Stochastic Model for the Cumulative Effects of Absenteeism-SCBZ Property

Dr. T. Chitrakalarani[1], A. Yogeswari[2]

[1]Associate Professor of Mathematics, K.N.G. Arts College for women (Aut), Thanjavur, India

[2]Department of Mathematics, Sengamala Thayaar Educational Trust Women's College, Mannargudi, India

Abstract: ***The aim of this paper is to obtain the expected time to reach the uneconomic status of an organization by assuming that the threshold variable is a random variable which satisfies the 'setting the clock back to zero (SCBZ)' property.***

Keywords: Absenteeism, cumulative damage model, SCBZ property

1. Introduction

Employee absenteeism ranks among the nations most widespread and costly human resource problems. Absenteeism is a real irritant for managers, forcing them to make alternative arrangements to cover for employees who do not come to work. The economic impact of employee absenteeism derives mainly from the costs of decreased productivity because of absence from work, less experienced replacement staff and the additional expense of hiring substitute labour. Employee's workload increases due to the absence of coworker. Their efficiency may not be like earlier. When substitutes are employed, such laborers have no proper training. They have to be trained suitably which again economically burdens the industrial units. In both the there is a loss of manpower which leads to delay in the work. Successive absenteeism of workers to the organization, change over from a more productive to less productive or high economic status to low economic status by losing more and more man hours in successive absenteeism.

Man power loss is an important aspect of study of Man power Planning. Cumulative damage process is related to the shock models in reliability theory. The basic idea is that accumulating random amount of man power loss due to successive absenteeism which leads to the uneconomic status of the organization when the total loss crosses the threshold level. Several absenteeism models [ ] have discussed the factors associated with absenteeism. Hameed, M.S.A. and F. Proschan [] discussed the shock models underlying the Birth Process. Raja Rao ,B.[] studied the life expectancy for a class of life distributions having the ScBZ property. Chithrakalarani, T. and Ganesan [ ] obtained the expected time to breakdown by assuming that the threshold variable has an exponential distribution. The aim of this paper is to obtain the expected time to reach the uneconomic status of an organization by assuming that the threshold variable is a random variable which satisfies the 'setting the clock back to zero (SCBZ)' property.

2. The Model

2.1 Assumptions

1) Absenteeism occurs at k random epochs and at every epoch the organization has a random loss in manhours
2) Each absenteeism causes a random loss in manhours and the losses on successive absenteeism are i.i.d. random variables
3) The process of loss is linear and cumulative
4) The inter-occurrence times between the successive absenteeism are i.i.d. random variables
5) If the total man hours loss exceeds a random threshold level Y, the organization faces the uneconomic status
6) The process generating the sequence of losses and thresholds are mutually independent

2.2. Notations

- Xi - a continous random variable denoting the amount of man hours loss to the Organization due to absenteeism on the i th occasion and X^i are i. i. d.
- Y - a continuous random variable denoting the threshold having SCBZ property
- g(.)- the probability density function of X
- g_k(.)- the k fold convolution of g (.)
- T - a continuous random variable denoting the time to breakdown point of an Organization or time to cross the threshold level
- τ_0 - truncation point of the random variable Y
- g*(.) - Laplace transform of g(.)
- g_k*(.) - Laplace transform of g_k(.)
- h(.) - p.d.f. of the random threshold level which has SCBZ property and
- H(.) is the corresponding c.d.f.
- U_i - a continuous random variable denoting the interoccurence time between the (i-1)th and ith absenteeism
- f(.) - the p.d.f. of the random variable and the corresponding c.d.f.
- S(.) - the survival function
- $V_k(t)$ - probability that there are exactly k absenteeism occurring in (0,t]

2.3 Results

Let Y be the random variable which has c.d.f. defined as H(y)=1- $e^{-\theta y}$:y≤ τ_0 = 1 - $e^{-\theta 1}$ τ_0 . $e^{-\theta 2(y-\tau 0)}$: $\tau_0 < y$ (2.1)

Assuming the truncation level τ_0 itself being a random variable which follows exponential distribution with parameter λ

$$H(y) = 1 - \frac{\theta 1-\theta 2}{\lambda+\theta 1+\theta 2} e^{-(\theta 1+\lambda)y} - \frac{\lambda e^{-\theta 2}}{\lambda+\theta 1-\theta 2}$$
$$= 1 - pe^{-(\theta 1+\lambda)y} - qe^{-\theta 2} \quad (2.2)$$

Where $p = \frac{\theta 1-\theta 2}{\lambda+\theta 1+\theta 2}$ and $q = \frac{\lambda}{\lambda+\theta 1-\theta 2}$ such that p+q = 1

$$h(y) = p(\theta_1 + \lambda)\, e^{-(\theta 1+\lambda)y} + q\, e^{-\theta 2y} \quad (2.3)$$

