Gravitational Lens

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Abstract: ***Out of so many properties of gravitation, gravitational lens is more important. It contains all aspects of gravitation. It has the power of prediction of gravitation of newly born stars, also predicts about the life of stars including the duration of being dwarf star or black hole. As gravitational pull of a star determines the size, strength and life, so, gravitational lens can view all dimension in its formation. The quadrupole moment (or the l-th time derivative of the l-th multipole moment) of an isolated system's stress-energy tensor must be nonzero in order for it to emit gravitational radiation. This is analogous to the changing dipole moment of charge or current necessary for electromagnetic radiation.***

Keywords: Gravitational lens, stress energy tensor, multipole moment, quadrupole, prediction, etc

1. Introduction

Power radiated by orbiting bodies

Gravitational waves carry energy away from their sources and, in the case of orbiting bodies, this is associated with an in spiral or decrease in orbit. Imagine for example a simple system of two masses — such as the Earth-Sun system — moving slowly compared to the speed of light in circular orbits. Assume that these two masses orbit each other in a circular orbit in the x-y plane. To a good approximation, the masses follow simple Keplerian orbits. However, such an orbit represents a changing quadrupole moment. That is, the system will give off gravitational waves.

Suppose that the two masses are m_1 and m_2, and they are separated by a distance r. The power given off (radiated) by this system is:

$$P = \frac{dE}{dt} = -\frac{32}{5}\frac{G^4}{c^5}\frac{(m_1 m_2)^2(m_1+m_2)}{r^5} \qquad \text{------(1.1)}$$

where G is the gravitational constant, c is the speed of light in vacuum and where the negative sign means that power is being given off by the system, rather than received. For a system like the Sun and Earth, r is about 1.5×10^{11} m and m_1 and m_2 are about 2×10^{30} and 6×10^{24} kg respectively. In this case, the power is about 200 watts. This is truly tiny compared to the total electromagnetic radiation given off by the Sun (roughly 3.86×10^{26} watts).

In theory, the loss of energy through gravitational radiation could eventually drop the Earth into the Sun. However, the total energy of the Earth orbiting the Sun (kinetic energy plus gravitational potential energy) is about 1.14×10^{36} joules of which only 200 joules per second is lost through gravitational radiation, leading to a decay in the orbit by about 1×10^{-15} meters per day or roughly the diameter of a proton. At this rate, it would take the Earth approximately 1×10^{13} times more than the current age of the Universe to spiral onto the Sun. This estimate overlooks the decrease in *r* over time, but the majority of the time the bodies are far apart and only radiating slowly, so the difference is unimportant in this example. In only a few billion years, the Earth is predicted to be swallowed by the Sun in the red giant stage of its life.

A more dramatic example of radiated gravitational energy is represented by two solar mass neutron stars orbiting at a distance from each other of 1.89×10^{8} m (only 0.63 light-seconds apart). [The Sun is 8 light minutes from the Earth.] Plugging their masses into the above equation shows that the gravitational radiation from them would be 1.38×10^{28} watts, which is about 100 times more than the Sun's electromagnetic radiation.

2. Orbital Decay from Gravitational Radiation

Gravitational radiation robs the orbiting bodies of energy. It first circularizes their orbits and then gradually shrinks their radius. As the energy of the orbit is reduced, the distance between the bodies decreases, and they rotate more rapidly. The overall angular momentum is reduced however. This reduction corresponds to the angular momentum carried off by gravitational radiation. The rate of decrease of distance between the bodies versus time is given by:[7]

$$\frac{dr}{dt} = -\frac{64}{5}\frac{G^4}{c^5}\frac{(m_1 m_2)(m_1+m_2)}{r^3} \qquad \text{-----------(2.1)}$$

where the variables are the same as in the previous equation.

The orbit decays at a rate proportional to the inverse third power of the radius. When the radius has shrunk to half its initial value, it is shrinking eight times faster than before. By Kepler's Third Law, the new rotation rate at this point will be faster by $\sqrt{8} = 2.828$, or nearly three times the previous orbital frequency. As the radius decreases, the power lost to gravitational radiation increases even more. As can be seen from the previous equation, power radiated varies as the inverse fifth power of the radius, or 32 times more in this case.

If we use the previous values for the Sun and the Earth, we find that the Earth's orbit shrinks by 1.1×10^{-20} meter per second. This is 3.5×10^{-13} m per year which is about 1/300 the diameter of a hydrogen atom. The effect of gravitational radiation on the size of the Earth's orbit is negligible over the age of the universe. This is not true for closer orbits.

A more practical example is the orbit of a Sun-like star around a heavy black hole. Our Milky Way has a 4 million solar-mass black hole at its center in Sagittarius A. Such supermassive black holes are being found in the center of almost all galaxies. For this example take a 2 million solar-mass black hole with a solar-mass star orbiting it at a radius of 1.89×10^{10} m (63 light-seconds). The mass of the black hole will be 4×10^{36} kg and its gravitational radius will be 6×10^{9} m. The orbital period will be 1,000 seconds, or a little under 17 minutes. The solar-mass star will draw closer to the black hole by 7.4 meters per second or 7.4 km per orbit. A collision will not be long in coming.

