at various scales.

3.2 Vessel segmentation and Catheter extraction

3.2.1 Geodesic Distance

In graph theory, the distance between two vertices in a graph is the number of edges in a shortest path connecting them. This is also known as the geodesic distance because it is the length of the graph geodesic between those two vertices. If there is no path connecting the two vertices, i.e., if the edges belong to different connected components, then conventionally the distance is defined as infinite.

The diameter d of a graph is the maximum eccentricity of any vertex in the graph. That is, it is the greatest distance between any pair of vertices. To find the diameter of a graph, first find the shortest path between each pair of vertices. The greatest length of any of these paths is the diameter of the graph.

3.2.2 Dijkstra's shortest path algorithm

Dijkstra's shortest path algorithm is a graph search algorithm, which computes short paths in a graph with nonnegative edge weights. For a given source vertex (node) in the graph, the algorithm finds the path with lowest cost (i.e. the shortest path) between that vertex and every other vertex. It can also be used for finding costs of shortest paths from a single vertex to a single destination vertex by stopping the algorithm once the shortest path to the destination vertex has been determined. For example, if the vertices of the graph represent cities and edge path costs represent driving distances between pairs of cities connected by a direct road, Dijkstra's algorithm can be used to find the shortest path first is widely used in network routing protocols, most notably IS-IS and OSPF (Open Shortest Path First).

3.2.3 Canny edge detector

The Canny operator was designed to be an optimal edge.It takes an input as gray scale image, and produces output as image showing the positions of tracked intensity discontinuities. The Canny operator works in a multi-stage process. First of all the image is smoothed by Gaussian convolution. Then a simple 2-D first derivative operator is applied to the smoothed image to highlight the regions of the image with high spatial derivatives. Edges give rise to ridges in the gradient magnitude image. The algorithm then tracks along the top of these ridges and sets to zero all pixels that are not actually on the ridge top so as to give a thin line in the output, a process known as non-maximal suppression. The tracking process exhibits hysteresis controlled by two thresholds: T1 and T2, with T1 > T2. Tracking can only begin at a point on a ridge higher than T1. Tracking then continues in both directions out from that point until the height of the ridge falls below T2. This hysteresis helps to ensure that noisy edges are not broken up into multiple edge fragments. The effect of the Canny operator is determined by three parameters - the width of the Gaussian kernel used in the smoothing phase, and the upper and lower thresholds used by the tracker. Increasing the width of the Gaussian kernel reduces the detector's sensitivity to noise, at the expense of losing some of the finer detail in the image. The localization error in the detected edges also increases slightly as the Gaussian width is increased.

Usually, the upper tracking threshold can be set quite high, and the lower threshold quite low for good results. Setting the lower threshold too high will cause noisy edges to break up. Setting the upper threshold too low increases the number of spurious and undesirable edge fragments appearing in the output. The Gaussian smoothing in the Canny edge detector fulfills two purposes: first it can be used to control the amount of detail that appears in the edge image and second, it can be used to suppress noise. The Canny edge detector allows to find the intensity discontinuities in an image, it is not guaranteed that these discontinuities correspond to actual edges of the object.

3.2.4 Graph cut segmentation

Segmentation is an important part of image analysis. It refers to the process of partitioning an image into multiple segments. More precisely, image segmentation is the process of assigning a label to every pixel in an image such that pixels with the same label share certain visual characteristics. The goal of segmentation is to simplify and/or change the representation of an image into something that is more meaningful and easier to analyse. Segmentation by computing a minimal cut in a graph is a new and quite general approach for segmenting images. This approach guarantees global solutions, which always a best solution, and in addition these solutions are not depending on a good initialization. Graph theory is the study of graphs. A graph is an abstract representation of a set of objects, where several pairs of the objects are connected by links. It is a mathematical structure and is used to model pairwise relations between objects from a certain collection. In a graph G = (V, E), V and E denote the set of vertices and edges of G, respectively.

A weighted graph associates a positive label (weight) with every edge in the graph. A directed graph G consists of a set of vertices V and a set of ordered pairs of edges.

An s-t graph is a weighted directed graph with two identied nodes, the source s and the sink t. An s-t cut, c (s;t), in a graph G is a set of edges E cut such that there is no path from the source to the sink when E cut is removed from G. The cost of a cut E cut is the sum of the edge weights in E cut.

In graph theory, a cut is a partition of the vertices of a graph into two disjoint subsets that are joined by at least one edge. The cut-set of the cut is the set of edges whose end points are in different subsets of the partition. Edges are said to be crossing the cut if they are in its cut-set. In an unweighted undirected graph, the size or weight of a cut is the number of edges crossing the cut. In a weighted graph, the same term is defined by the sum of the weights of the edges crossing the cut.In a flow network, an s-t cut is a cut that requires the source and the sink to be in different subsets, and its cut-set only consists of edges going from the source's side to the sink's side. The capacity of an s-t cut is defined as the sum of capacity of each edge in the cut-set.

Graph-cut is an algorithm that finds a globally optimal segmentation solution. Energy-based segmentation methods can be distinguished by the type of energy function they use and by the optimization technique for minimizing it.Energy function plays a major role in segmentation using graph cuts.