P($\sum_{i=1}^{k} Xi < y$) = P(the system does not reach the uneconomic status after k Absenteeism)

$$= \int_0^\infty gk(x)[1-H(x)]dx$$
$$= p\, g_k^*(\theta_1+\lambda) + q\, g_k^*(\theta_2) \quad (2.4)$$

$$P(T>t) = \sum_{k=0}^{\infty} V_k(t)\, P\left(\sum_{i=1}^{k} Xi < y\right)$$

$$= p \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\, [g^*(\theta_1+\lambda)]^k + q \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)][g^*(\theta_2)]^k$$
$$= S(t) \quad (2.5)$$

$$L(t) = p[1-g^*(\theta_1+\lambda)] \sum_{k=1}^{\infty} [g^*(\theta_1+\lambda)]^{k-1} F_k(t)$$

$$+ q[1-g^*(\theta_2)] \sum_{k=1}^{\infty} g^*(\theta_2)]^{k-1} F_k(t)$$

Taking Laplace transform of L(t), we get

$$L^*(s) = \frac{p[1-g^*(\theta 1+\lambda)]f^*(s)}{[1-g^*(\theta 1+\lambda)f^*(s)]} + \frac{q[1-g^*(\theta 2)]f^*(s)}{[1-g^*(\theta 2)f^*(s)]} \quad (2.6)$$

$E(T) = -\frac{d}{ds} L^*(s)$ s=0

$E(T^2) = \frac{d^2}{ds^2} L^*(s)$ s=0 from which variance of T is obtained.

2.4. Special Cases

Case i)

Assume that the threshold level y ~ exp(θ) , X ~ exp(λ) and the random variable denoting the inter occurence time ~ exp(c).

In this case $L^*(s) = \frac{c\theta}{c\theta+s(\alpha+\theta)}$

$E(T) = \frac{\alpha+\theta}{c\theta}$ $V(T) = \frac{(\alpha+\theta)^2}{c^2\theta^2}$

Case ii)

Assume that y satisfies the SCBZ property with parameter θ_1 and θ_2 and assume as before X ~ exp(α) and U ~exp(c) .From (2.6) we have

$L^*(s) = \frac{p(\lambda+\theta_1)c}{c(\lambda+\theta_1)+s(\alpha+\lambda+\theta_1)} + \frac{q\theta_2 c}{c\theta_2+s(\alpha+\theta_2)}$ Yielding

$E(T) = \frac{p(\alpha+\lambda+\theta_1)}{c(\lambda+\theta_1)} + \frac{q(\alpha+\theta_2)}{c\theta_2}$

And $V(T) = p\left(\frac{\alpha+\lambda+\theta_1}{\lambda+\theta_1}\right)^2\left(\frac{2-p}{c^2}\right) + q\left(\frac{\alpha+\theta_2}{\theta_2}\right)^2(2-q)$

$-2pq\left(\frac{\alpha+\lambda+\theta_1}{\lambda+\theta_1}\right)\left(\frac{\alpha+\theta_2}{\theta_2}\right)$

In the next section we compute E(T) and V(T) for varying values of c, given some specific choices of other parameters in both SCBZ and non SCBZ (i.e. exponential) cases.

2.5 Numerical Illustrations

	E(T)		V(T)	
C	SCBZ	Non SCBZ	SCBZ	Non SCBZ
	λ=0.6 θ_1=0.4 α = 1.0 θ_2=0.2	θ = 0.4 α = 1.0	λ=0.6 θ_1=0.4 α = 1.0 θ_2=0.2	θ = 0.4 α = 1.0
1	5.0000	3.5000	6.0001	12.2500
2	2.5000	1.7500	1.5000	3.0630
3	1.6667	1.6667	0.6667	1.3611
4	1.2500	0.8750	0.3750	0.7658
5	1.0000	0.7000	0.2400	0.4900
6	0.8333	0.5833	0.1670	0.3403
7	0.7143	0.5000	0.2245	0.2500
8	0.6250	0.4375	0.0937	0.1914
9	0.5556	0.3889	0.0741	0.1512
10	0.5000	0.3500	0.0600	0.1225

From the table above it may be concluded that when the threshold distribution has SCBZ property, the expected values are becoming larger compared to the threshold with non SCBZ property with increasing value of c. The var(T) in the non SCBZ property is initially higher than the corresponding value of SCBZ case. However as c increases the variance decreases.

References

[1] **Bhatia, S.K. and Valecha**, G.K. "An empirical Study of Factors associated with Absenteeism", Indian Management, New Delhi, April, 1979.

[2] **Chithrakalarani, T. and Ganesan**, M.S. (2002) "Stochastic Model for the Cumulative effects of Industrial Accidents" Stochastic Modelling and Applications 5(1), 53 – 56.

[3] **Hameed, M.S.A. and F. Proschan** (1975), Shock models with underlying Birth Process, J. Appl. Prob. Vol.12, 18-28.

[4] **Raja Rao, B.** (1990) Life expectancy for a class of Life Distributions having the SCBZ property, Mathematical Bio Sciences 98, 251- 71.