Assume that a pair of solar-mass neutron stars are in circular orbits at a distance of 1.89×10^{8} m (189,000 km). This is a little less than 1/7 the diameter of the Sun or 0.63 light-seconds. Their orbital period would be 1,000 seconds. Substituting the new mass and radius in the above formula gives a rate of orbit decrease of 3.7×10^{-6} m/s or 3.7 mm per orbit. This is 116 meters per year and is not negligible over cosmic time scales.

Suppose instead that these two neutron stars were orbiting at a distance of 1.89×10^{6} m (1890 km). Their period would be 1 second and their orbital velocity would be about 1/50 of the speed of light. Their orbit would now shrink by 3.7 meters per orbit. A collision is imminent. A runaway loss of energy from the orbit results in an ever more rapid decrease in the distance between the stars. They will eventually merge to form a black hole and cease to radiate gravitational waves. This is referred to as the in spiral.

The above equation cannot be applied directly for calculating the lifetime of the orbit, because the rate of change in radius depends on the radius itself, and is thus non-constant with time. The lifetime can be computed by integration of this equation (see next section).

3. Orbital Lifetime Limits from Gravitational Radiation

Orbital lifetime is one of the most important properties of gravitational radiation sources. It determines the average number of binary stars in the universe that are close enough to be detected. Short lifetime binaries are strong sources of gravitational radiation but are few in number. Long lifetime binaries are more plentiful but they are weak sources of gravitational waves. LIGO is most sensitive in the frequency band where two neutron stars are about to merge. This time frame is only a few seconds. It takes luck for the detector to see this blink in time out of a million year orbital lifetime. It is predicted that such a merger will only be seen once per decade or so.

The lifetime of an orbit is given by

$$t = \frac{5}{256}\frac{c^5}{G^3}\frac{r^4}{(m_1 m_2)(m_1 + m_2)} \quad \text{------(3.1)}$$

Where, r is the initial distance between the orbiting bodies. This equation can be derived by integrating the previous equation for the rate of radius decrease. It predicts the time for the radius of the orbit to shrink to zero. As the orbital speed becomes a significant fraction of the speed of light, this equation becomes inaccurate. It is useful for inspirals until the last few milliseconds before the merger of the objects.

Substituting the values for the mass of the Sun and Earth as well as the orbital radius gives a very large lifetime of 3.44×10^{30} seconds or 1.09×10^{23} years (which is approximately 10^{15} times larger than the age of the universe). The actual figure would be slightly less than that. The Earth will break apart from tidal forces if it orbits closer than a few radii from the sun. This would form a ring around the Sun and instantly stop the emission of gravitational waves.

If we use a 2 million solar mass black hole with a solar mass star orbiting it at 1.89×10^{10} meters, we get a lifetime of 6.50×10^{8} seconds or 20.7 years.

Assume that a pair of solar mass neutron stars with a diameter of 10 kilometers is in circular orbits at a distance of 1.89×10^{8} m (189,000 km). Their lifetime is 1.30×10^{13} seconds or about 414,000 years. Their orbital period will be 1,000 seconds and it could be observed by LISA if they were not too far away. A far greater number of white dwarf binaries exist with orbital periods in this range. White dwarf binaries have masses on the order of our Sun and diameters on the order of our Earth. They cannot get much closer together than 10,000 km before they will merge and cease to radiate gravitational waves. This results in the creation of either a neutron star or a black hole. Until then, their gravitational radiation will be comparable to that of a neutron star binary. LISA is the only gravitational wave experiment which is likely to succeed in detecting such types of binaries.

If the orbit of a neutron star binary has decayed to 1.89×10^{6}m (1890 km), its remaining lifetime is 130,000 seconds or about 36 hours. The orbital frequency will vary from 1 revolution per second at the start and 918 revolutions per second when the orbit has shrunk to 20 km at merger. The gravitational radiation emitted will be at twice the orbital frequency. Just before merger, the in spiral can be observed by LIGO if the binary is close enough. LIGO has only a few minutes to observe this merger out of a total orbital lifetime that may have been billions of years. Its chances of success are quite low despite the large number of such mergers occurring in the universe. No mergers have been seen in the few years that LIGO has been in operation. It is thought that a merger should be seen about once per decade of observing time.

4. Discussion & Result

So, gravitational lens predicts the all possible dimension of newly born star, its life cycle and finally the ultimate end. A gravitational radiation source determines the average number of binary stars in the universe that are close enough to be detected. Short lifetime binaries are strong sources of gravitational radiation but are few in number. . At this rate, it would take the Earth approximately 1×1013 times more than the current age of the Universe to spiral onto the Sun. This estimate overlooks the decrease in r over time, but the majority of the time the bodies are far apart and only radiating slowly, so the difference is unimportant in this example. In only a few billion years, the Earth is predicted to be swallowed by the Sun in the red giant stage of its life.

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Author Profile

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