A graph $G = \{V, E\}$ is defined as a set of nodes or vertices V and a set of edges E connecting "neighboring" nodes. A graph cut is the process of partitioning a directed or undirected graph into disjoint sets. The concept of optimality of such cuts is usually introduced by associating an energy to each cut.Graph cut methods have been successfully applied to stereo, image restoration, texture synthesis and image segmentation.

3.2.5 Non Maximal Suppression

Non-maximum Suppression technique is used to extract the centerline of the segmented vessel. Non-maximum supression is a technique combined with the edge detection algorithms it is used. Along the image gradient direction, the images are scanned and if pixels are not included in the local maxima portion those pixels are set to the value zero. This leads to the effect of suppressing the whole information of the images which are not included in local maxima portion. This level is said to be Non-maximum suppression. Then at this stage, edge points set is obtained, as a binary image. At some situation it is said to be thin edges. Non Maximum Suppression technique can set all the pixels in the present neighborhood window which are lesser than the maximum value in that current window to zero (or black). The technique is same as like the Max Filter. In Max Filter, the highest value for the mentioned window size (current ROI area) is evaluated. Then the current pixel is compared to the highest value. If it is lesser value, then it is set to black otherwise the value doesn't get changed.

3.3 Catheter Detection

3.3.1 Adaboost Classifier

Algorithm based on multi-scale stacked sequential learning(Sequential learning is the discipline of machine learning that deals with dependent data such that neighboring labels exhibit some kind of relationship), which includes pointwise classification method using adaboost classifier and contextual information extracted and refined by stacked classifier. A well known algorithm for classification includes Adaboost algorithm.

Adaboost expands as adaptive since it uses n number of iterations to produce a single strong learner. Adaboost generates the classifier which is well-correlated to the true classifier (i.e., strong learner) by consequently adding a classifier which is lightly correlated to the true classifier (i.e., weak classifier). During each and every stage of training the images, a newly produced weak learner is get included to the ensemble and adjust the weighting vector to impose on misclassified things in past iterations.

Adaptive Boosting (Adaboost) is a machine learning algorithm.It is a meta-algorithm, and can be used in conjunction with many other learning algorithms to improve their performance. AdaBoost is adaptive in the sense that subsequent classifiers built are tweaked in favour of those instances misclassified by previous classifiers.AdaBoost is sensitive to noisy data and outliers. In some problems, however, it can be less susceptible to the overfitting problem than most learning algorithms. The classifiers it uses can be weak (i.e., display a substantial error rate), but as long as their performance is slightly better than random (i.e. their error rate is smaller than 0.5 for binary classification), they will improve the final model. Even classifiers with an error rate higher than would be expected from a random classifier will be useful, since they will have negative coefficients in the final linear combination of classifiers and hence behave like their inverses.

Contextual means focusing on the relationship of the nearby pixels and misaligned pixels has been classified by the classifier. There are two main approaches to lesion detection by classification: One can either train a classifier to identify image subwindows containing lesions or classify single voxels as belonging to a lesion or not. Focus on difficult data points. The data points that have been misclassified most by the previous weak classifier. Focus on difficult data points. The data points that have been misclassified most by the previous weak classifier.

4. Simulation Result

4.1 Input image

The angiographic image is taken as input for vessel segmentation, centerline extraction and catheter detection.



Figure 2: Input image

4.2 Output images

4.2.1 Preprocessing

Input image is smoothened by Gaussian filter.



Figure 3: Filtered image

4.2.2 Vessel and Background separation

From the filtered output the Vessel structure and Background are separated, since further processing needs only the vessel structure.



Figure 4: Vessel and Background separation

4.2.3 Geodesic distance map

The geodesic distance map contains the distance of each pixel. The distance is the maximum difference of image gradients and the path is computed by Dijkstra's shortest path algorithm. The vessel region is represented in white and the background region is represented in black on Geodesic distance map. The unary term in Graph cut energy formulation is considered from Geodesic distance map



Figure 5: Geodesic Distance Map

4.2.4 Edgeness map

Canny edge detector algorithm is used to detect the edges. The Boundary term for Graph cut energy formulation is considered from Edgeness map.



Figure 6: Edgeness map

4.2.5 Graph cut Segmentation

By graph cut algorithm, the image is decomposed into N cuts. Each N cuts are taken as vertices and their edges have minimum edge weight. Eigen vectors and Eigen values are taken into account for vessel curvature determination.



Figure 7: Graph cut segmentation

4.2.6 Centerline extraction and Catheter detection The vessel centerline is extracted by using Non maximal suppression and the non maximal points are joined by Ridge traversal method. The catheter position is detected by distinguishing features on centerline by using Adaboost classifier.



Figure 8: Final segmentation, Centerline extraction and Catheter detection

5. Simulation Result

Segmentation of coronary arteries in angiographic images is a fundamental tool to evaluate arterial diseases and choose proper coronary treatment. An accurate segmentation of coronary artery allows the physician to access different medical imaging information.Segmentation of vessels in angiographic images is done by Graph cut algorithm on analysis of vessel appearance, local connectivity to other vessels(Geodesic distance) and edgeness detection by canny edge detection algorithm. The interventional tool to remove blockages is catheter. Catheter can be of detected using pointwise classification method using adaboost classifier.

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